15 Jan 2009

1.

a) $\dim M = 2$, $\dim N = 2$, $\dim M \cap N = 1$

b) $p = (x, y, z)$, $T_p(M \cap N) = span\{(x/y(y+v), v, 1)\}$, $v = -(2x^2 - 1)y/(2x^2 - 2y^2)$, $T_p(M \cap N)^\perp = span\{(-1/x, 1/y, 1), (2x, -2y, -1)\}$

2.

a) (0,1/2,0)

b) $\pi^{1/2}$

c) mais perto: $(2-2a, 3-3a, 4, 4a)$, mais longe: $(2+2a, 3+3a, 4+4a)$; $a = 29^{-1/2}$

3.

a) 78π

b) 0

4. não é aditiva

5.

a) 2

30 Jan 2009

1.

b) $\phi(\varphi) = (\cos\varphi, \cos\varphi, \sqrt{2}\sin\varphi)$, $\varphi \in]-\pi/2, \pi/2[$

2.

a) $\sqrt{3}(e^{2\pi} - 1)$

b) $e^{2\pi - 1}$

3. $x = y = z = 1$

4.

a) 0

b) 0

5. $\{\emptyset, \Omega, A_o, A_1, A_0^c, A_1^c, A_0 \cup A_1, (A_0 \cup A_1)^c\}$, $\mu(B) = \#B/\#\Omega$

6.

a) 0

b) $2 + f(0)$

6 Jan 2010

1.

a) $\dim M = 3$

b) $T_pM = \{(x, y, z, w)\colon x + w = 0\}$, $T_pM^\perp = \{(x, 0, 0, w)\colon x - w = 0\}$

2.

a) mais perto (2/3,0,2/3), mais longe (2,0,-2)

b) (0,1/2,0)

c) 1/2

d) $(5 - \cos 2)/8$

3.

a) $\{\emptyset, \Omega, A_0, A_1, A_0 \cup A_1, A_0^c, A_1^c, (A_0 \cup A_1)^c\}$

27 Jan 2010

1.

a) $\theta \neq 0$, $\dim M_\theta = 2$

b) $T_pM = span\{(1, 0, 0, -1), (0, \theta, 1, 0)\}$, $T_pM^\perp = span\{(0, -1, \theta, 0), (1, 0, 0, 1)\}$

2.

b) $1/(e^\pi - 1) + \pi^3/24 - 1$

3.

a) $]0,1[\times]-1,0[\times]0,2[$
c) $\pi/2$
4.
a) 0, $m(E)$, decrescente
b) sse $m(f^{-1}(a))=0$
5 Jan 2011
2.
a) $2\pi(1-1/e)$
b) $45/56$
c) 2π
3. $(1-e^{-1})/16$
4. $\pi\alpha^2/6$
5.
b) $1/2$
26 Jan 2011
1.
a) $(0,3/8,0)$
b) $\sqrt{\pi}$
c) $(-1,0,0)$
2. $\sqrt[4]{2}$
3. 0
4.
b) $e^{-\lambda}(1+2\lambda+3\lambda^2/2)$
c) $(1-e^{-\lambda})/\lambda$
6. integrável
9 Jan 2012
2.
a) $T_pM = span\{(1,1,\sqrt{2}),(1,-1,0)\}$, $T_pM^\perp = span\{(1,1,-\sqrt{2})\}$
b) $\sqrt{2}/3$
3.
a) $2R/3$
b) $x^2+y^2=1/2$, $z=1/2$
c) $\frac{3}{8}\frac{R^2-r^2}{R^{3/2}-r^{3/2}}$
4.
a) 3
b) $1+1/4+1/9$
25 Jan 2012
2.
a) $3R/4$
b) $(0,3/4,0)$
c) $\sqrt{\pi}$
3.
a) $(2x,2y,-1)/\sqrt{4z+1}$
b) 0
4.
a) 55
b) 10
11 Jan 2013

1.
b) $2\pi^2$
2.
b) $\frac{\pi}{2}\frac{e-1}{e^2}$
3.
a) $3R/4$
b) $(1,1,1,0)$
4.
a) dim=2
b) $(0,0,\pm 1)$ em $\bar{D}\cap\{(x,y,z)\in\mathbb{R}^3\colon z=0\}$ e $\bar{D}\cap\{(x,y,z)\in\mathbb{R}^3\colon z=1\}$, $\pm(x,y,0)$ em S, $(0,\pm 1,0)$ em $\bar{D}\cap\{(x,y,z)\in\mathbb{R}^3\colon y=0\}$
c) $\pi/2$

