Risk and Return

Gestão Financeira II

Undergraduate Courses

2010-2011

Outline

- Common Measures of Risk and Return
- Historical Returns of Stocks and Bonds
- Historical Tradeoff between Risk and Return

Returns

- How do me measure the return from investment in one asset?
  - We compare initial value of the investment in the asset with the final value, at the end of the investment period.
- For Stocks we have:
  - Dollar return: $P_{t+1} + D_{t+1} - P_t$
  - Percentage return: $r_{t+1} = \frac{P_{t+1} + D_{t+1} - P_t}{P_t}$

where

P_t : price at beginning of period

P_{t+1} : price at end of period

D_{t+1} : dividend (cash flow) paid during period

Returns: Example

- Suppose you bought 100 shares of Wal-Mart one year ago at $25. You received $20 in dividends (20 cents per share 100 shares). At the end of the year, stock sells for $30. How well did you do?
  - Investment $25 100 = $2,500; At the end of the year, stock is worth $3,000 and dividends of $20
  - Dollar return: 520 = 20 + (3,000 – 2,500)
  - Percentage return:

$$r_t = \frac{3{,}000 + 20 - 2{,}500}{2{,}500} = 20.8\% = 20\% + 0.8\%$$

Holding Period Returns

- The holding period return is the return that an investor would get when holding an investment over a period of *n* years (assumes immediate reinvestment of dividends):

$$\text{Holding period return} = (1+r_1)\times(1+r_2)\times\cdots\times(1+r_n)-1$$

where r_i is the return during year *i*

Holding Period Returns: Example

- Suppose your investment provides the following returns over a four-year period:
  - Year 1: 10%
  - Year 2: -5%
  - Year 3: 20%
  - Year 4: 15%

$$\text{Holding period return}$$
$$= (1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_n) - 1$$
$$= 1.1 \times 0.95 \times 1.2 \times 1.15 - 1 = 0.4421 = 44.21\%$$

Average Return

- Arithmetic average: return earned in an average year over a particular period
- Geometric average: average compound return per year over a particular period
- Geometric average will be less than the arithmetic average unless all the returns are equal
- Which is better?
  - Geometric average is an excellent measure of past realized performance and good estimate of annual return to be obtained over extended periods of time in the future
  - Arithmetic average is best estimate of the expected return in a single period in the future

Average Return: Example

- What is the geometric average return?

$$(1+r)^4 = (1+r_1)\times(1+r_2)\times(1+r_3)\times(1+r_4)$$

$$r = \sqrt[4]{1.1\times 0.95\times 1.2\times 1.15} - 1 = 9.58\%$$

- So, our investor made an average of 9.58% per year, realizing a holding period return of 44.21%
- Arithmetic average return is higher:

$$r = \frac{10\% + (-5\%) + 20\% + 15\%}{4} = 10\%$$

Return Statistics

- History of capital market returns can be summarized by describing the:
  - average arithmetic return $\bar{r} = \frac{r_1 + r_2 + \cdots + r_T}{T}$
  - standard deviation of those returns

$$\sigma = \sqrt{Var} = \sqrt{\frac{(r_1 - \bar{r})^2 + (r_2 - \bar{r})^2 + \cdots + (r_T - \bar{r})^2}{T - 1}}$$

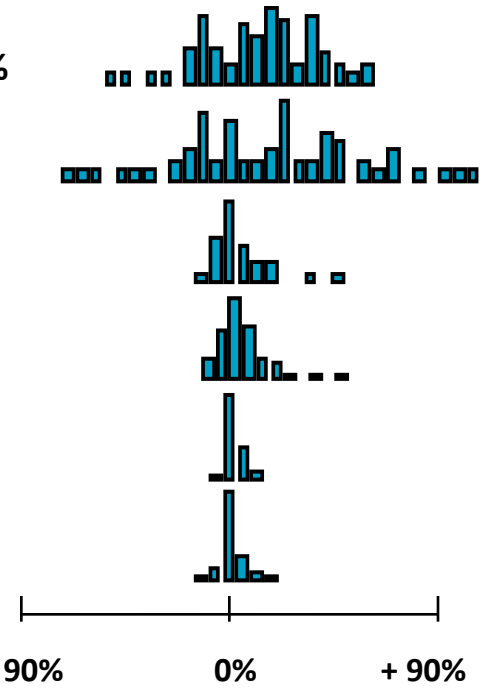
  - the frequency distribution of the returns

