Instituto Superior de Economia e Gestão

Masters in Economics and Masters in Monetary and Financial Economics

Microeconomics

Midterm Test - Solution Topics

(Topics only!)

Maximum duration: 1h30 6th of November of 2014

Question 1

(4 marks) Show that if preferences ≿ are represented by a utility function, then ≿ satisfy transitivity.

Consider any 3 consumption bundles x, y, and z such that x ≿ y and y ≿ z. Let ≿ be represented by the utility function u(.). Then, we have u(x) ≥ u(y) and u(y) ≥ u(z). Since ≥ (defined on the set of real numbers) is transitive, we have u(x) ≥ u(z). Given that u(.) represents ≿, it follows that x ≿ z.

Question 2

A consumer has preferences over goods *x* and *m* represented by the utility function:

$$u(x,m) = ln(x) + m.$$

Let *p* be the price of *x*, let the price of *m* be equal to 1, and let income be equal to *y*. Assume that the consumption set is $(-\infty,\infty)\times$🛤, i.e., *m* may be negative.

1. (3 marks) Derive the Marshallian demands for *x* and *m*. Note that the demand for *x* is independent of income.

   The consumer problem is: Max $u(x,m) = ln(x) + m$. s.t. $m + p.x \leq y$ and $x \geq 0$. Since u(.) is increasing in x and in m, the solution must satisfy budget balancedness. It follows that $m = y - p.x$ and the problem becomes: Max $ln(x) + y - p.x$. s.t. $x \geq 0$.
   The solution to this problem has $x^* > 0$ (given the *ln*), namely, $x^* = 1/p$ and $m^* = y - 1$.

2. (1.5 marks) Derive the indirect utility function.

   The indirect utility function is $v(p,y) = ln(1/p) + y-1$.

3. (1.5 marks) Use the Slutsky equation to decompose the effect of an own-price change on the demand for *x* into income and substitution effects.

Marshallian demand is $x(p,y) = 1/p$. Since it does not depend on income, the derivative of this function with respect to income is 0. Using the Slutsky equation, we then know that the effect of an own-price change on the Marshallian demand for x equals the substitution effect (i.e., the effect of an own-price change on the Hicksian demand for x).

Now assume that m can only assume non-negative values.

4. (2 marks) Is the demand for x still independent of income? Why or why not?

No. In this case, we have to solve the utility maximization problem again, but considering an additional restriction: $m \geq 0$. The problem is: Max $u(x,m) = ln(x) + m$. s.t. $m + p.x \leq y$ and $x \geq 0$, $m \geq 0$. Again, the budget constraint must be satisfied with equality and (given the ln), we must have $x^* > 0$. Then, there are two types of solutions:

a) $\lambda^* = 1$: $x^* = 1/p$ and $m^* = y - 1$ and

b) $\lambda^* > 1$: $m^* = 0$, $x^* = y/p$, and we must have $\lambda^* = 1/y > 1$ or $y < 1$

Question 3

Consider a Leontief production function of the form $f(x_1,x_2) = min\{ax_1, bx_2\}$, with $a > 0$ and $b > 0$.

1. (1 mark) Sketch the isoquant map for this technology.
2. (4 marks) Solve the cost minimization problem and derive the cost function.

The solution must have $min\{ax_1^*, bx_2^*\} = y$ and $ax_1^* = bx_2^*$. Then, the problem Min $w_1. x_1 + w_2. x_2$. s.t. $min\{ax_1, bx_2\} \geq y$ and $x_1, x_2 \geq 0$ becomes:

$$\text{Min } w_1. x_1 + w_2.a.x_1/b \text{ s.t. } ax_1 = y \text{ and } x_1, x_2 \geq 0.$$

So that the solution is $x_1^* = y/a$ and $x_2^* = y/b$. And the cost function is $c(w_1, w_2, y) = (w_1/a + w_2/b)y$.

3. (1 mark) Without trying to solve the profit maximization problem, can you tell whether there is a solution for this problem? Justify.

No, unless the profit function is equal to zero (which happens for certain price levels).

Question 4

(3 marks) Show that if a production function is homogeneous of degree 2, then it exhibits increasing returns to scale.

Let f(.) be a production function that is homogeneous of degree 2, i.e., $f(t.x) = t^2 f(x)$ for all $t > 0$ and all x. Since $t^2 f(x) > tf(x)$, for all $t > 1$, then f(.) satisfies $f(t.x) > tf(x)$ for all $t > 1$ and all x, i.e., f(.) exhibits increasing returns to scale.