31 Jan 2013

1.
a) $\gamma(t)=(\sin(t),\sin(2t),0)$, $t\in[0,2\pi]$, $\Gamma=\gamma([0,2\pi])$ não é uma variedade
b) $p=\gamma(\pi/2)=(1,0,0)$, $T_p\Gamma=span\{(0,1,0)\}$, $T_p\Gamma^{\perp}=span\{(1,0,0),(0,0,1)\}$
2.
b) $(\pi/2)^3$
3.
a) $(3/2,3/2)$
b) $2\sqrt{R}/3$
4.
a) dim=2
b) $(0,0,\pm 1)$ em $\bar{D}\cap\{(x,y,z)\in\mathbb{R}^3\colon z=0\}$ e $\bar{D}\cap\{(x,y,z)\in\mathbb{R}^3\colon z=1/2\}$, $(0,\pm 1,0)$ em $\bar{D}\cap\{(x,y,z)\in\mathbb{R}^3\colon y=0\}$, $\pm(x,y,1-z)/[\sqrt{2}(1-z)]$ em S
c) $\pi/4$

13 Jan 2014

1.
a) dim=1
b) $T_{(1,1)}M^{\perp}=span\{(1,0)\}$, $T_{(1,1)}M=span\{(0,1)\}$
2.
a) $(1-\sqrt{30}/30)(1,2,3,4)$
b) $\sqrt{\pi}$
3.
a) A componente com $x>0$
b) 1
c) sim
d) 3/16

27 Jan 2014

1.
a) dim=2
b) $T_{(1,1,1)}H^{\perp}=span\{(1,1,-1)\}$, $T_{(1,1,1)}H=span\{(1,0,1),(0,1,1)\}$
c) $(\sqrt{10}/5,2\sqrt{10}/5,1)$
2.
a) 1/2
b) $(0,0)$
3.

a) Em $\{(x_1,x_2,x_3)\in\mathbb{R}^3\colon x_1=\pm1,\max_{i=2,3}|x_i|\le 1\}$, $\nu(x)=(\pm1,0,0)$; em $\{(x_1,x_2,x_3)\in\mathbb{R}^3\colon x_2=\pm1,\max_{i=1,3}|x_i|\le 1\}$, $\nu(x)=(0,\pm1,0)$; em $\{(x_1,x_2,x_3)\in\mathbb{R}^3\colon x_3=\pm1,\max_{i=1,2}|x_i|\le 1\}$, $\nu(x)=(0,0,\pm1)$
b) 0
4.
b) 0
5. $2^{\#\Omega}$
12 Jan 2015
1.
a) dim=2, $T_{(x,y,z)}M^{\perp}=span\{(x,y,z)\}$, $T_{(x,y,z)}M=span\{(-y,x,0),(-z,0,x)\}$
b) $\max_M f=\frac{1}{4}$, $\min_M f=\frac{1}{12}$
c) $3\pi/5$
2.
a) $\phi^{-1}(u,v)=(\sqrt{uv},\log\sqrt{\frac{u}{v}})$
c) $\pi/2$
3.
a) $\{\emptyset,\mathbb{R},\{\alpha\},\{\alpha\}^c\}$
b) $\{\emptyset,\mathbb{R}\}$
4.
a) $\mu(D)$
26 Jan 2015
1.
a) div$\nabla f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$, $\nu(x,y,z)=(x,y,z)$
b) 2π
c) $(1,0)$ min local, $(-1,0)$ min global, $(1/2,\pm\sqrt{3}/2)$ max global
2.
a) $\phi^{-1}(u,v)=(\sqrt{uv},\log\sqrt{\frac{u}{v}})$
c) $1/\pi$
3.
b) 0
4. $\mu=\sum_{n=1}^{10}\delta_{1/n}$
15 Jan 2016
1.
a) $dim M=2$, $T_{(x,y,z)}M=span\{(z,0,x),(0,z,y)\}$, $T_{(x,y,z)}M^{\perp}=span\{(x,y,-z)\}$
b) Não existem
c) 0
2.
a) $(5-\cos 2)/8$
b) $\sqrt{\pi}$
3.
a) $\sqrt{2}\pi e^{-\pi^2/4}$
b) 0
4.
b) 0
5. 2
2 Fev 2016
1.
a) S: $(x,y,0)$; V_1: $(0,0,1)$; V_2: $(0,0,-1)$

b) 0
c) S: não existem; C: $(\sqrt{2}/2, \sqrt{2}/2, 0)$ max, $(-\sqrt{2}/2, -\sqrt{2}/2, 0)$ min
d) $1/2$
e) S: $1/2$; C: 0
2.
a) $\sqrt{2}\pi$
b) $e - 1/e$
3. 0
4. constante
5. 0

9 Jan 2017

1.
a) $\dim M_2 = 1$, $T_{(x,y)}M_2 = span\{(-4y, x)\}$
b) Para $(0, 1)$

$$\phi(t) = \begin{cases} (2\cos(t/2 + \pi/4), \sin(t/2 + \pi/4), & 0 < t \leq \pi/2 \\ (\cos t, \sin t), & \pi/2 < t < 3\pi/4 \end{cases}$$

Mesma ideia para $(0, -1)$
c) $dim M = 1$,

$$T_{(x,y)}M = \begin{cases} T_{(x,y)}M_2, & (x, y) \in M_2 \\ span\{(-y, x)\}, & (x, y) \in M_1 \end{cases}$$

d) $(4\sqrt{5}/5, \sqrt{5}/5)$
e) $3\pi/2$
2.
a) $1/2 + \sin^2 1$
b) $\sqrt{\pi}$
3.
a) $3/2$
b) $\log 2$
c) 1

