Capital Asset Pricing Model (CAPM)

Gestão Financeira II

Undergraduate Courses

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Motivation

- The Capital Asset Pricing Model (CAPM) is an equilibrium model that establishes a relationship between the price of a security and its risk.
- In particular, the CAPM is used to determine the cost of capital: minimum return required by investors for a certain level of risk.
- The CAPM assumes investors are well diversified. Hence the risk premium is proportional to a measure of market (or systematic) risk, known as Beta.

CAPM: Assumptions

- Frictionless markets
  - No trading costs
  - No taxes
- Unlimited borrowing and lending
  - No restrictions on short sales
- Lending and borrowing rates are the same
- Investors care only about means and variances
- All investors are fully rational and have the same information (homogeneous expectations)

Market Equilibrium

- Since everyone has the same efficient frontier, then:
  – Everyone holds the same risky tangency portfolio.

- Then

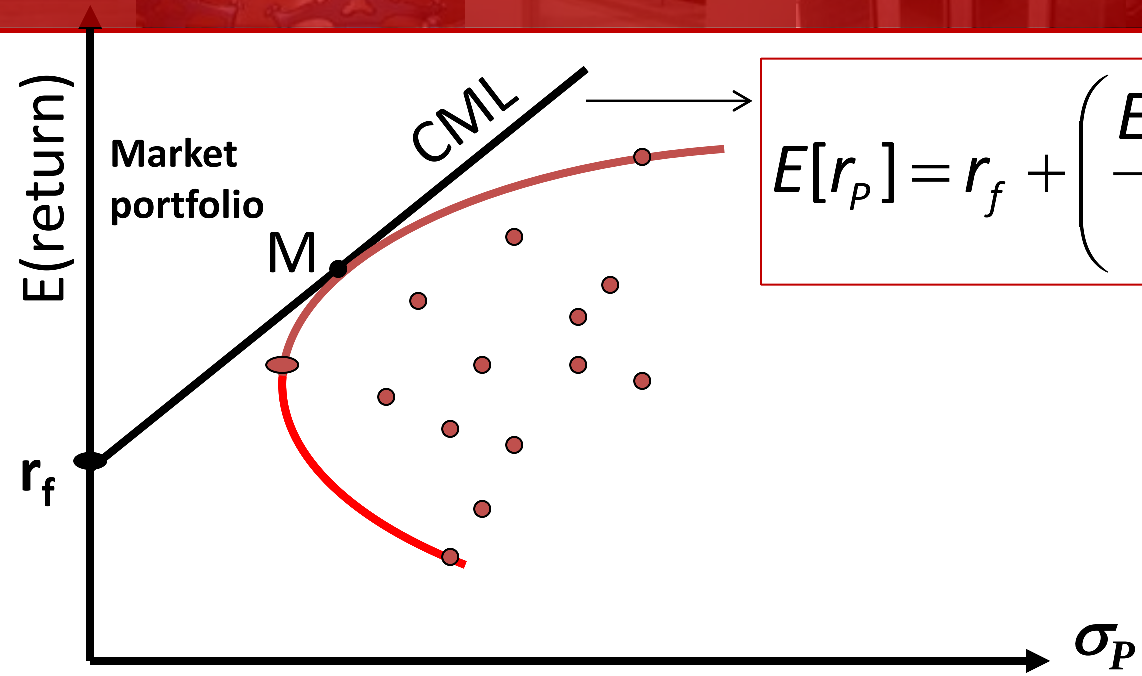
THE TANGENCY PORTFOLIO IS

THE MARKET PORTFOLIO

Market Equilibrium

- Every investor solves the mean-variance problem and holds some combination of risk-free asset and portfolio of risky assets (market).
- The sum of all investors' risky portfolios will have the same weights as tangency.
- In equilibrium the sum of all investors' desired portfolios must equal the supply of assets.
- Aggregate supply and demand of assets is the market portfolio.
- Market portfolio is the tangency.

