LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

January 19, 2017

Answer to 4 Groups only

2 hours

Group 1

1. Consider a consumer whose utility function is $u(x_1, x_2) = \min\{x_1, 3x_2\}$, where x_1 represents the quantity of good 1 and x_2 represents the quantity of good 2.

a) (0.5 marks) Formulate the consumer choice problem.

A: Find x_1, x_2 to maximize $u(x_1, x_2) = \min\{x_1, 3x_2\}$ s.t. $p_1x_1 + p_2x_2 = m$ and $x_1, x_2 \geq 0$.

b) (2 marks) Find this consumer's demand for goods 1 and 2.

A: At the solution we have $x_1 = 3x_2$, so that, substituting in the budget restriction and solving: $x_1(y, p_1, p_2) = 3m/(3p_1 + p_2)$ and $x_2(y, p_1, p_2) = m/(3p_1 + p_2)$.

c) (0.5 marks) Determine the indirect utility function.

A: $v(y, p_1, p_2) = 3m/(3p_1 + p_2)$.

c) (1 mark) Determine the expenditure function.

A: $e(u, p_1, p_2) = u(3p_1 + p_2)/3$.

2. (1 mark) Let $\succcurlyeq$ be a preference relation on R^n_+ and suppose $u(\cdot)$ is a utility function that represents it. Show that u(x) is strictly increasing if and only if $\succcurlyeq$ is strictly monotonic.

Group 2

Willy owns a factory on the banks of a river that occasionally floods. He has no other assets. If there is no flood this spring, Willy's factory will be worth €500,000. If there is a flood, the factory will be worthless. Willy is an expected utility maximizer with von Neumann Morgenstern utility function u(w) = ln(w) where w is his wealth. Willy believes that the probability of a flood is 1/10. Willy is offered a chance to buy as much flood insurance as he likes at a cost of €c per Euro's worth of insurance. The way this policy works is that if he buys €X worth of flood insurance and if there is no flood, he must pay a total of €cX in insurance premiums. If there is a flood, he doesn't have to pay his insurance premium, and he receives a payment of €X from the insurance company.

a) (1.5 marks) Formulate Willy's utility maximization problem.

A: Find the x to Max $0{,}9 \ln(500000-cx) + 0{,}1 \ln(x)$ s.t. $0 \leq x \leq 500000$.

b) (3 marks) Write down a formula for the amount of insurance that Willy will buy as a function of the cost €c per euro of insurance.

A: Solving the utility maximization problem, the first order solution for an interior maximum gives $x = 50000/c$.

c) (0.5 marks) At what price c, will Willy buy just enough insurance so that his wealth is the same, whether or not there is a flood?

A: The actuarially fair price, i.e., the price at which the insurance firm gets 0 expected profits: $0{,}9cx + 0{,}1(-x) = 0 \Leftrightarrow c = 1/9$.

Group 3

1. In a perfectly competitive market, a firm has the production function $f(x_1, x_2)=(x^a_1 + x^a_2)^{s/a}$, where $a = -1$ and where $0 < s < 1$.

a) (1,5 marks) Find the conditional input demand functions for inputs 1 and 2 with prices w_1 and w_2, respectively, and output y.

A: $x_1(w_1, w_2,y) = y^{1/s}[1+(w_2/w_1)^{½}]$ and $x_2(w_1, w_2,y) = y^{1/s}[1+(w_1/w_2)^{½}]$.

b) (0,5 marks) Find the cost function $c(w_1,w_2,1)$ for producing 1 unit of output.

A: $c(w_1, w_2,1) = (w_1^{½} + w_2^{½})^2$ as $c(w_1, w_2,y) = y^{1/s}(w_1^{½} + w_2^{½})^2$.

c) (1,5 marks) Suppose that s = 1/2, w_1 = 4 and w_2 =1. If the firm can sell its output at a competitive price of €72 per unit, how many units should it produce to maximize its profits?

A: The firm solves Max 72y - c(4, 1,y) to obtain y = 4.

2. (1,5 marks) In a perfectly competitive market, a firm's technology can be described by the production function $f(x_1, x_2) = x^a{}_1x^b{}_2$. For which values of a and b does the long run profit maximization problem have a solution for all prices? Explain.

A: When a + b < 1, the production function has decreasing returns to scale and the profit maximization problem has a solution.

Group 4

1. (2 marks) Suppose that a consumer's indirect utility function is $v(m,p_1,p_2) = m^2/(4p_1p_2)$ and assume that initially m = 10, and $p_1 = p_2 = 1$. What is the compensating variation (CV) of an increase in p_2 to $p'_2 = 2$?

A: Initially v(10,1,1) = 25. Then, from v(10+CV,1,2)=25, we obtain CV = sqrt(200)-10.

2. A monopolist faces linear demand p = a -bq and has cost C = cq + F, where all parameters are positive, a > c, and $(a -c)^2 > 4bF$.

a) (1,5 marks) Solve for the monopolist's output, price, and profits.

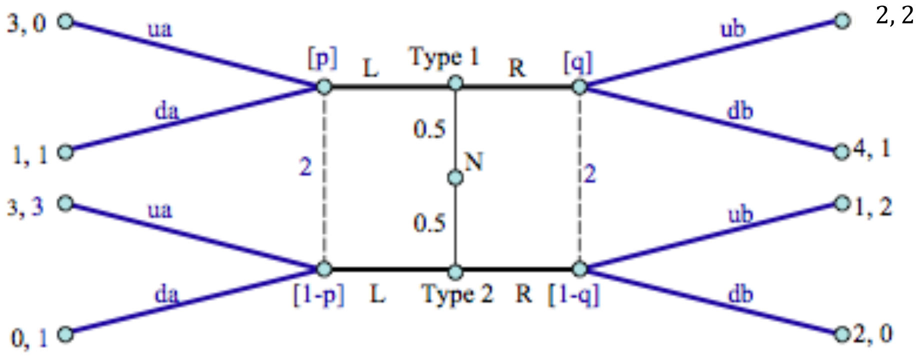
A: The monopolist finds q such that Max q(a -bq) - (cq+ F). Therefore, q =(a-c)/2b, p = (a+c)/2, and Profit = $(a-c)^2/4b$ -F.

b) (1, 5 marks) Calculate the deadweight loss.

A: in a perfectly competitive market, p = c and q=(a-c)/b. Then, DWL = $(a-c)^2/8b$.

Group 5

1. (5 marks) Compute the weak perfect Bayesian Nash equilibria of the following game.



A: PBE = {[LL, uaub, p = 0.5, 0 ≤ q ≤ 1], [RR, daub, p ≥ 2/3, q = 0,5]}

Group 6

1. (2 marks) Comment on the following sentence: "In a strategic form game G, all players have a strictly dominant strategy if and only if the Nash equilibrium is unique."

A: If each player has a strictly dominant strategy, this strategy gives a strictly higher payoff against any strategy profile of the others. Thus, it is his/her unique best reply. It follows that there is a unique Nash equilibrium. Nevertheless, it is easy to find a counterexample for the converse statement, so that we may have a unique Nash equilibrium even though there is at least one player that does not have a strictly dominant strategy.

2. (3 marks) Players 1 and 2 face an incomplete information game. Player 1 does not know the type of player 2, believing that he is type I with probability 1/2 and type II with probability 1/2. Compute all Bayes-Nash equilibria in pure strategies.

Type I

	L	R
U	1,2	1,3
D	-2,3	2,4

Type II

	L	R
U	1,2	1,4
D	-2,2	2,1

A: There are no Bayes-Nash equilibria I pure strategies.