Return Statistics

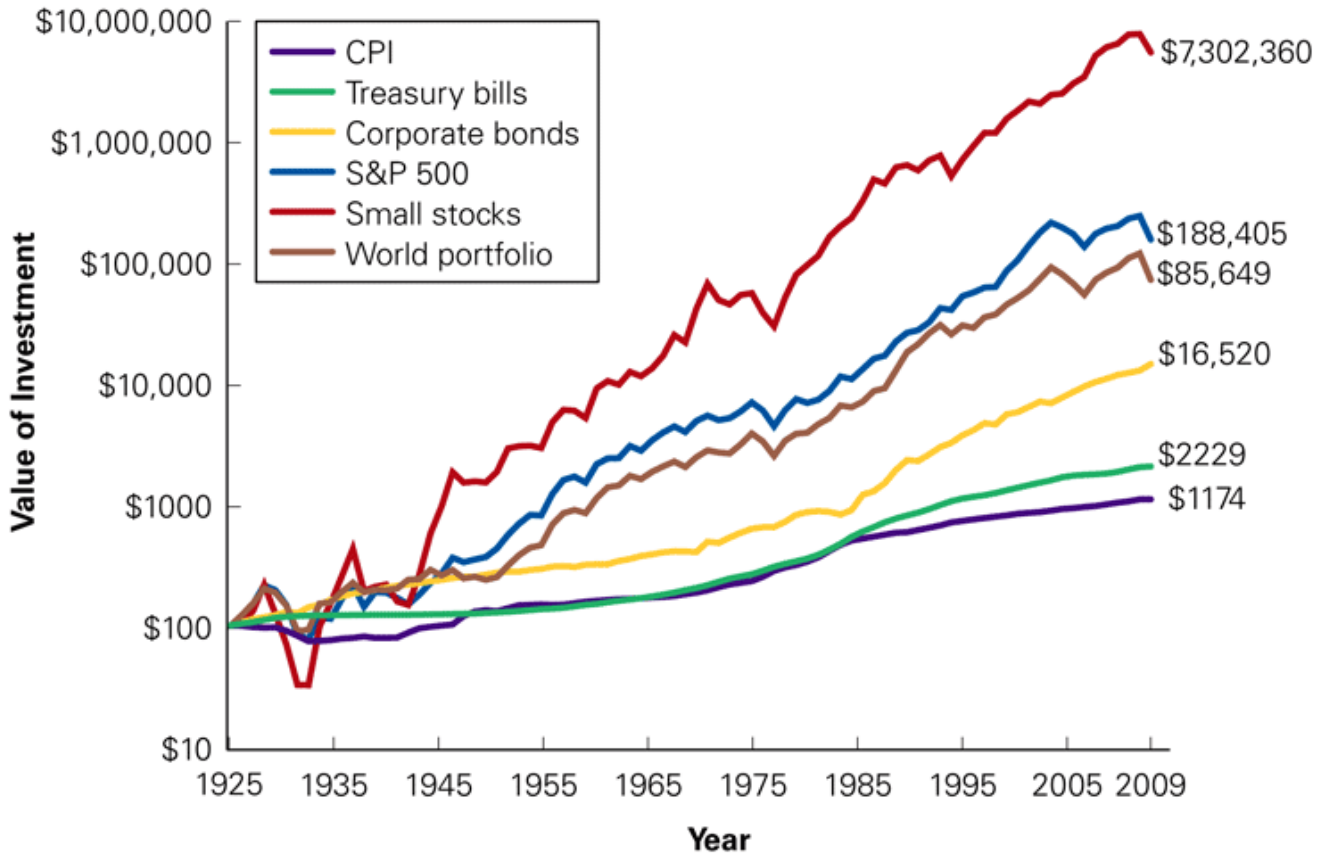
- Historical rates of returns on common stocks, bonds, and Treasury bills (Ibbotson and Sinquefeld).
- Year-by-year historical rates of return starting in 1926 in the United States:
  - Large-company Stocks
  - Small-company Stocks
  - Long-term Corporate Bonds
  - Long-term U.S. Government Bonds
  - U.S. Treasury Bills

Historical Returns U.S. 1926-2005

Series	Average Annual Return	Standard Deviation	Distribution
Large Company Stocks	12.3%	20.2%	
Small Company Stocks	17.4	32.9	
Long-Term Corporate Bonds	6.2	8.5	
Long-Term Government Bonds	5.8	9.2	
U.S. Treasury Bills	3.8	3.1	
Inflation	3.1	4.3	



