LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

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Solution Topics

Group 1

1. Consider a consumer whose utility function is $u(x_1, x_2) = \sqrt{2x_1 + x_2}$, where x_1 represents the quantity of good 1 and x_2 represents the quantity of good 2.

a) (0.5 marks) Formulate the consumer choice problem.

R: Max $u(x_1, x_2) = \sqrt{2x_1 + x_2}$ s.t. $p_1 x_1 + p_2 x_2 \leq m$, $x_1 \geq 0$, $x_2 \geq 0$.

b) (2 marks) Find this consumer's demand for goods 1 and 2.

R: The goods are perfect substitutes. $x(p_1, p_2, m) = (m/p_1, 0)$ if $p_1 < 2p_2$; $(0, m/p_2)$ if $p_1 > 2p_2$; (x_1, x_2) s. t. $p_1 x_1 + p_2 x_2 = m$ if $p_1 = 2p_2$

c) (0.5 marks) Determine the indirect utility function.

R: $v(p, m) = \sqrt{2m/p_1}$ if $p_1 < 2p_2$; $v(p, m) = \sqrt{m/p_2}$ if $p_1 \geq 2p_2$.

d) (1 mark) Determine the expenditure function.

R: $e(p, u) = p_1 u^2/2$ if $p_1 < 2p_2$; $e(p, u) = p_2 u^2$ if $p_1 \geq 2p_2$.

2. (1 mark) Let $\succcurlyeq$ be a preference relation on R^n_+ and suppose $u(\cdot)$ is a utility function that represents it. Let $v(x) = f(u(x))$ for every $x \in R^n_+$, where $f: R \to R$ is strictly increasing on the set of values taken on by u. Show that $v(x)$ represents $\succcurlyeq$.

R: Since $u(\cdot)$ is a utility function that represents $\succcurlyeq$, we have $x \succcurlyeq y$ if and only if $u(x) \geq u(y)$. Since $f' > 0$, $u(x) \geq u(y)$ if and only if $f(u(x)) \geq f(u(y))$ or $v(x) \geq v(y)$ by definition of v(.). Therefore, we have $x \succcurlyeq y$ if and only if $v(x) \geq v(y)$ and $v(\cdot)$ represents $\succcurlyeq$.

Group 2

1. Maria's utility function is given by $u(x_1, x_2) = x_1^2 x_2$, where x_1 represents the quantity of good 1 and x_2 represents the quantity of good 2. Maria's income is 1500€, the price of good 1 is €200, and the price of good 2, initially equal to 50€, rises to 75€.

a) (1.25 marks) Compute the decrease in consumer surplus that Maria derives from the consumption of good 2 due to the increase in the price of good 2.

R: Solve the consumer problem (utility maximization subject to budget constraint; non-negativity constraints can be ignored because the utility function is a Cobb-Douglas) to obtain the demand function of goods 1 and 2: $x_1(p_1,p_2) = 2m/3p_1$ and $x_2(p_1,p_2) = m/3p_2$. The decrease in consumer surplus is given by the integral of the demand of good 2, i.e., $m/3p_2$, when p_2 varies between 50 and 75, which is $m\ln(3/2)/3$.

b) (1.25 marks) Compute the compensating variation (CV) associated to this change in the price of good 2. Represent the compensating variation graphically.

R: Find the solution to: $200x_1 + 75x_2 = 1500 + CV$; $x_2 = 200x_1/150$; and $x_1^2 x_2 = 5^2 10^2$ to obtain the value of CV.

2. (2.5 marks) A risk-averse individual with initial wealth w_0 and vNM utility function $u(\cdot)$ must decide whether and for how much to insure his car. The probability that he will have an

accident and incur a dollar loss of L in damages is $\alpha \in (0, 1)$. Let p denote the rate at which each euro of insurance can be purchased (i.e., when x units of insurance are purchased, the agent pays px) and assume that insurance is available at an actuarially fair price (i.e., one that yields insurance companies zero expected profits). How much insurance, x, should he purchase?

R: The agent finds x to solve Max $\alpha u(w-L+x-px) + (1-\alpha)u(w-px)$ s.t. $x \geq 0$. Since insurance is actuarially fair, we have $\alpha = p$ and the FOC corresponding to an interior solution is $p(1-p)u'(w-L+x-px) = p(1-p)u'(w-px)$, which, since $u''<0$, implies $w-L+x-px = w-px$ or $x = L$.

Group 3

1. In a perfectly competitive market, let a firm's production function be given by $f(k,l) = 2kl$, where k denotes the quantity of capital and l denotes the quantity of labour used in the production process.

a) (2 marks) Compute the conditional input demand function and the cost function.
R: Solve the cost minimization problem, i.e., find l, k ≥ 0 that solve Min $wl + rk$ s. t. $2kl \geq y$, to obtain: $l(y,w,r) = \sqrt{ry/2w}$ and $k(y,w,r) = \sqrt{wy/2r}$. The cost function is $c(y,w,r) = \sqrt{2rwy}$.

b) (0.5 marks) Evaluate this technology's returns to scale.
R: Since $f(tk,tl) = 2(tk)(tl) = t^2 f(k,l)$, we have $f(tk,tl) > tf(k,l)$, for all $t > 1$, so that returns to sacle are increasing.

c) (1 mark) Can we solve the profit maximization problem? Why or why not?
R: No, because the technology exhibits increasing returns to scale.