31 Jan 2017

1.
a) 1-variedade
b) $2(x, y)$
c) 0
d) π
2. 4π
3.
a) $(37/6\pi, 37/6\pi, 0)$
b) 0
4.
a) $\mathcal{X}_B$
b) $\{\emptyset, \Omega, A \cup B, (A \cup B)^c\}$
5. 6π

8 Jan 2018

1.
a) $\dim M_2 = 1$, $T_{(x,y)}M_2 = span\{(yb^2, -xa^2)\}$

b) Para $(0,a)$

$$\phi(t)=\begin{cases}(b\cos(at/b+\pi/2(1-a/b)),a\sin(t)), & 0<t\leq\pi/2\\ a(\cos t,\sin t), & \pi/2<t<\pi\end{cases}$$

Mesma ideia para $(0,-a)$

c) $\dim M=1$,

$$T_{(x,y)}M=\begin{cases}T_{(x,y)}M_2, & (x,y)\in M_2\\ span\{(-y,x)\}, & (x,y)\in M_1\end{cases}$$

d) 4
e) $3\pi/2$
2.
a) (1,2,3)
b) $(\sin^2(1)+1)/2$
c) $\sqrt{\pi}$
d) $\sum_{n\geq 1}1/n^2$

30 Jan 2018

1.
a) Não
b) Não
2.
a) (0,0,4/(3π)), $vol=3\pi^2$
b) 0
3.
a) Não
b) 2
4.
a) $\sqrt{\pi}$
b) $-\sum_{n=1}^{+\infty}n^{-2}=-\pi^2/6$
5.
a) $2x+10y=1$
b) $\varphi(1/4,1/20)=1/80$

4 Jan 2019

1.
a) $\dim M=2$, $T_pM^{\perp}=span\{(1,1,1)\}$
b) $\phi(x,y)=(x,y,-x-y)$, $(x,y)\in\mathbb{R}^2$, $T_pM=span\{(1,0,-1),(0,1,-1)\}$
c) 2π
d) não é variedade
e) minimizantes: $N\cap\{(x,y,z)\in\mathbb{R}^3\colon z=1\}$
2.
a) $2x+10y=1$
b) $\varphi(1/4,1/20)=1/80$
3.
a) $(0,0,1/2)$
b) $\pi/2^n$
c) 0

1 Fev 2019

1.

a) não é variedade
b) 2
2.
a) $(0, 0, 3/4)$
b) $(0, 0, 1/2)$
c) $\pi/2^n$
d) 0
3.
a) não
b) colecção dos subconjuntos de Ω que são numeráveis ou com complementar numerável.
4.
a) Verdadeira
b) Falsa

7 Jan 2020

1.
b) $\dim(M) = 1$
c) $+\infty$
d) 0
e) $\phi'(t) \cdot \nu(\phi(t)) = 0$, $\|\nu(\phi(t))\| = 1$
f) $(1 - e^{-4\pi})/2$
g) sim
2.
a) $\mu(D)$
3.
$(1, 1, 1)$

3 Fev 2020

1.
a) $\sqrt{3}(e^{2\pi} - 1)$
b) $e^{2\pi} - 1$
2.
a) $3R/4$
b) $(1, 1, 1, 0)$
3.
a) $S =]-1, 1[^n$.

$$massa = \int_S \rho = \left(\int_{-1}^{1} |x| \, dx \right)^n = 1$$

centro de massa $(z_1, \ldots, z_n)$:

$$z_i = \int_S x_i \rho(x_1, \ldots, x_n) \, dx_1 \ldots dx_n = \left(\int_{-1}^{1} |x| \, dx \right)^{n-1} \int_{-1}^{1} x_i |x_i| \, dx_i = 0$$

b) $\partial S = \{x \in \mathbb{R}^n \colon \max_i |x_i| = 1\}$. $(n-1)$-variedades $A_i^{\pm} = \{x \in \mathbb{R}^n \colon x_i = \pm 1, |x_j| < 1, j \neq i\} \subset \{x \in \mathbb{R}^n \colon F_i^{\pm}(x) = 0\}$ com $F_i^{\pm}(x) = x_i \mp 1$. Logo $\nabla F_i^{\pm}(x) = e_i$, $x \in A_i^+ \cup A_i^-$. ∂S não é variedade nos cantos:

$\partial S \setminus \bigcup_i (A_i^+ \cup A_i^-)$. Nos restantes pontos

$$\nu(x) = \begin{cases} e_i, & x \in A_i^+ \\ -e_i, & x \in A_i^- \end{cases}$$

c) $n2^{n-1}[f(1) - f(-1)]$
4. $f(x) = 0$
5.
a) $\mathcal{X}_{[0,1]\setminus\mathbb{Q}}$
b) 1