

GAME THEORY

Exercise list 2

Exercise 1

Find all Bayesian-Nash equilibria of the following game with incomplete information:

- (a) Nature chooses J_1 and J_2 with 50% probability.
- (b) Player 1 observes Nature's choice, but player 2 does not.
- (c) Player 1 chooses C or B ; simultaneously, player 2 chooses E or D .

J_1	E	D
C	1, 1	0, 0
B	0, 0	0, 0

J_2	E	D
C	0, 0	0, 0
B	0, 0	2, 2

Exercise 2

Consider a Cournot duopoly with market demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2$. Firm 1's cost function, given the quantity produced, is $C_1(q_1) = cq_1$ and firm 2's cost function is $C_2(q_2) = c_H q_2$ with probability a and $C_2(q_2) = c_L q_2$ with probability $1 - a$. All of this is common knowledge. However, information is asymmetric: firm 2 knows its cost function, but firm 1 does not.

- i. Formulate this situation as game in strategic form.
- ii. Compute a Bayesian-Nash equilibrium.

Exercise 3

Consider the Battle of Sexes:

	<i>Bach</i>	<i>Stravinski</i>
<i>Bach</i>	3, 1	0, 0
<i>Stravinski</i>	0, 0	1, 3

- i. Find all Nash equilibria of this game.
- ii. Now assume that this game has incomplete information:

	<i>Bach</i>	<i>Stravinski</i>
<i>Bach</i>	$3 + t_c, 1$	$0, 0$
<i>Stravinski</i>	$0, 0$	$1, 3 + t_p$

Where t_c and t_p follow a uniform distribution in $[0, x]$. Determine the Bayesian-Nash equilibrium in pure strategies and show that as x goes to 0, the Bayesian-Nash equilibrium tends to the mixed strategies equilibrium of the complete information game.