2. (1,5 marks) Comment on the following statement: "A Cobb-Douglas production function exhibits decreasing returns to scale if and only if the marginal product of labour is decreasing in the amount of labour used."
R: If a Cobb-Douglas production function exhibits decreasing returns to scale, then the marginal product of labour is decreasing in the amount of labour used. However, the converse is not true: the marginal product of labour may be decreasing in the amount of labour used, but the technology may exhibit constant or increasing returns to scale.

Group 4

1. In a perfectly competitive market there are J firms. Each firm produces output q according to an identical long run cost function $c(q) = k + q^2$, $k > 0$, for $q > 0$ and $c(0) = 0$. Market demand is given by $Q_d = a - p$.

a) (1.25 marks) Determine the long run supply function of an individual firm.
R: P = MgC gives $p = 2q$ or $q = p/2$. The long run supply curve of an individual firm is $q = p/2$ as long as $p \geq$ min AC $= 2\sqrt{k}$; otherwise, $q = 0$.

b) (1.25 marks) Consider k = 1. Determine the long run equilibrium: price, quantity produced, and number of firms in the market.
R: Using $q^* = p^*/2$, $a - p^* = J^*q^*$, and $p^*q^* - (1 + q^{*2}) = 0$, we obtain $p^* = 2$, $q^* = 1$, $J^* = a - 2$ e $Q^* = a-2$.

2. Consider a market structure with J identical firms with marginal cost $c \geq 0$. Let the inverse

market demand be given by p = a – bQ_d for total market output Q_d.

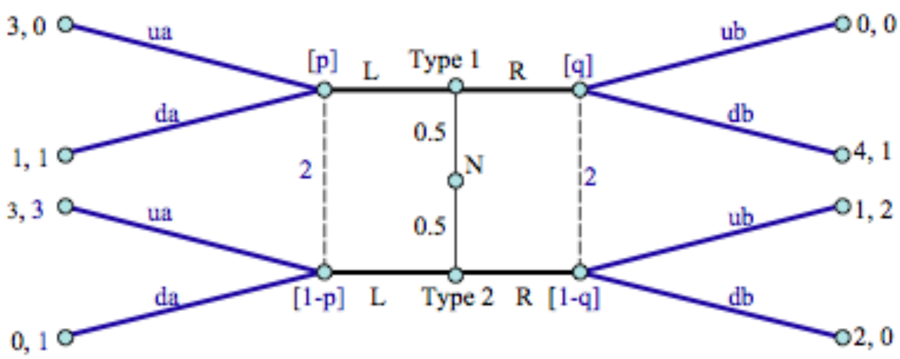
a) (1 mark) Compute total surplus, W, as a function of Q_d, when each firm produces the same output Q_d/J.
R: Solve Max (a-b Q_d) Q_d/J - c Q_d/J to find $Q_d = (a-c)/2b$. Then, $W = (a-c)^2/4b$.

b) (1 mark) Compute the maximum potential total surplus W*.
R: We obtain maximum surplus $W^* = (a-c)^2/2b$ when p = c, i.e., for $Q_d = (a-c)/b$.

c) (0.5 marks) In which market structure do we achieve maximum total surplus? Explain briefly.
R: Perfect competition.

Group 5

1. (5 marks) Compute the weak perfect Bayesian Nash equilibria of the following game.



R: {[(L,R), (da,ub), p=1, q=0], [(R, L), (ua,db), p=0, q=1], [(L, L), (ua,ub), p = 0,5, q ≤ 2/3]}

Group 6

1. (2.5 marks) Players 1 and 2 simultaneously choose a positive integer smaller or equal than K. If they choose the same number, player 2 pays 1€ to player 1; otherwise, no payment is made. Determine the unique Nash equilibrium of the game.

2. (2.5 marks) Players 1 and 2 face an incomplete information game. Player 1 does not know the type of player 2, believing that he is type I with probability 1/3 and type II with probability 2/3. Considering the payoff matrices below, show that (U, (R,R)) is not a Bayes-Nash equilibrium.

Type I

	L	R
U	1,2	2,4
D	0,3	3,1

Type II

	L	R
U	1,3	2,2
D	0,2	3,3

R: For Player 2-Type I to play R, the probability with which Player 1 plays U (p) must be $p \geq 0.5$. For Player 2-Type II to play R, we must have $p \leq 0.5$. Therefore, for (R,R) to be a BNE, we must have $p = 0.5$, i.e., Player 1 must play a mixed strategy, which implies E(U) = E (D). However, when Player 2 plays (R,R), we do not have E(U) = E (D).