Microeconomics

Joana Pais

Fall 2016

Primitive notions

There are four building blocks in any model of consumer choice. They are the consumption set, the feasible set, the preference relation, and the behavioural assumption. Each is conceptually distinct from the others, though it is quite common sometimes to lose sight of that fact. This basic structure is extremely general, and so, very flexible. By specifying the form each of these takes in a given problem, many different situations involving choice can be formally described and analysed.

The notion of a consumption set is straightforward. We let the consumption set, X, represent the set of all alternatives, or complete consumption plans, that the consumer can conceive - whether some of them will be achievable in practice or not. The consumption set is sometimes also called the choice set.

Primitive notions

Let each commodity be measured in some infinitely divisible units. Let $x_i \in R$ represent the number of units of good i. We assume that only non–negative units of each good are meaningful and that is is always possible to conceive of having no units of any particular commodity. Further, we assume there is finite, fixed, but arbitrary number n of different goods. We let $x = (x_1, \ldots, x_n)$ be a vector containing different quantities of each of the n commodities and call x a consumption bundle or a consumption plan. A consumption blundle $x \in X$ is thus represented by a point $x \in R_n^+$. Usually, we'll simplify things and just think of the consumption set as the entire non–negative orthant, $X = R_n^+$. In this case, it is easy to see that each of the following basic requirements is satisfied.

Primitive notions

Properties of the consumption set, X:

- $X \subseteq \mathbb{R}_n^+$.
- X is closed.
- X is convex.
- $0 \in X$.

Preference relations

Consumer preferences are characterised axiomatically. In this method of modelling as few meaningful and distinct assumptions as possible are set forth to characterise the structure and properties of preferences.

The rest of the theory then builds logically from these axioms, and predictions of behaviour are developed through the process of deduction.

These axioms of consumer choice are intended to give formal mathematical expression to fundamental aspects of consumer behaviour and attitudes towards the objects of choice. Together, they formalise the view that the consumer can choose and that choices are consistent in a particular way.

Formally, we represent the consumer's preferences by a binary relation defined on the consumption set, X. If $x_1 \succsim x_2$, we say that x_1 is **at least as good as** x_2, for this consumer.

Preference relations

- **Axiom 1: Completeness**. For all x_1 and x_2 in X, either $x_1 \succeq x_2$ or $x_2 \succeq x_1$. Axiom 1 formalises the notion that the consumer can make comparisons, that is, that he has ability to discriminate and the necessary knowledge to evaluate alternatives.
- **Axiom 2: Transitivity**. For any three elements x_1, x_2, and x_3 in X, if $x_1 \succeq x_2$ and $x_2 \succeq x_3$, then $x_1 \succeq x_3$. Axiom 2 gives a very particular form to the requirement that the consumer's choices be consistent. Although we require only that the consumer be capable of comparing two alternatives at a time, the assumption of transitivity requires that those pairwise comparisons be linked together in a consistent way.

Preference relations

Definition 1.1: Preference Relation
The binary relation $\succeq$ on the consumption set X is called a preference relation if it satisfies Axioms 1 and 2.

Definition 1.2: Strict Preference Relation
The binary relation $\succ$ on the consumption set X is defined as follows: $x_1 \succ x_2$ if and only if $x_1 \succeq x_2$ and $x_2 \not\succeq x_1$.

The relation $\succ$ is called the strict preference relation induced by $\succeq$, or simply the strict preference relation when $\succeq$ is clear. The phrase $x_1 \succ x_2$ is read "x_1 is strictly preferred to x_2".

Definition 1.3: Indifference Relation
The binary relation $\sim$ on the consumption set X is defined as follows: $x_1 \sim x_2$ if and only if $x_1 \succeq x_2$ and $x_2 \succeq x_1$.

The relation $\sim$ is called the indifference relation induced by $\succeq$, or simply the indifference relation when $\succeq$ is clear. The phrase $x_1 \sim x_2$ is read "x_1 is indifferent to x_2".

Definition 1.4: Sets derived from the Preference Relation
Let x_0 be any point in the consumption set, X. Relative to any such point, we can define the following subsets of X:

- $\succeq (x_0) = \{x | x \in X, x \succeq x_0\}$, called the "at least as good as" set.
- $\preceq (x_0) = \{x | x \in X, x_0 \succeq x\}$, called the "no better than" set.
- $\succ (x_0) = \{x | x \in X, x \succ x_0\}$, called the "preferred to" set.
- $\prec (x_0) = \{x | x \in X, x_0 \succ x\}$, called the "worse than" set.

