Mathematical Description of an Extensive-Form Game with Perfect Information

Γ = (I, K, P, C, u), where:

1. I is the set of players (finite by assumption); I={1,2,…n};
2. K is the game-tree, i.e., the structure of the decision process: a set of ordered nodes without a curl, where
    a. x_1 represents the initial node;
    b. X is the set of non-terminal nodes;
    c. Z is the set of terminal nodes;
    d. IP (Immediate Predecessor) is a function on X∪Z with IP: $X \cup Z \rightarrow X \cup \emptyset$ and IP(x)= $\emptyset$ iff x= x_1;
    e. IF (Immediate Followers) is a correspondence with IF: $X \rightarrow X \cup Z$ and IP(x)={x'$\in$ X∪Z: IP(x')=x};
3. P is a partition of X that assigns each node to a player, with P: $X \rightarrow I$ and $X^i=\{x\in X: P(x)=i\}$, $X= X^1 \cup \ldots \cup X^n$;
4. C is a family of sets $C=\{C_x\}_{x\in X}$, where C_x is the set of actions available to player P(x) at x;
5. u_i is agent i's utility function, i.e., u_i: $Z \rightarrow \mathbb{R}$

Remarks:

1. For consistency, there has to be a one-to-one identification between IF(x) and C_x;
2. A play of the game is a sequence of nodes starting at the initial node and finishing at a particular terminal node: $x_1, x_2, \ldots, x_{k\ (=z_t)}$, with $x_1 = IP^{k-1}(z_t)=IP(IP(\ldots.IP(z_t)))$

Mathematical Description of an Extensive-Form Game with Imperfect Information

Γ = ((I,N), K, P, B, C, p, u), where:

1. P={ $X^1,\ldots, X^n, X^N$} and $X^N=\{x_1\}$;
2. B=($B_1,\ldots B_n$), where B_i is an information sets for player I, a partition of X^i (the set of nodes belonging to i); b_i is an element of B_i
3. p is the probability distribution over C_{x1} (represents how Nature decides between its actions);

Remarks:

1. If x, x'$\in$ b_i, player I cannot distinguish between the two nodes;
2. If x, x'$\in$ b_i, then $C_x= C_{x'}$. It follows that, what we actually have is $\{C_b\}_{b\in B}$;
3. u_i, agent i's utility function, has to be Von Neuman-Morgenstern, i.e., u_i: $\mathcal{L}(Z) \rightarrow \mathbb{R}$since now agents compare probability distributions over the terminal nodes when deciding which strategy to use.