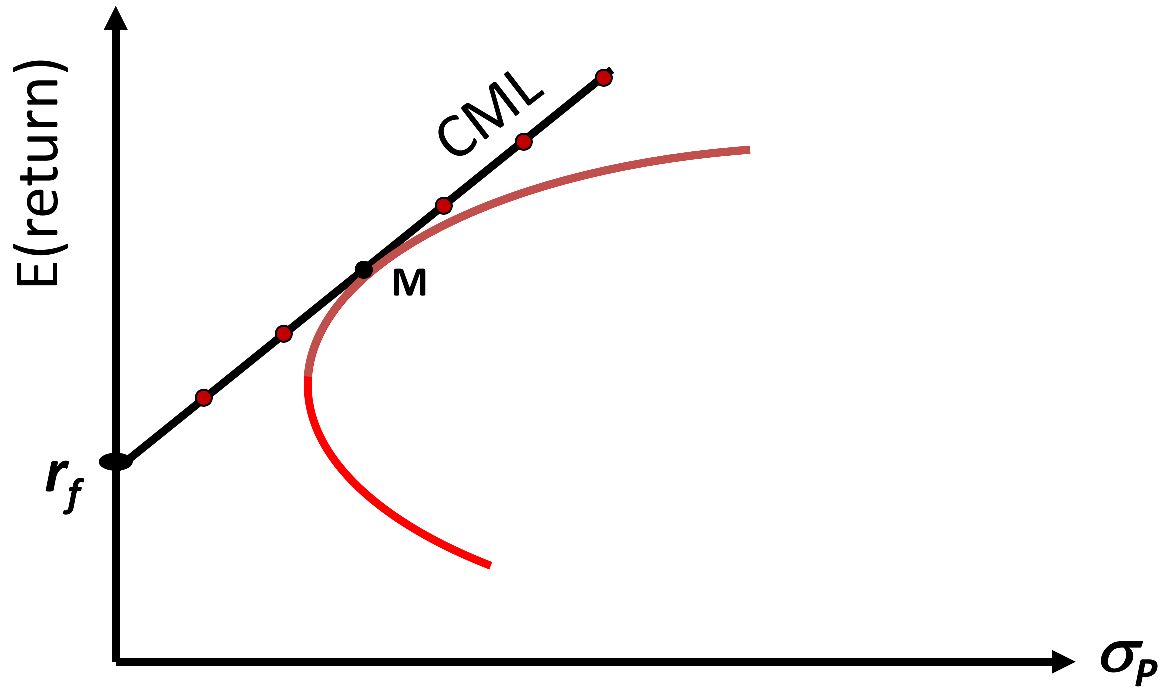
Capital Market Line



$$E[r_P] = r_f + \left(\frac{E[r_M] - r_f}{\sigma_M} \right) \sigma_P$$

- Investors choose a point along the line – Capital Market Line (CML)
- Efficient portfolios are combination of the risk-free asset and the market portfolio *M.*

Capital Market Line



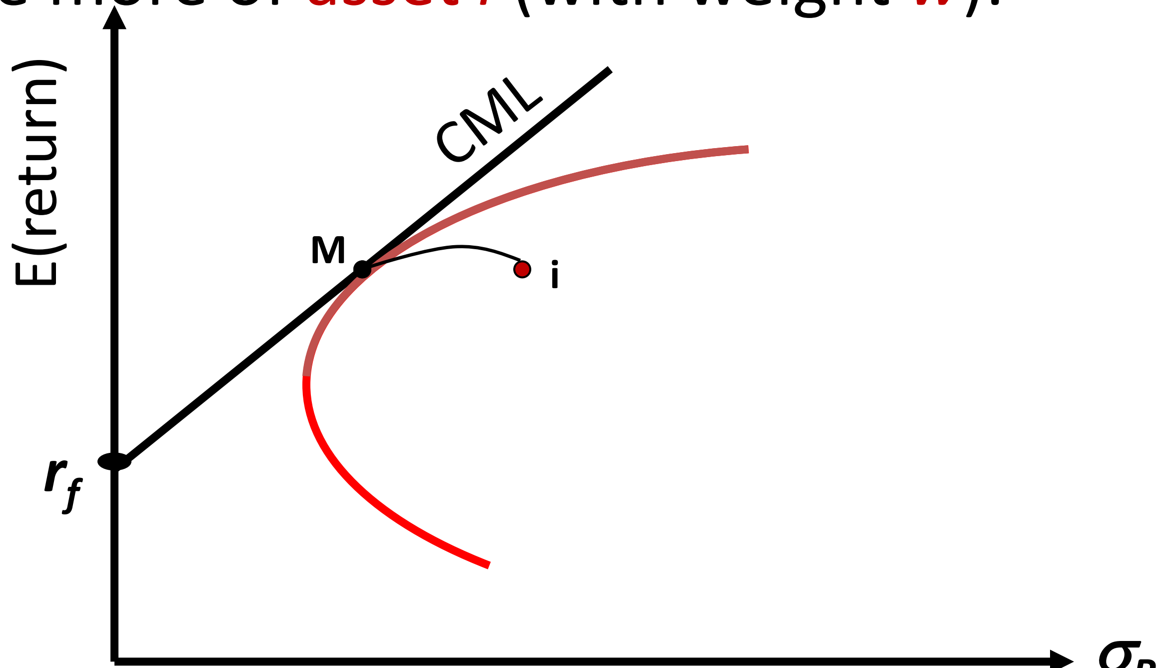
Where the investor chooses to be along the CML depends on his risk aversion – but all investors face the same CML

