GAME THEORY

Exercise list 3

Exercise 1

Two players, 1 and 2, share 1€ using the following procedure: each player i chooses a number s_i, $s_i \in [0, 1]$, $i =1,2$. The choices are simultaneous. If $s_1 + s_2 \leq 1$, each player gets the amount chosen; if $s_1 + s_2 > 1$ both get 0.

  i. Determine the set of pure Nash equilibria.

Suppose that player 2, before choosing s_2, observes the number chosen by player 1 and this fact is common knowledge.

  ii. Find a few examples of pure Nash equilibria of the modified game.

 iii. Determine the set of pure subgame perfect Nash equilibria.

Exercise 2

Player 1 may choose Stop or Continue. If he chooses Stop, the game ends and each player gets 1€. If he chooses Continue, both players simultaneously choose non-negative integers and each player gets the product of the chosen numbers.

  i. Formulate this situation as an extensive-form game with imperfect information.

  ii. Determine the set of pure subgame perfect Nash equilibria.

  iii. How does this set change if the non-negative integers are at most equal to $M > 0$?

Exercise 3

Consider the following extensive-form game with imperfect information Γ:
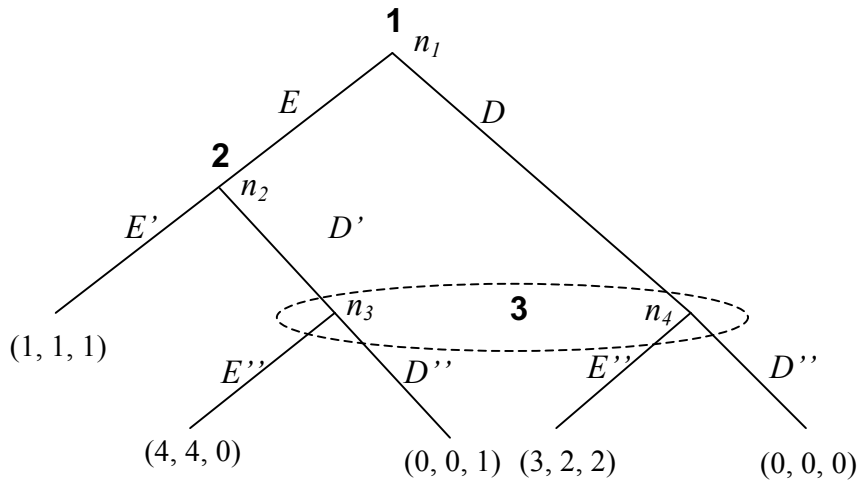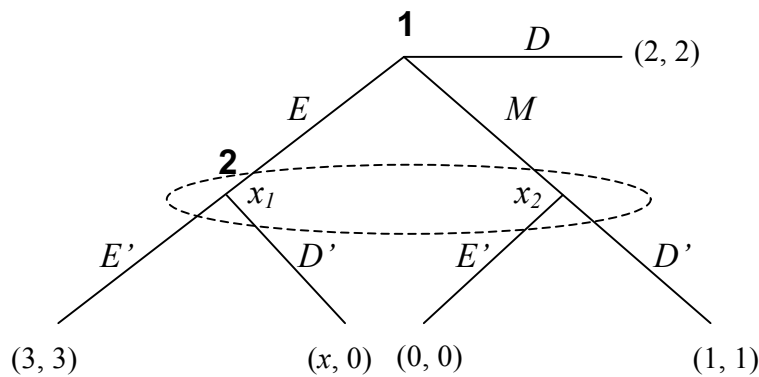
This game has two types of Nash equilibria:

Type 1: $x_1(E)=1$, $x_2(E')=1$ and $x_3(E'') \in [0, 1/4]$.

Type 2: $x_1(E)=0$, $x_2(E') \in [1/3, 1]$ and $x_3(E'')=1$.

i. Show that equilibria of Type 1 are perfect Bayesian equilibria of Γ.

ii. Show that no equilibrium of Type 2 is a perfect Bayesian equilibria of Γ.