Preference relations

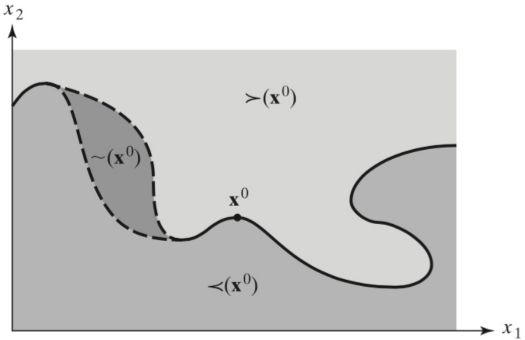

Figure: Hypothetical preferences satisfying Axioms 1 and 2.

Preference relations

- **Axiom 3: Continuity**. For all $x \in \mathbb{R}_n^+$, the "at least as good as" set $\succeq (x)$, and the "no better than" set, $\preceq (x)$ are closed in $\mathbb{R}_+^n$.
- **Axiom 4': Local non–satiation**. For all $x_0 \in \mathbb{R}_n^+$, and for all $\varepsilon > 0$, there exist some $x \in B_\varepsilon(x^0) \cap \mathbb{R}_n$ such that $x \succ x_0$. Axiom 4' says that within any vicinity of a given point x_0, no matter how small that vicinity is, there will always be at least one other point x that the consumer prefers to x_0.

Preference relations

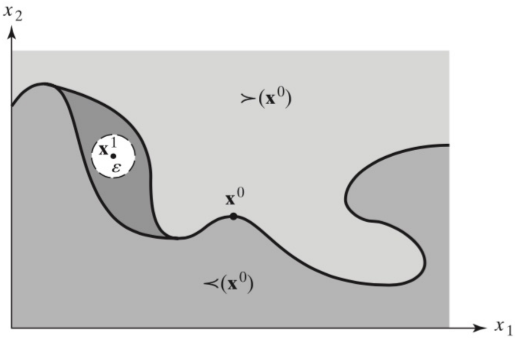

Figure: Hypothetical preferences satisfying Axioms 1, 2 and 3

Preference relations

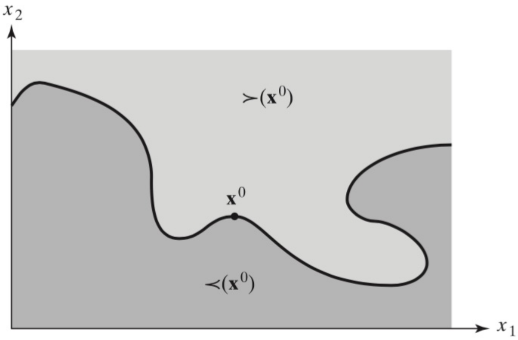

Figure: Hypothetical preferences satisfying Axioms 1, 2, 3, and 4'

- **Axiom 4: Monotonicity**. For all $x_0, x_1 \in \mathbb{R}_n^+$, if $x_0 \geq x_1$, then $x_0 \succeq x_1$, while if $x_0 \gg x_1$, then $x_0 \succ x_1$.

  Axiom 4 says that if one bundle contains at least as much of every commodity as another bundle, then the one is at least as good as the other. Moreover, it is strictly better if it contains strictly more of every good.

Preference relations

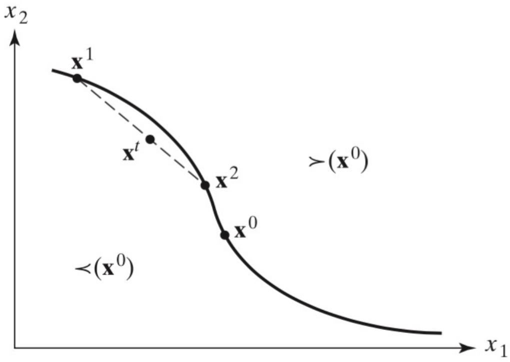

Figure: Hypothetical preferences satisfying Axioms 1, 2, 3, and 4'

Preference relations

- **Axiom 5': Convexity**. If $x_1 \succeq x_0$, then $tx_1 + (1-t)x_0 \succeq x_0$ for all $t \in [0,1]$.
- **Axiom 5: Strict Convexity**. If $x_1 \neq x_0$ and $x_1 \succeq x_0$, then $tx_1 + (1-t)x_0 \succ x_0$ for all $t \in (0,1)$.