Security Market Line

- The CML gives risk-return trade-off for efficient portfolios.
- In equilibrium, what is the relation between expected return and risk for individual stocks?
  - Individual stocks are below CML.
  - This relation is named Security Market Line (SML).
  - Individual stock risk is measured by its covariance with market portfolio because it is the *marginal variance.*
    - How does a small increment to the weight of a stock change the variance of the portfolio?
    - As in Economics, it is the marginal cost of goods that determines their prices, not their total or average cost.

Security Market Line

- Suppose you hold portfolio *M* and are considering adding a little more of asset *i* (with weight *w*):



$$E(r_P) = wE(r_i) + (1-w)E(r_M)$$

$$\sigma_P^2 = (w\sigma_i)^2 + [(1-w)\sigma_M]^2 + 2w(1-w)\sigma_{iM}$$

Security Market Line

- Now, evaluate the change in return and risk of your portfolio when you increase *w*:

$$\frac{\partial \mathrm{E}(r_P)}{\partial w} = \mathrm{E}(r_i) - \mathrm{E}(r_m)$$

$$\frac{\partial \sigma_P^2}{\partial w} = 2w\sigma_i^2 - 2(1-w)\sigma_M^2 + 2(1-2w)\sigma_{iM}$$

- Remember that, in the equilibrium portfolio (*M*), the excess demand for stock *i* is zero (*w=0*). Evaluate the changes in return and risk for equilibrium level *w = 0*:

$$\left.\frac{\partial \mathrm{E}(r_P)}{\partial w}\right|_{w=0} = \mathrm{E}(r_i) - \mathrm{E}(r_M)$$

$$\left.\frac{\partial \sigma_P}{\partial w}\right|_{w=0} = \left.\frac{\partial \sigma_P^2}{\partial w}\frac{\partial \sigma_P}{\partial \sigma_P^2}\right|_{w=0} = \frac{-2\sigma_M^2 + 2\sigma_{iM}}{2\sigma_M}$$

Security Market Line

- Risk-return trade off in equilibrium (M) is then:

$$\frac{\partial E(r_P)/\partial w}{\partial \sigma_P/\partial w} = \frac{E(r_i)-E(r_M)}{\frac{-2\sigma_M^2+2\sigma_{iM}}{2\sigma_M}} = \frac{E(r_i)-E(r_M)}{\frac{\sigma_{iM}-\sigma_M^2}{\sigma_M}}$$

- Risk-return trade off in equilibrium (M) given by CML is the same:

$$\frac{E(r_M)-r_f}{\sigma_M} = \frac{E(r_i)-E(r_M)}{\frac{\sigma_{iM}-\sigma_M^2}{\sigma_M}} \Rightarrow E(r_i) = r_f + \frac{\sigma_{iM}}{\sigma_M^2}\left[E(r_M)-r_f\right]$$