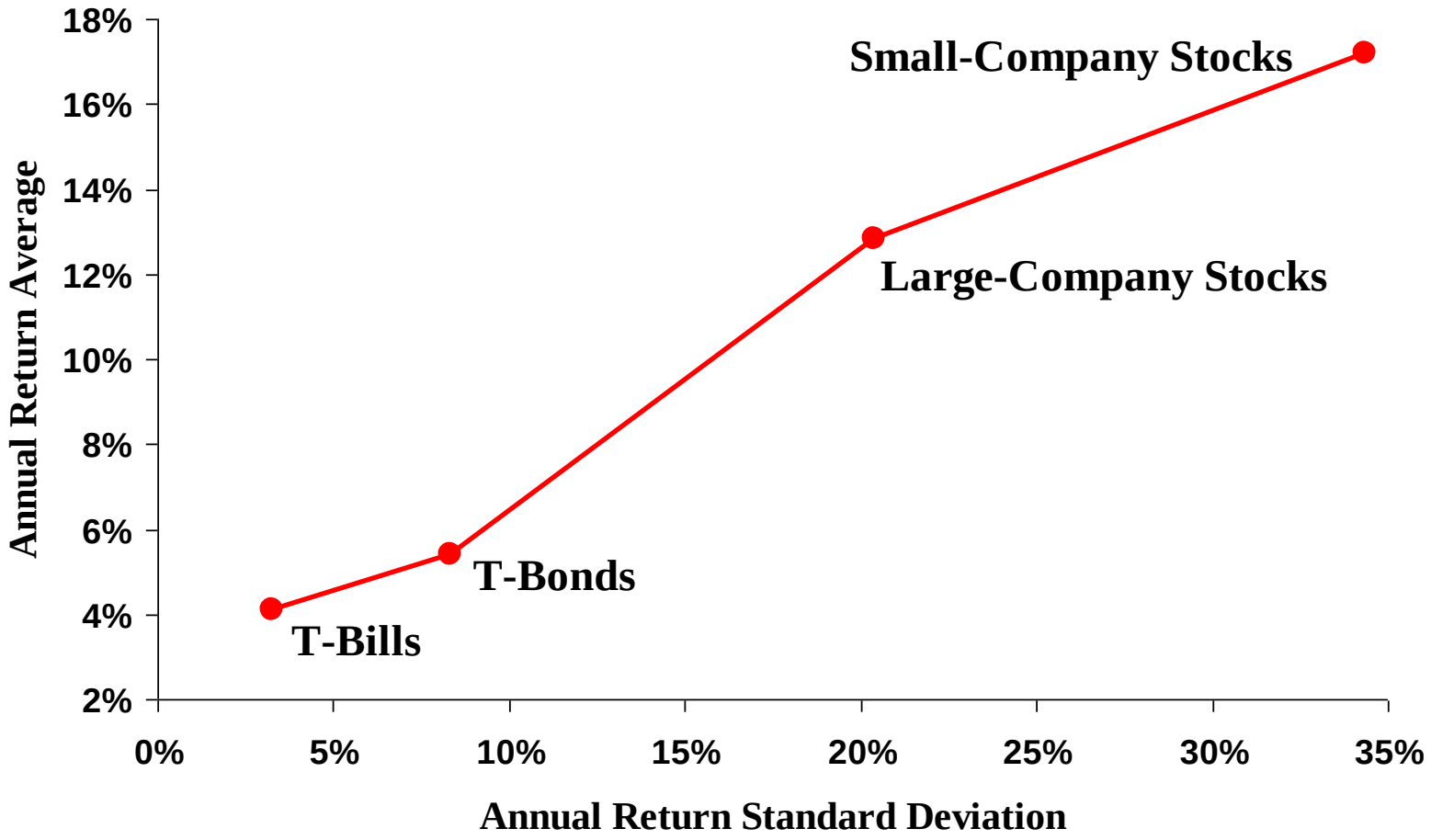
Cumulative Returns U.S. 1925-2009



Risk Premium

- *Risk Premium* is the added return (over and above the risk-free rate) resulting from bearing risk
- Stock markets offer long-run excess of stock return over the risk-free return:
  - Average excess return from large company common stocks: 8.5% = 12.3% – 3.8%
  - Average excess return from small company common stocks: 13.6% = 17.4% – 3.8%
  - Average excess return from long-term corporate bonds: 2.4% = 6.2% – 3.8%
- Given the superior performance of stocks over such long period, why does anyone hold bonds?

