Microeconomics

Masters in Economics and Masters in Monetary and Financial Economics

Solution Topics to the Midterm Test

Maximum duration: 1h30 3rd of November of 2016

Question 1

A consumer has lexicographic preferences over R^2_+ if the relation $\succsim$ satisfies $(x_1, x_2) \succsim (y_1, y_2)$ whenever $x_1 > y_1$, or $x_1 = y_1$ and $x_2 \geq y_2$.

a. (2 marks) Sketch an indifference map for these preferences.

A: Each bundle is an indifference curve.

b. (3 marks) Can these preferences be represented by a utility function? Explain.

A: No. There is a result that says that if preferences are complete, transitive, and continuous, they can be represented by a continuous utility function. Since these preferences are not continuous, we cannot apply the result. However, this does not mean that there exists no utility function that represents these preferences. Still, given the definition of the preferences, each indifference curve is a single point. Therefore, each consumption vector must have a value, different from all others, i.e., no two points in R2 can have the same value. But, R2 has many more points than R from which the numerical values must be assigned. Concluding, there are not enough real numbers to assign to all consumption vectors, so that there exists no utility function that represents the preferences.

Question 2

A consumer has preferences over goods 1 and 2 represented by the utility function:

$$u(x_1, x_2) = x_1 + 2x_2.$$

Let p_1 be the price of good 1, let p_2 be the price of good 2, and let income be equal to y.

a. (3 marks) Derive the Marshallian demands for goods 1 and 2.

A: Goods 1 and 2 are perfect substitutes. Solving the utility maximization with non-negativity constraints, one obtains $x(p_1, p_2, m) = (m/p_1,0)$ if $2p_1 < p_2$; $(0, m/p_2)$ if $2p_1 > p_2$; (x_1, x_2) s. t. $p_1 x_1+p_2 x_2 = m$ if $2p_1 = p_2$.

b. (1 mark) Derive the indirect utility function.

A: $v(p_1, p_2, m) = m/ p_1$ if $2p_1 < p_2$; $v(p, m) = 2m/p_2$ if $2p_1 \geq p_2$

c. (1 mark) Decompose the effect of an increase in the price of good 1 on its demand into income and substitution effects. Explain.

A: Consider the effect of an increase in p_1 from p_1' to p_1'' when p_2 and m remain constant. There are 3 cases to consider:

(1) If p_1' is such that 2 $p_1' > p_2$, the change in the consumption of good 1 when the price of good 1 increases is null, so that both income and substitution effects are 0.

(2) If p_1' is such that 2 $p_1' \leq p_2$, and when the price increases, we have 2 $p_1'' > p_2$, the consumption of good 1 drops from m/p_1' to 0. The substitution effect is responsible for this change (and the income effect is 0).

(3) If p_1' is such that 2 $p_1' \leq p_2$, and when the price increases, we still have 2 $p_1'' \leq p_2$, the consumption of good 1 drops from m/p_1' to m/p_1''. In this case, the income effect is responsible for the change (and the substitution effect is 0).

Question 3

Suppose preferences are represented by the Cobb-Douglas utility function, $u(x_1, x_2) = Ax_1^{\alpha}x_2^{1-\alpha}$, $0 < \alpha < 1$, and $A > 0$. Let p_1 be the price of good 1, let p_2 be the price of good 2, and let income be equal to y.

a. (1 mark) Compute this consumer's marginal rate of substitution (MRS) and show that this consumer's preferences are strictly monotonic and strictly convex.

A: MRS= $-\frac{\alpha}{1-\alpha}\frac{x_2}{x_1}$. Given that the MRS is negative and strictly decreasing (in absolute value) in x_1, preference are strictly monotonic and strictly convex.

b. (3 marks) Derive the Hicksian (or compensated) demands for goods 1 and 2.

A: Solving the expenditure minimization problem, we obtain: $x^h_1(p_1, p_2, u) = \frac{u}{A}\alpha(\frac{p_2}{p_1})^{1-\alpha}(\frac{1}{\alpha})^{\alpha}(\frac{1}{1-\alpha})^{1-\alpha}$ and $x^h_2(p_1, p_2, u) = \frac{u}{A}(1-\alpha)\left(\frac{p_1}{p_2}\right)^{\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}$.

c. (1 mark) Determine the expenditure function.

A: $e(p_1, p_2, u) = \frac{u}{A}(\frac{p_1}{\alpha})^{\alpha}(\frac{p_2}{1-\alpha})^{1-\alpha}$.

Question 4

1. (2.5 marks) Comment on the following sentence: "If a good is a normal good, its demand curve must be downward sloping."

A: True. For a normal good, the substitution and income effects reinforce each other: in both cases, when price increases, quantity demanded decreases. It follows that demand must be downward sloping.

2. (2.5 marks) Assume a two-good world (n=2). Show that the Engel aggregation holds, i.e., show that $s_1\eta_1 + s_2\eta_2 = 1$, where $s_i \equiv \frac{p_i x_i(p_1,p_2,y)}{y}$ and $s_i \equiv \frac{\partial x_i(p_1,p_2,y)}{\partial y}\frac{y}{x_i(p_1,p_2,y)}$, *i* = 1, 2.

A: See Jehle and Reny (2011), p. 61.

Question 5

A risk-averse person has wealth y and faces a risk of loss $L < y$ with probability p. An insurance company offers cover of the loss at a premium $k > pL$. It is possible to take out partial cover on a pro-rata basis, so that an amount tL of the loss can be covered at cost tk where $0 < t < 1$.

a. (3 marks) Explain why the person will not choose full insurance.

A: With probability p, wealth is y-L+tL-tk; with probability 1-p, wealth is t-tk. Therefore, expected utility is: $E(u) = (1 - p)u(y - tk) + pu(y - tk- (1-t)L)$. Computing the derivative with respect to t, we obtain: $dE(u)/dt = -(1 - p)ku'(y - tk) + (L-k)pu(y-tk-(1-t)L)$. In the neighbourhood of t=1 (full insurance), we have: $dE(u)/dt|_{t=1} = (Lp - k)\, u'(y - k)$. We know that $u'(y - k) > 0$ and, by assumption, $Lp < k$. Therefore, this expression is negative, which means that in the neighbourhood of full insurance, the individual could increase expected utility by reducing the insurance cover.

b. (2 marks) Find the conditions that will determine t*, the optimal value of t.

A: For an interior maximum, we have $dE(u)/dt = 0$. Therefore, the optimal t is the solution to the equation: $-(1 - p)ku'(y - t^*k) + (L-k)pu(y-t^*k-(1-t^*)L) = 0$.