Beta

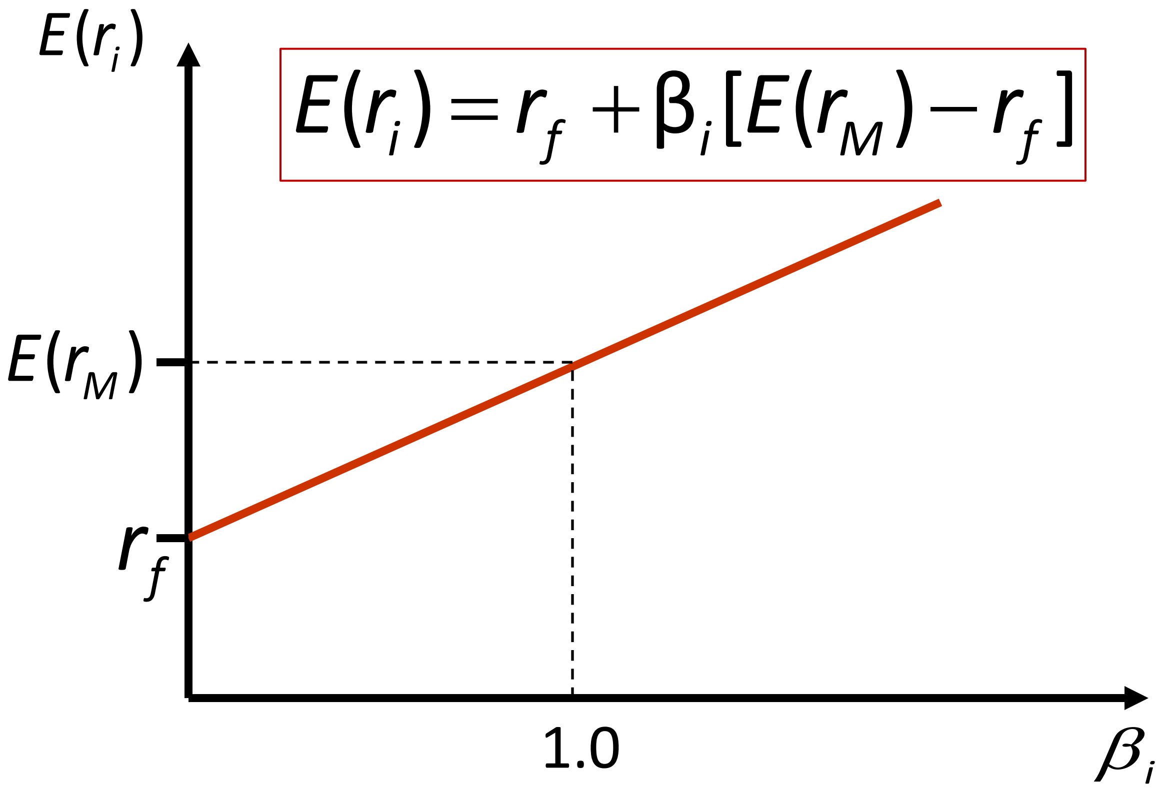
- According to the Security Market Line, for any security *i*: $$E(r_i) = r_f + \beta_i \, [E(r_M) - r_f]$$

where $$\beta_i = \frac{Cov(r_{i,} r_M)}{Var(r_M)}$$

- Beta measures the responsiveness of a stock to movements in the market portfolio (i.e., systematic risk).

