UNIVERSIDADE TÉCNICA DE LISBOA

INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

MICROECONOMICS

2010/2011

References:

- Mas-Collel, A., M. Whinston, and J. Green (1995), *Microeconomic Theory*, Oxford University Press, New York (MWG)
- Varian, H. (1992), *Microeconomic Analysis*, Norton, New York (V)

PRODUCTION

(Lectures 1, 2, and 3)

Exercise 1

Draw production sets that a) violate and b) satisfy each of the following properties:

a) No free lunch;
b) Possibility of inaction;
c) Free disposal;
d) Nonincreasing returns to scale ($y \in Y \Rightarrow \alpha y \in Y$, for all $\alpha \leq 1$);
e) Irreversibility ($y \in Y$ and $y \neq 0 \Rightarrow -y \notin Y$);
f) Additivity ($y, y' \in Y \Rightarrow y + y' \in Y$).

Exercise 2 (MWG, Ex. 5.B.2 and 5.B.3)

Let $f(\cdot)$ be the production fuction associated with a single-output technology and let Y be its production set. Show that:

a) Y statisfies constant returns to scale if and only if $f(\cdot)$ is homogeneous of degree 1;
b) Y is convex if and only if $f(\cdot)$ is concave;
c) Y convex rules out the existence of economies of scale when there is possibility of inaction.

Exercise 3

Show that if the production function is homogeneous of degree 1, the marginal rate of substitution is independent of the scale of production.

Exercise 4

Suppose that the production function takes the form $f(x) = (b_1 x_1^a + b_2 x_2^a)^{1/a}$.

a) Show that when a=1, isoquants become linear;

b) Show that as $a \to 0$, this function comes to represent the Cobb-Douglas production function $f(x)= x_1^{b1}x_2^{b2}$;

c) Show that as $a \to -\infty$, this function has in the limit the Leontief production function $f(x)= \min\{x_1, x_2\}$;

d) Compute the marginal rate of substitution and the elasticity of substitution for $f(\cdot)$.

Exercise 5

Derive the profit function and the supply correspondence for the following production functions:

a) $f(x)= x_1 + x_2$;
b) $f(x)= \min\{x_1, x_2\}$;
c) $f(x)= x_1^a x_2^b$, for $a, b>0$.

Exercise 6

Let $f(x)=10x-x^2/2$. Determine:

a) the factor demand function;
b) the profit function.

Exercise 7

Establish all the properties of the cost function.

Exercise 8

Derive the cost function and conditional factor demand functions of the technologies given by:

a) $f(x)= x_1 + x_2$;
b) $f(x)= \min\{x_1, x_2\}$;
c) $f(x)= (x_1^a + x_2^a)^{1/a}$, for $a \leq 1$.

Exercise 9

Let $f(x_1, x_2, x_3, x_4)=\min\{x_1, x_2\}+ \min\{x_3, x_4\}$ and let $g(x_1, x_2, x_3, x_4)=\min\{x_1+x_2, x_3+x_4\}$.

a) Determine the cost functions and the conditional factor demands for both production functions;
b) What kind of returns to scale does each of these technologies exhibit?

Exercise 10

V, Ex. 4.4, p. 63

Exercise 11

V, Ex. 4.6, p. 63

Exercise 12

V, Ex. 5.2, p. 77

Exercise 13

V, Ex. 5.4, p. 77

Exercise 14

V, Ex. 5.6, p. 78

Exercise 15

V, Ex. 5.16, p. 79

Exercise 16

V, Ex. 5.17, p. 80

Exercise 17

Company A produces a single output q from two inputs x_1 and x_2. The following table contains two monthly observations concerning A's technology:

w_1	w_2	x_1	x_2	p	q
3	1	50	50	5	50
2	2	65	40	5	50

Can you recover A's technology?

Exercise 18

Determine the production functions and the conditional factor demands for the following cost functions:

a) $c(w_1,w_2,y)=y(w_1+2\,w_2)$;
b) $c(w_1,w_2,y)=yw_1^a w_2^b$;
c) $c(w_1,w_2,y)=y\min\{2w_1,w_2\}$.