

## CONSUMPTION

(Lectures 4, 5, and 6)

**Remark:** (\*) signals those exercises that I consider to be the most important

### Exercise 20 (MWG, Ex. 1.B.1, 1.B.2)

Show that if  $\succsim$  is rational, then:

1. if  $\mathbf{x} \succ \mathbf{y} \succsim \mathbf{z}$ , then  $\mathbf{x} \succ \mathbf{z}$ ;
2.  $\succ$  is both irreflexive ( $\mathbf{x} \succ \mathbf{x}$  never holds) and transitive (if  $\mathbf{x} \succ \mathbf{y}$  and  $\mathbf{y} \succ \mathbf{z}$ , then  $\mathbf{x} \succ \mathbf{z}$ );
3.  $\sim$  is reflexive ( $\mathbf{x} \sim \mathbf{x}$  for all  $\mathbf{x}$ ), transitive (if  $\mathbf{x} \sim \mathbf{y}$  and  $\mathbf{y} \sim \mathbf{z}$ , then  $\mathbf{x} \sim \mathbf{z}$ ), and symmetric (if  $\mathbf{x} \sim \mathbf{y}$ , then  $\mathbf{y} \sim \mathbf{x}$ ).

### Exercise 21

Prove that strong monotonicity implies local nonsatiation, but not vice versa.

### Exercise 22

Assume that there are only two goods in one economy. Draw indifference curves that (a) satisfy and (b) violate each of the following properties:

1. transitivity;
2. strict convexity;
3. convexity;
4. monotonicity.

### Exercise 23

Show that if there exists a utility function that represents  $\succsim$ , then  $\succsim$  must be rational.

**Exercise 24** (MWG, Ex. 3.C.1)

Assume that there exist only two goods, good 1 and good 2. Define  $\mathbf{x} \succsim \mathbf{y}$  if either ' $x_1 > y_1$ ' or ' $x_1 = y_1$  and  $x_2 \geq y_2$ '. This is known as the *lexicographic preference relation*.

Verify that the lexicographic ordering is complete, transitive, strongly monotone, and strictly convex.

**Exercise 25**

Show that lexicographic preferences are not continuous.

**Exercise 26**

Show that the expenditure function  $e(\mathbf{p}, u)$  satisfies the following properties:

1. non increasing in  $p_i$ ,  $i=1, \dots, n$ ;
2. homogeneous of degree 1 in  $\mathbf{p}$ ;
3. concave in  $\mathbf{p}$ ;
4. continuous in  $\mathbf{p}$ ,  $\mathbf{p} \gg 0$ .

**Exercise 27** (\*)

Let  $u(x_1, x_2) = kx_1^a x_2^{1-a}$ , for  $0 < a < 1$ .

1. Solve the utility maximization problem and find the demand functions;
2. Verify that  $x_i(\mathbf{p}, m)$ ,  $i=1, 2$ , satisfies homogeneity of degree 0 in  $(\mathbf{p}, m)$  and Walras' law.

**Exercise 28** (\*)

Consider the CES utility function  $u(\mathbf{x}) = (x_1^a + x_2^a)^{1/a}$ , for  $a \leq 1$ .

1. Compute the demand functions;
2. Derive the indirect utility function;
3. Derive the demand correspondences and indirect utility functions for the linear utility and the Leontief utility cases. Show that the functions computed in 1. and 2. approach these as  $a \rightarrow 1$  and as  $a \rightarrow -\infty$ , respectively;

4. Compute the Hicksian demand functions and verify their properties.

**Exercise 29 (\*)**

Let  $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$  denote the consumption set and let the utility function be  $u(\mathbf{x}) = x_1 + w(x_2, x_3, \dots, x_L)$ .

1. Show that the demand functions for goods 2, ..., L are independent of income;
2. Argue that the indirect utility function can be written in the form  $v(\mathbf{p}, m) = m + \phi(\mathbf{p})$  for some function  $\phi(\cdot)$ ;

Now, for simplicity, let  $L=2$  and suppose  $x_1 \geq 0$ .

3. Are the demand functions still independent of income?
4. For a given fixed level of prices  $\mathbf{p}$ , examine how demand changes as income increases.