Preference relations

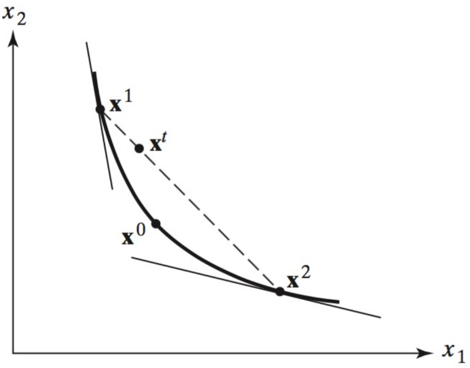

Figure: Hypothetical preferences satisfying Axioms 1, 2, 3, 4, and 5' or 5

The utility function

A real–valued function $u : \mathbb{R}^n_+ \to \mathbb{R}$ is called a **utility function representing the preference relation** $\succeq$ if, for all $x_0, x_1 \in \mathbb{R}^n_+$, $u(x_0) \geq u(x_1) \Leftrightarrow x_0 \succeq x_1$.

Theorem 1.1: Existence of a Real-Valued Function Representing the Preference Relation $\succeq$:

If the binary relation $\succeq$ is complete, transitive, continuous, and strictly monotonic, there exist a continuous real–valued function, $u : \mathbb{R}^n_+ \to R$ that represents $\succeq$.

Theorem 1.2: Invariance of the Utility Function to Positive Monotonic Transformations:

Let $\succeq$ be a preference relation on $\mathbb{R}^n_+$ and suppose $u(x)$ is a utility function that represents it. Then $v(x)$ also represents $\succeq$ if only if $v(x) = f(u(x))$ for every x, where $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing on the set of values taken on by u.

Theorem 1.3: Properties of Preference and Utility Functions:

Let $\succeq$ be represented by $u : \mathbb{R}^n_+ \rightarrow \mathbb{R}$. Then:

- $u(x)$ is strictly increasing if and only if $\succeq$ is strictly monotonic.
- $u(x)$ is quasiconcave if and only if $\succeq$ is convex.
- $u(x)$ is strictly quasiconcave if and only if $\succeq$ is strictly convex.

The utility function

There is a certain vocabulary we use when utility is differentiable, so we should learn it. The first–order partial derivative of $u(x)$ with respect to x_i is called the marginal utility of good i.
For the case of two goods, we defined the **marginal rate of substitution of good 2 for good 1** as the absolute value of the slope of an indifference curve. We can derive an expression for this terms of the two goods' marginal utilities. To see this, consider any bundle $x = (x_1, x_2)$. Because the indifference curve through x is just a function in the (x_1, x_2) plane, let $x_2 = f(x_1)$ be the function describing it. Therefore, as x_1 varies, the bundle $(x_1, x_2) = (x_1, f(x_1))$ traces out the indifference curve through x.

Assumption 1.2: Consumer Preferences:

The consumer's preference relation $\succeq$ is complete, transitive, continuous, strictly monotonic, and strictly convex on $\mathbb{R}^n_+$. Therefore, by theorems 1.1 and 1.3 it can be represented by a real–valued utility function u that is continuous, strictly increasing, and strictly quasiconcave on $\mathbb{R}^n_+$.

The consumer's problem

The consumer's **budget set** is $B = \{x | x \in \mathbb{R}^n_+, p \cdot x \leq y\}$.

Formally, the consumer's utility–maximisation problem is written:

$$max\, u(x) \text{ s.t. } x \in B.$$

Note that if x^* solves this problem, then $u(x^*) \geq u(x)$ for all $x \in B$, which means that $x* \succeq x$ for all $x \in B$. The converse is also true.

The consumer's problem

The consumer's **Marshallian demand functions** $x_i^* = x_i(p, y)$, $i = 1, 2, \ldots, n$ are the solutions to the utility–maximization problem.

The consumer's problem

When $u(x)$ is differentiable, at the optimum consumption bundle, we have:

$$\frac{\partial u(x^*)/\partial x_j}{\partial u(x^*)/\partial x_k} = \frac{p_j}{p_k}.$$

So, at the optimum, the marginal rate of substitution between any two goods must be equal to the ratio of the good's prices.

Theorem 1.4: Sufficiency of Consumer's First-Order Conditions:

Suppose that $u(x)$ is continuous and quasiconcave on $\mathbb{R}^n_+$, and that $(p, y) \gg 0$. If u is differentiable at x^*, and $(x^*, \lambda^*) \gg 0$ solves (1.10), then x^* solves the consumer's maximisation problem at prices p and income y.