CONSUMPTION

(Lectures 4, 5, and 6)

Remark: (*) signals those exercises that I consider to be the most important

Exercise 20 (MWG, Ex. 1.B.1, 1.B.2)

Show that if $\succsim$ is rational, then:

1. if $\mathbf{x} \succ \mathbf{y} \succsim \mathbf{z}$, then $\mathbf{x} \succ \mathbf{z}$;
2. $\succ$ is both irreflexive ($\mathbf{x} \succ \mathbf{x}$ never holds) and transitive (if $\mathbf{x} \succ \mathbf{y}$ and $\mathbf{y} \succ \mathbf{z}$, then $\mathbf{x} \succ \mathbf{z}$);
3. $\sim$ is reflexive ($\mathbf{x} \sim \mathbf{x}$ for all $\mathbf{x}$), transitive (if $\mathbf{x} \sim \mathbf{y}$ and $\mathbf{y} \sim \mathbf{z}$, then $\mathbf{x} \sim \mathbf{z}$), and symmetric (if $\mathbf{x} \sim \mathbf{y}$, then $\mathbf{y} \sim \mathbf{x}$).

Exercise 21

Prove that strong monotonicity implies local nonsatiation, but not vice versa.

Exercise 22

Assume that there are only two goods in one economy. Draw indifference curves that (a) satisfy and (b) violate each of the following properties:

1. transitivity;
2. strict convexity;
3. convexity;
4. monotonicity.

Exercise 23

Show that if there exists a utility function that represents $\succsim$, then $\succsim$ must be rational.

Exercise 24 (MWG, Ex. 3.C.1)

Assume that there exist only two goods, good 1 and good 2. Define **x** $\succsim$**y** if either '$x_1 >$ y_1' or '$x_1 = y_1$ and $x_2 \geq y_2$'. This is known as the *lexicographic preference relation*.

Verify that the lexicographic ordering is complete, transitive, strongly monotone, and strictly convex.

Exercise 25

Show that lexicographic preferences are not continuous.

Exercise 26

Show that the expenditure function e(**p**,u) satisfies the following properties:

1. non increasing in p_i, i=1,…n;
2. homogeneous of degree 1 in **p**;
3. concave in **p**;
4. continuous in **p**, **p**$\gg$0.

Exercise 27 (*)

Let $u(x_1,x_2)= kx_1^a x_2^{1-a}$, for 0<a<1.

1. Solve the utility maximization problem and find the demand functions;
2. Verify that x_i(p,m), i=1,2, satisfies homogeneity of degree 0 in (**p**,m) and Walras' law.

Exercise 28 (*)

Consider the CES utility function $u(x)= (x_1^a + x_2^a)^{1/a}$, for a≤1.

1. Compute the demand functions;
2. Derive the indirect utility function;
3. Derive the demand correspondences and indirect utility functions for the linear utility and the Leontief utility cases. Show that the functions computed in 1. and 2. approach these as a→1 and as a→-∞, respectively;

4. Compute the Hicksian demand functions and verify their properties.

Exercise 29 (*)

Let $(-\infty,\infty)\times R_+^{L-1}$ denote the consumption set and let the utility function be u(**x**)= $x_1+w(x_2, x_3,..x_L)$.

1. Show that the demand functions for goods 2,…,L are independent of income;
2. Argue that the indirect utility function can be writtten in the form v(**p**,m)= w+ϕ(**p**) for some function ϕ(·);

Now suppose $x_1 \geq 0$.

3. Are the demand functions still independent of income?
4. Let L=2 and, for a given fixed level of prices **p**, examine how demand changes as income increases.

Exercise 30 (from MWG, Ex. 3.D.6)

Consider the three-good setting in which the consumer has utility function
$u(x)= (x_1- b_1)^a(x_2- b_2)^c (x_3- b_3)^d$.

1. Why can you assume that a+c+d=1 without loss of generality? Do so for the rest of the problem.
2. Write down the first-order conditions for the UMP, and derive the consumer's Walrasian demand and indirect utility functions. This system of demand is known as the *linear expenditure system* and it is due to Stone (1954);
3. Verify that these demand functions satisfy the following properties: homogeneity of degree 0 in (**p**,m), Walras' Law, convexity/uniqueness.

Exercise 31

V, Ex. 7.2, p. 114

Exercise 32

V, Ex. 7.4, p. 114

Exercise 33

V, Ex. 7.6, parts a) and b), p. 115

Exercise 34 (from MWG, Ex. 3.G.3)

Consider the linear expenditure system utility function given in Exercise 30.

1. Derive the Hicksian demand and expenditure functions. Check their properties;
2. Verify that the Slutsky equation holds;
3. Verify that the own-substitution effects are negative and that the compensated cross-price effects are symmetric.

Exercise 35

The consumer buys bundles $\mathbf{x_i}$ at prices $\mathbf{p_i}$, i=0,1. Justify whether each of the following choices staisfies the weak axiom of revealed preference:

1. $\mathbf{p_0}$=(1,3), $\mathbf{x_0}$=(4,2), $\mathbf{p_1}$=(3,5), $\mathbf{x_1}$=(3,1);
2. $\mathbf{p_0}$=(1,6), $\mathbf{x_0}$=(10,5), $\mathbf{p_1}$=(3,5), $\mathbf{x_1}$=(8,4);
3. $\mathbf{p_0}$=(1,2), $\mathbf{x_0}$=(3,1), $\mathbf{p_1}$=(2,2), $\mathbf{x_1}$=(1,2);
4. $\mathbf{p_0}$=(2,6), $\mathbf{x_0}$=(20,10), $\mathbf{p_1}$=(3,5), $\mathbf{x_1}$=(18,4).

Exercise 36 (MWG, Ex. 2.F.16) (*)

Consider a setting where L=3 and a consumer whose consumption set is R^3. Suppose that his demand function x(**p**,w) is

$$x_1(p,w) = \frac{p_2}{p_3},\ x_2(p,w) = -\frac{p_1}{p_3},\ x_3(p,w) = \frac{w}{p_3}.$$

1. Show that x(**p**,w) is homogeneous of degree 0 in (**p**,w) and satisfies Walras' law;
2. Show that x(**p**,w) violates the weak axiom;
3. Show that **v**. S(**p**,w).**v** = 0 for all vÎR^3.

Exercise 37 (MWG, Ex. 2.F.17)

In an L-commodity world, a consumer's Walrasian demand function is

$$x_k(p,w) = \frac{w}{\sum_{l=1}^{L} p_l} \text{ for k=1,…,L.}$$

1. Is this demand function homogeneous of degree 0 in (**p**,w)?
2. Does it satisfy Walras' law?
3. Does it satisfy the weak axiom?

Exercise 38

V, Ex. 8.2, p. 140

Exercise 39

V, Ex. 8.5, p. 141

Exercise 40

V, Ex. 8.6, p. 141

Exercise 41

V, Ex. 8.7, p. 141

Exercise 42

V, Ex. 8.10, p. 141

Exercise 43

V, Ex. 8.12, p. 142

Exercise 44

V, Ex. 8.16, p. 142

Exercise 45

V, Ex. 9.10, p. 159

Exercise 46

V, Ex. 9.11, p. 159

Exercise 47

V, Ex. 10.2, p. 171