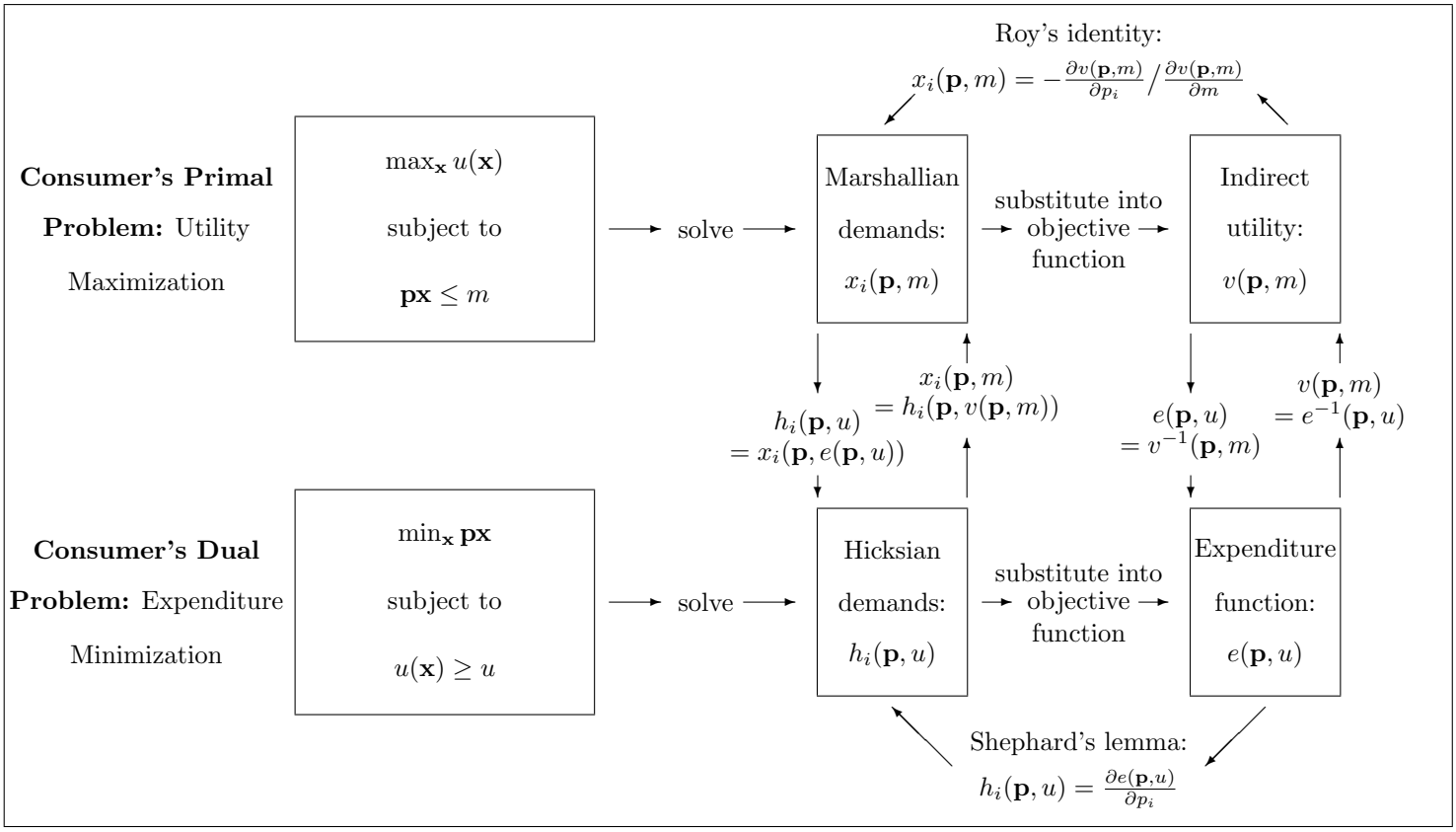
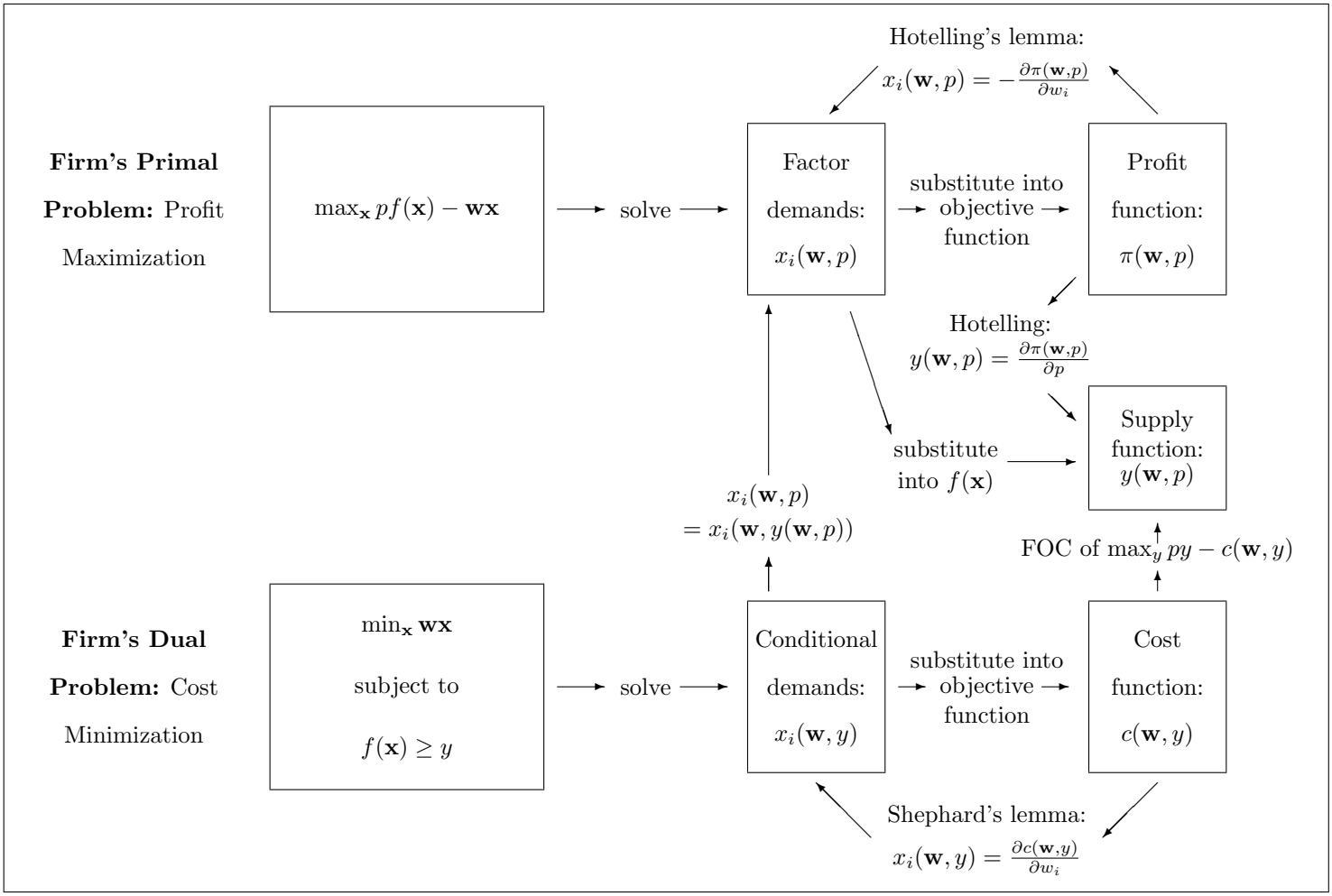

Roy's identity:
$x_i(\mathbf{p}, m) = -\frac{\partial v(\mathbf{p},m)}{\partial p_i} / \frac{\partial v(\mathbf{p},m)}{\partial m}$
Consumer's Primal Problem: Utility Maximization
$\max_{\mathbf{x}} u(\mathbf{x})$ subject to $\mathbf{px} \leq m$
solve
Marshallian demands: $x_i(\mathbf{p}, m)$
substitute into objective function
Indirect utility: $v(\mathbf{p}, m)$
$h_i(\mathbf{p}, u) = x_i(\mathbf{p}, e(\mathbf{p}, u))$
$x_i(\mathbf{p}, m) = h_i(\mathbf{p}, v(\mathbf{p}, m))$
$e(\mathbf{p}, u) = v^{-1}(\mathbf{p}, m)$
$v(\mathbf{p}, m) = e^{-1}(\mathbf{p}, u)$
Consumer's Dual Problem: Expenditure Minimization
$\min_{\mathbf{x}} \mathbf{px}$ subject to $u(\mathbf{x}) \geq u$
solve
Hicksian demands: $h_i(\mathbf{p}, u)$
substitute into objective function
Expenditure function: $e(\mathbf{p}, u)$
Shephard's lemma:
$h_i(\mathbf{p}, u) = \frac{\partial e(\mathbf{p},u)}{\partial p_i}$


Hotelling's lemma:
$x_i(\mathbf{w}, p) = -\frac{\partial \pi(\mathbf{w},p)}{\partial w_i}$
Firm's Primal Problem: Profit Maximization
$\max_{\mathbf{x}} pf(\mathbf{x}) - \mathbf{wx}$
solve
Factor demands: $x_i(\mathbf{w}, p)$
substitute into objective function
Profit function: $\pi(\mathbf{w}, p)$
Hotelling:
$y(\mathbf{w}, p) = \frac{\partial \pi(\mathbf{w},p)}{\partial p}$
substitute into $f(\mathbf{x})$
Supply function: $y(\mathbf{w}, p)$
$x_i(\mathbf{w}, p) = x_i(\mathbf{w}, y(\mathbf{w}, p))$
FOC of $\max_y py - c(\mathbf{w}, y)$
Firm's Dual Problem: Cost Minimization
$\min_{\mathbf{x}} \mathbf{wx}$ subject to $f(\mathbf{x}) \geq y$
solve
Conditional demands: $x_i(\mathbf{w}, y)$
substitute into objective function
Cost function: $c(\mathbf{w}, y)$
Shephard's lemma:
$x_i(\mathbf{w}, y) = \frac{\partial c(\mathbf{w},y)}{\partial w_i}$