Topics on the Profit Function and Cost Minimization

Lecture2; V, chs. 3 and 4; MWG, ch. 5

1. The profit function

1.1 Properties

a. $\Pi(\cdot)$ is increasing (decreasing) in p_i if *i* is an output (input);
b. $\Pi(\cdot)$ is homogeneous of degree 1 in $\boldsymbol{p}$;
c. $\Pi(\cdot)$ is convex in $\boldsymbol{p}$; intuition: by responding to the price change and adjusting the output level or input mix, the firm is at least as well of as if it did not respond;
d. $\Pi(\cdot)$ is continuous in $\boldsymbol{p}$;
e. *Hotelling's lemma*: $\frac{\partial \Pi(p)}{\partial p_i} = y_i(p)$, for all i=1,…,n, assuming that the derivative exists and $p_i > 0$.

Proofs: V, p. 41, 43, and 44

1.2 Implications

a. a. above implies: $\frac{\partial \Pi(p)}{\partial p_i} > (<) 0$ iff *i* is an output (input);
b. b. above implies y() and x() are homogeneous of degree 0;
c. c. above has comparative statics implications, namely it implies that $\frac{\partial y_i}{\partial p_i} > 0$ and $\frac{\partial y_i}{\partial p_j} = \frac{\partial y_j}{\partial p_i}$, for all i,j=1,…,n.

2. Cost minimization

2.1 Cost minimization problem

Assume that there is a single output to be produced and consider the problem of minimizing the cost of producing an output level y given a vector of input prices **w**: $c(y,w) = \underset{x}{Min}\, wx \ s.t.\ f(x) = y,\ x_i \geq 0,\ i = 1,...n-1$. The *cost function* c(·) gives the minimum cost of producing y for input prices **w**. The FOC's imply $w_i \geq \lambda \frac{\partial f(x^*)}{\partial x_i}, x_i^* \geq 0, (w_i - \lambda \frac{\partial f(x^*)}{\partial x_i}) x^*_i = 0, i = 1,...,n-1$ and $f(x^*) = y$. In an interior solution, the TRS equals the ratio of input prices, i.e., the economic rate of substitution.
The SOC requires the so called bordered Hessian for the Lagrangian to be positive semi-definite. (This means that the determinant and the principal minors should have the same sign, which, however, alternates with the number of constraints and is positive if the number is even and negative if it is odd.)

The solutions of the above problem are the *conditional input demands*, denoted by $x_i(\boldsymbol{w}, y)$. It also follows that $\lambda^* = w_i \Big/ \frac{\partial f}{\partial x_i}(x^*)$. This tells us how much the cost increases if we tighten the constraint by requiring an extra unit of y, i.e., λ equals the marginal cost.

Example: Cobb-Douglas, V, p.54.

2.2 Implications of cost minimization: comparative statics

Given a list of input price vectors $\mathbf{w}^t$ and the associated optimal factor levels $\mathbf{x}^t$, t=1,…,T, an obvious necessary condition for cost minimization is that $\mathbf{w}^t\, \mathbf{x}^t \leq \mathbf{w}^t\, \mathbf{x}^s$ for all t,s such that $\mathbf{y}^s \geq \mathbf{y}^t$. This is the Weak Axiom of Cost Minimization (WACM) and implies that $\triangle\mathbf{w}\triangle\mathbf{x} \leq 0$.