CAPM: Expected Return of a Stock

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$



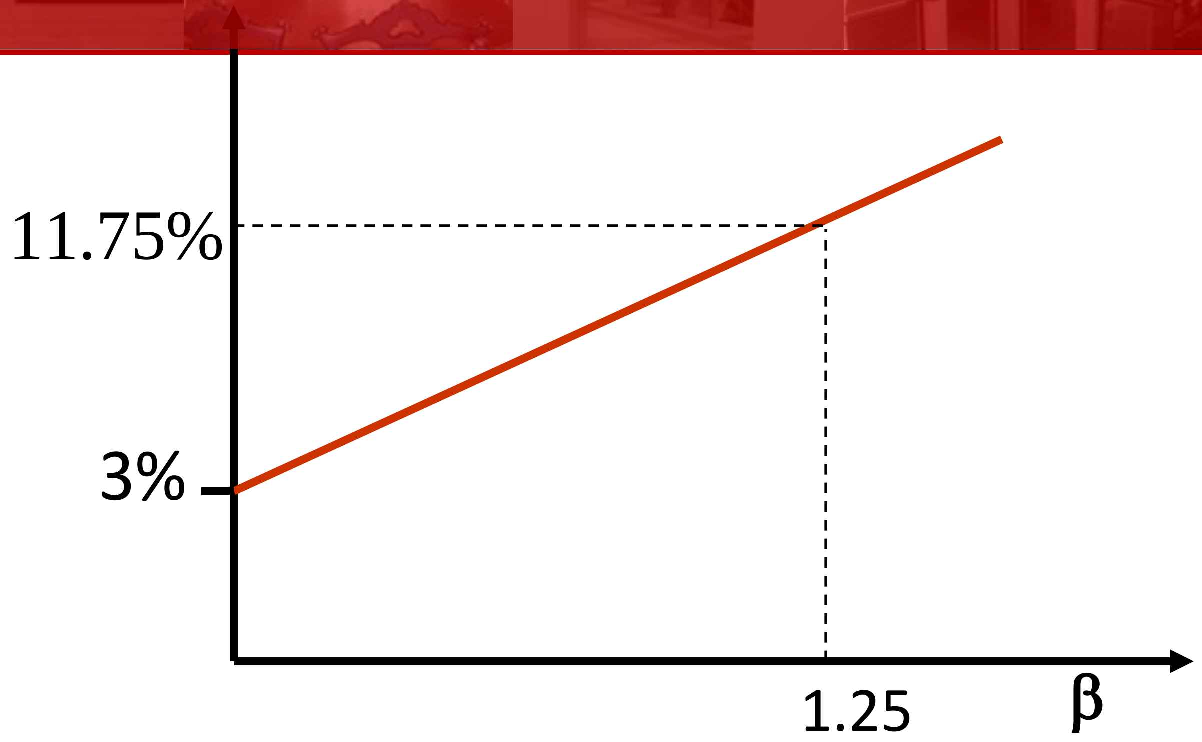
CAPM: Expected Return of a Stock

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

Expected return on a stock	=	Risk-free rate	+	Beta of stock	Market risk premium

- Assume $\beta_i = 0$, then $E(r_i) = r_f$
- Assume $\beta_i = 1$, then $E(r_i) = E(r_M)$
- Assume $\beta_i < 1$, then $E(r_i) < E(r_M)$
- Assume $\beta_i > 1$, then $E(r_i) > E(r_M)$

CAPM: Example



$$\beta_i = 1.25 \qquad r_f = 3\% \qquad E(r_M) = 10\%$$

$$E(r_i) = 3\% + 1.25 \times (10\% - 3\%) = 11.75\%$$

Beta of a Portfolio

- Beta of a portfolio is portfolio-weighted average of individual assets:

$$\beta_P = \sum_{i=1}^{N} w_i \beta_i$$

- Thus, we can use SML for any portfolio:

$$E(r_P) = r_f + \beta_P [E(r_M) - r_f]$$

CML versus SML

- CML plots the relation between expected returns and standard deviation
- SML is the relation between expected returns and β
- All portfolios, whether efficient or not, must lie on the SML but only efficient portfolios are on the CML
  - with the same mean return can have different standard deviations, but must have the same β
  - in other words, the only relevant measure of risk for pricing securities is β (a measure of covariance with the market)

Why Beta?

- Because investors can diversify their portfolios, they only require a risk premium for non-diversifiable (market, systematic) risk. This is what Beta measures.
- High beta stocks are risky, and must therefore offer a higher return on average to compensate for the risk
- Why are high beta stocks risky?
  - Because they pay up just when you need the money least, when the overall market is doing well
  - And they loose money when you really need it – when the overall market is doing poorly
  - If anyone is to hold this security, it must offer a high expected return

Estimation of Beta

- β_i usually estimated using a time-series regression

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + \varepsilon_{i,t}$$

- Typical R^2=25%
- Estimation issues
  - Betas may change over time
  - Data might be too old
  - Five years of weekly or monthly data is reasonable
  - Use Data Analysis / Regression or Linest in Excel