Exercise 4

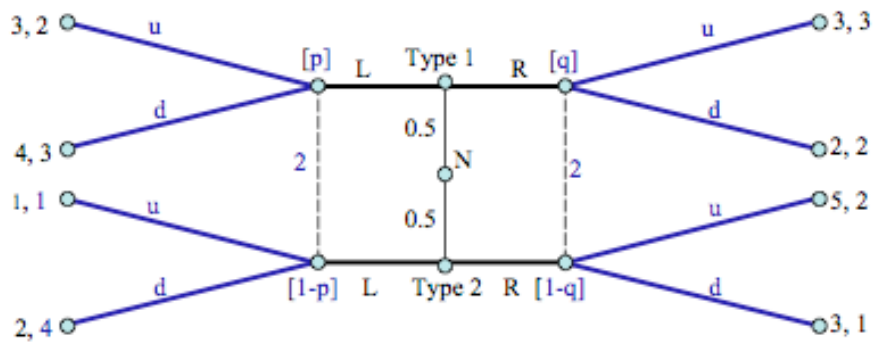Consider the following extensive-form game with imperfect information Γ:

Show that the strategy $x =(x_1, x_2)$, with $x_1(D)=1$ and $x_2(D')=1$, is a perfect Bayesian equilibrium for $x < 2$.
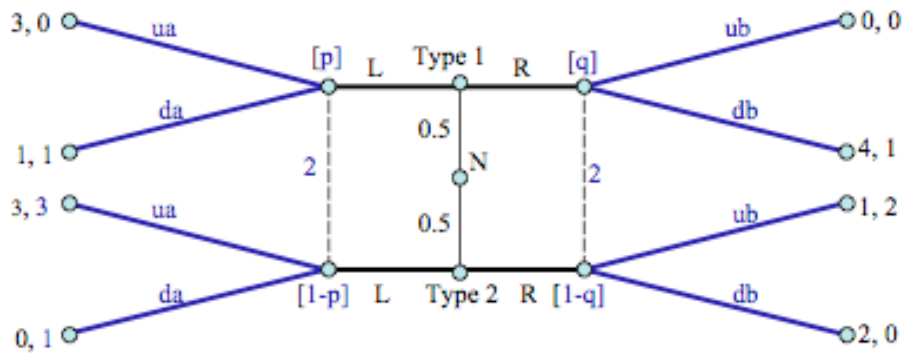
Exercise 5

Check whether Player 1's strategies (L,R), (R,L), (R,R) and (L,L) are part of perfect Bayesian equilibria of the following game:
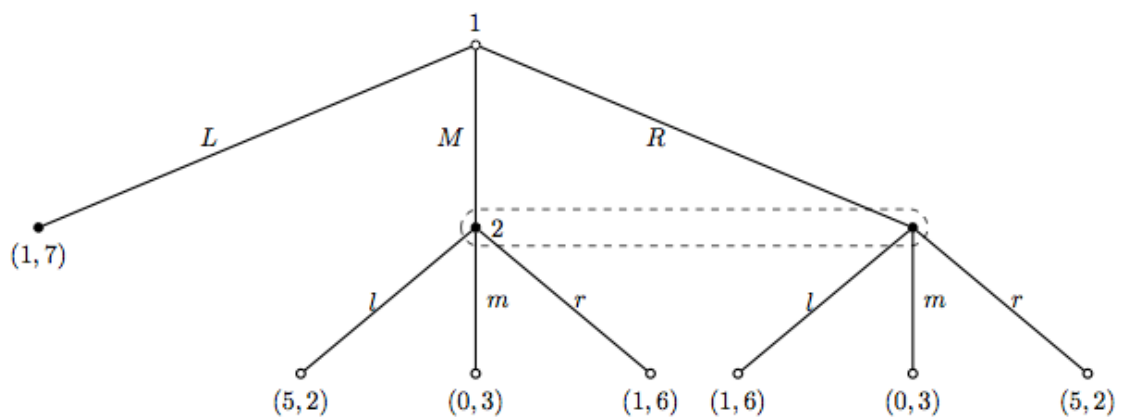
Exercise 6

Show that Player 1's strategy (L,R) is part of a perfect Bayesian equilibrium of the following game:

Exercise 7

Consider the following extensive-form game with imperfect information:

a) Write the game in normal form.

b) Determine the set of pure strategy Nash equilibria of the game.
c) How many subgames does this game have?
d) Determine the set of pure strategy subgame perfect Nash equilibria of the game.
e) Check whether the equilibria found in d) are perfect Bayesian equilibria.