**Exercise 30** (from MWG, Ex. 3.D.6)

Consider the three-good setting in which the consumer has utility function

$$u(\mathbf{x}) = (x_1 - b_1)^a (x_2 - b_2)^c (x_3 - b_3)^d.$$

1. Why can you assume that  $a+c+d=1$  without loss of generality? Do so for the rest of the problem.
2. Write down the first-order conditions for the UMP, and derive the consumer's Walrasian demand and indirect utility functions. This system of demand is known as the *linear expenditure system* and it is due to Stone (1954);
3. Verify that these demand functions satisfy the following properties: homogeneity of degree 0 in  $(\mathbf{p}, m)$ , Walras' Law, convexity/uniqueness.

**Exercise 31**

V, Ex. 7.2, p. 114

**Exercise 32**

V, Ex. 7.4, p. 114

**Exercise 33**

V, Ex. 7.6, parts a) and b), p. 115

**Exercise 34** (from MWG, Ex. 3.G.3)

Consider the linear expenditure system utility function given in Exercise 30.

1. Derive the Hicksian demand and expenditure functions. Check their properties;
2. Verify that the Slutsky equation holds;
3. Verify that the own-substitution effects are negative and that the compensated cross-price effects are symmetric.

**Exercise 35**

The consumer buys bundles  $\mathbf{x}_i$  at prices  $\mathbf{p}_i$ ,  $i=0,1$ . Justify whether each of the following choices satisfies the weak axiom of revealed preference:

1.  $\mathbf{p}_0=(1,3)$ ,  $\mathbf{x}_0=(4,2)$ ,  $\mathbf{p}_1=(3,5)$ ,  $\mathbf{x}_1=(3,1)$ ;
2.  $\mathbf{p}_0=(1,6)$ ,  $\mathbf{x}_0=(10,5)$ ,  $\mathbf{p}_1=(3,5)$ ,  $\mathbf{x}_1=(8,4)$ ;
3.  $\mathbf{p}_0=(1,2)$ ,  $\mathbf{x}_0=(3,1)$ ,  $\mathbf{p}_1=(2,2)$ ,  $\mathbf{x}_1=(1,2)$ ;
4.  $\mathbf{p}_0=(2,6)$ ,  $\mathbf{x}_0=(20,10)$ ,  $\mathbf{p}_1=(3,5)$ ,  $\mathbf{x}_1=(18,4)$ .

**Exercise 36** (MWG, Ex. 2.F.16) (\*)

Consider a setting where  $L=3$  and a consumer whose consumption set is  $\mathbb{R}^3$ . Suppose that his demand function  $\mathbf{x}(\mathbf{p},w)$  is

$$x_1(p, w) = \frac{p_2}{p_3}, \quad x_2(p, w) = -\frac{p_1}{p_3}, \quad x_3(p, w) = \frac{w}{p_3}.$$

1. Show that  $\mathbf{x}(\mathbf{p},w)$  is homogeneous of degree 0 in  $(\mathbf{p},w)$  and satisfies Walras' law;
2. Show that  $\mathbf{x}(\mathbf{p},w)$  violates the weak axiom;
3. Show that  $\mathbf{v} \cdot \mathbf{S}(\mathbf{p},w) \cdot \mathbf{v} = 0$  for all  $\mathbf{v} \in \mathbb{R}^3$ .

**Exercise 37** (MWG, Ex. 2.F.17)

In an  $L$ -commodity world, a consumer's Walrasian demand function is

$$x_k(p, w) = \frac{w}{\sum_{l=1}^L p_l} \quad \text{for } k=1, \dots, L.$$

1. Is this demand function homogeneous of degree 0 in  $(\mathbf{p}, w)$ ?
2. Does it satisfy Walras' law?
3. Does it satisfy the weak axiom?

**Exercise 38**

V, Ex. 8.2, p. 140

**Exercise 39**

V, Ex. 8.5, p. 141

**Exercise 40**

V, Ex. 8.6, p. 141

**Exercise 41**

V, Ex. 8.7, p. 141

**Exercise 42**

V, Ex. 8.10, p. 141

**Exercise 43**

V, Ex. 8.12, p. 142

**Exercise 44**

V, Ex. 8.16, p. 142

**Exercise 45**

V, Ex. 9.10, p. 159

**Exercise 46**

V, Ex. 9.11, p. 159

**Exercise 47**

V, Ex. 10.2, p. 171