Variables to use for the Market Return and the Risk-free rate

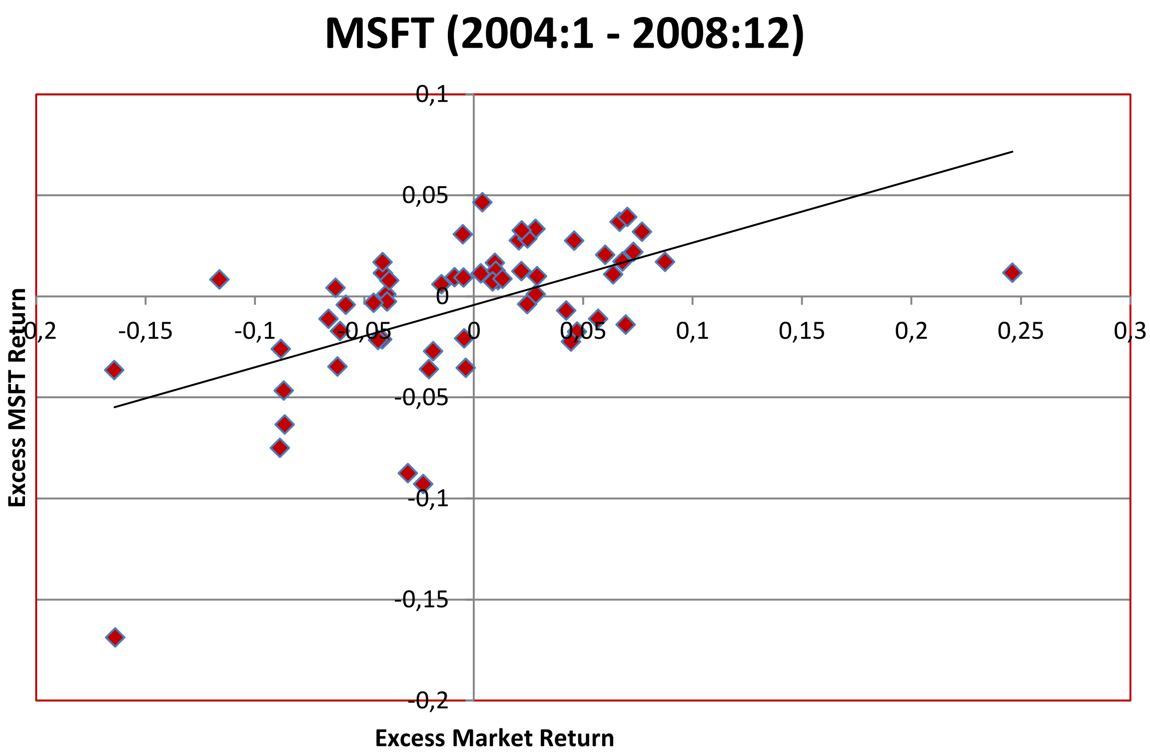
- What market proxy?
  - CAPM says it should be all the assets in the world
  - Typically people use *broad, value-weighted* stock market index (e.g. S&P 500)
- What risk-free rate?
  - CAPM says it should be riskless and match the horizon of the application
  - People use short-term sovereign debt: T-bills

Market Risk Premium

- This is the hardest input to measure in the CAPM equation
- From January 1926 to December 2005, the excess market return has been 6.7%
  - Depending on the sample and on whether we use the arithmetic or geometric mean, we can come up with numbers between 5% and 8%
- Can we trust this historical average?
  - Standard error of the estimate is 2.2%

Example: Estimating MSFT's Beta

$$r_{MSFT,t} - r_{f,t} = 0.002 + 0.993(r_{M,t} - r_{f,t}) + \varepsilon_{MSFT,t}$$



Example: Estimating MSFT's Expected Return

- Assuming a risk-free rate of 3% and an equity premium of 6%, the expected return (annual) on MSFT would be:

$$3\%+0.993 \times 6\%= 8.96\%$$

Jensen's alpha

- Excess return over that predicted by CAPM

$$\alpha_i = (\bar{r}_i - r_f) - \beta_i\,(\bar{r}_m - r_f)$$

- If alpha is positive
  - Security has earned a higher return on average than is required for its level of systematic risk
  - Could say that it was mispriced, but be careful drawing conclusions for the future…

  OR
  - The security might not be mispriced, but rather the CAPM is wrong! Or at least its practical implementation.
- Measure of portfolio performance

Testing the CAPM

- Take a large number of stocks or portfolios
- Over some long time period, e.g. 1950-2000, estimate alpha and beta for each of them by running a regression
- Then look at the alphas, are they statistically different from zero?

Testing the CAPM

- Fama and French (1993) find even weaker results
  - There does not seem to be any relation between β and average returns, once you control for other factors:
    - Size (market capitalization)
    - Ratio of book value of equity to market value (book-to-market)
  - Three-factor Fama-French model
    - Market return (r_M)
    - Small minus big (SMB) – small cap premium
    - High minus low (HML) – value premium

So, what do we know about the CAPM?

- Assumptions of CAPM are restrictive
- Gives a simple and elegant relation for expected returns and a nice measure of risk
- Research shows that it is not very accurate
- But, widely used in corporate finance and investments