Risk-Return Trade-off



Risk

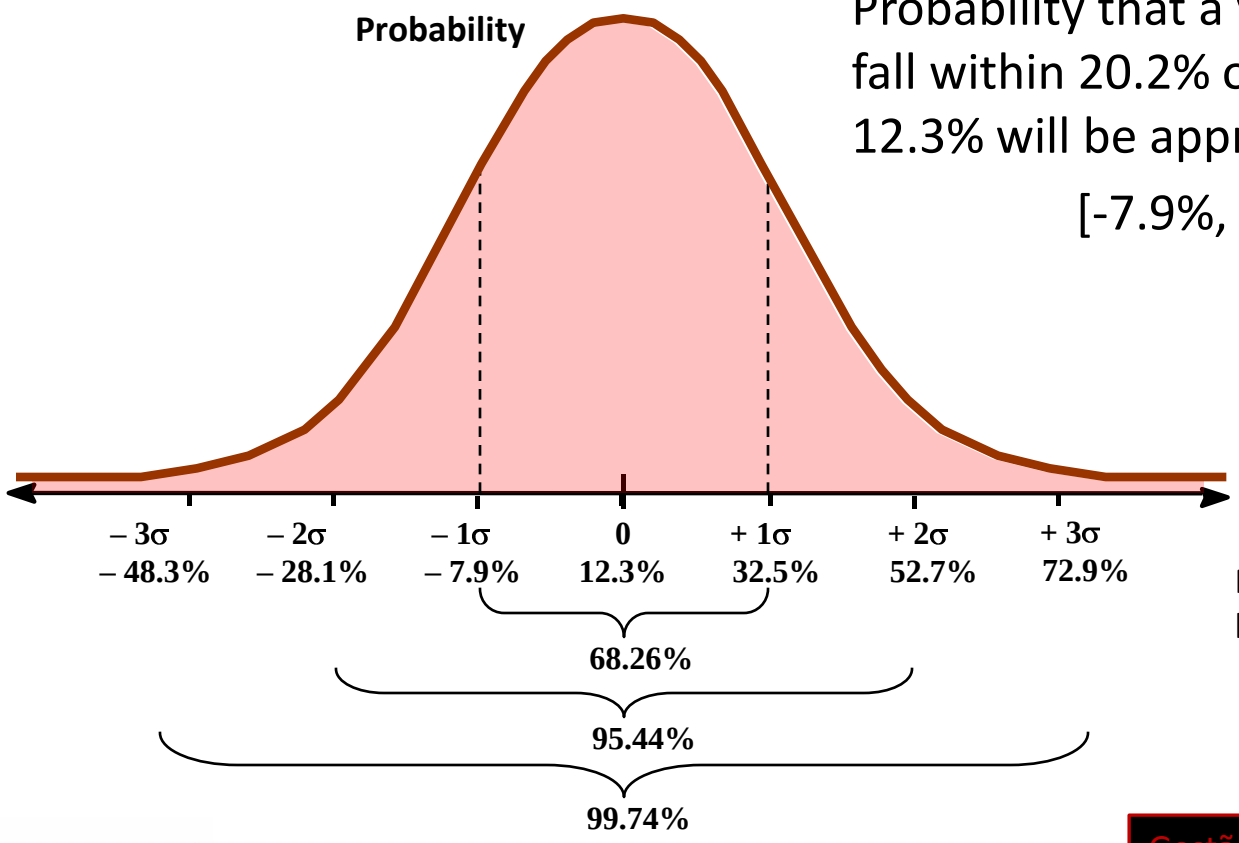
- Risk is uncertainty about the future
  - Stocks do better on average, but investors know that in any year, stocks may do much worse
- Summarize risk through standard deviation, σ, a measure of dispersion

1926-2005	Standard Deviation
Stocks	20.2%
T-bills	3.1%
Long-term Government Bonds	9.2%
Corporate Bonds	8.5%
Inflation	4.3%

Source: Ibbotson, CRSP

Normal Distribution

- Suppose stock returns are normally distributed



Probability that a yearly return will fall within 20.2% of the mean of 12.3% will be approximately 68%

[-7.9%, 32.4%]

What about the Future Returns and Risk?

- No one knows the future, but we form expectations about it.
- We will want a measure of Expected Return, based on probability distributions:

$$\text{Expected Return} = E\ R\ = \sum_R P_R \times R$$

- We will want a measure of Expected Risk, such as the Variance or the Standard Deviation of Expected Returns:

$$Var(R) = E\left[\ R - E\ R\ ^2\right] = \sum_R P_R \times \ R - E\ R\ ^2$$

$$SD(R) = \sqrt{Var(R)}$$

- In the end, we typically use past (historical) returns to predict the future. We'll see more when presenting portfolios.