game theory

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introduction

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references

main

Dutta, P. *Strategies and Games*, The MIT Press, 1999

Gibbons, R., *A Primer in Game Theory*, Pearson Education, 1992

additional

Osborne, M. and A. Rubinstein, *A Course in Game Theory*, MIT Press, 1994

(see http://theory.economics.utoronto.ca/books/index.php/index)

assessment

final exam 60%

midterm test 30% **(April 8)**

assignments 10%

syllabus

1 Normal Form Games with Complete Information

1-1 Equilibrium in Dominant Strategies > D3, G1.1A

1-2 Iterated Elimination of Dominated Strategies > D4, G1.1B

1-3 Nash Equilibrium in Pure Strategies > D5, D6, G1.1C, G1.2

1-4 Nash Equilibrium in Mixed Strategies > D8, G1.3A

2 Extensive Form Games with Complete Information

2-1 Extensive Form Games and Backward Induction > D11, G2.1A

2-2 Subgame Perfect Nash Equilibrium > D13, G2.2A, G2.4

2-3 Infinitely Repeated Games > D15, G2.3B

Midterm on April 8

syllabus

3 Games with Incomplete Information

3-1 Bayes-Nash Equilibrium > D20, G3.1

3-2 Perfect Bayesian Equilibrium > G4.1

4 Other Topics (if time allows)

4-1 Uncertainty

4-2 Asymmetric information:

- 4-2-1 Moral Hazard > D19
- 4-2-2 Adverse Selection > D24, G4.2

4-3 Mechanism Design and Auctions > D22, D23

what is game theory?

DECISIONS vs. GAMES

decision

may have several stages and need a sequential plan of action (strategy), but environment is neutral (individual decision problem)

game

interaction with others, who are similarly strategically aware purposive players, whose interests may conflict with yours; need at least two "players" to make a game

games are not always win-lose; can be win-win (e.g.: international trade) or lose-lose (e.g.: wars, strikes)

what is game theory?

AN EXAMPLE

A group of six friends goes to a restaurant.

Each person pays his/her own meal ➔ a simple decision problem

Before the meal, every person agrees to split the bill evenly among them ➔ a game

what is game theory?

THE MOST FAMOUS EXAMPLE

Two persons are arrested for a crime, but there is not enough evidence to convict either of them. Police would like the accused to testify against each other. The prisoners are put in different cells, with no communication possibility. Each suspect is told that if he testifies against the other (“Defect”), he is released and given a reward provided the other does not testify (“Cooperate”). If neither testifies, both are released (with no reward). If both testify, then both go to prison, but still collect rewards for testifying.

	C	D
C	1, 1	-1, 2
D	2,-1	0, 0

what is game theory?

THE MOST FAMOUS EXAMPLE

Each prisoner is better off defecting regardless of what the other does. Cooperation is a **strictly dominated** action for each prisoner.

The only feasible outcome is (D,D), which is Pareto dominated by (C, C).

This is the Prisoners' Dilemma.

what is game theory?

USES OF GAME THEORY

1. Explanation of outcomes of past strategic interactions
2. Prediction of outcomes of future interactions
3. Advice to players involved in such interactions
4. Provides the grounds to the design of mechanisms

Success so far enough to be encouraging but not perfect science and art are both evolving

Stories of success:

1. Explaining the past: Thomas Schelling analyses the Cold War
2. Predicting the future: Thomas Schelling and John Nash at RAND; Chris Ferguson who applied GT to online poker
3. Advice: Nokia Siemens Networks learns how to negotiate
4. Mechanism design: Paul Milgrom (and others) design auctions to award 3th generation mobile radio pectrum

what is game theory?

NOBEL PRIZES

1994 John Harsanyi, John Nash ("A Beautiful Mind") and Reinhard Selten

2005 Robert Auman and Thomas Schelling

2007 Leonid Hurwicz, Eric Maskin (in ISEG on March 30, 2012), and Robert Myerson

2012 Alvin Roth and Lloyd Shapley

More at
http://www.nobelprize.org/nobel_prizes/economics/laureates/

what is game theory?

DIMENSIONS OF ANALYSIS OF GAMES

1. Moves: sequential (e.g: Stackelberg) or simultaneous (e.g.: Cournot)

   Different kinds of interactive thinking:

   Sequential: If I do this, the other will do that, then I ...

   Simultaneous: I think that he thinks that ...

   Different techniques: "trees" versus "spreadsheets"

2. Pure conflict or some common alignment of interests

   Pure conflict in some sports; more generally mixed

3. One-shot or repeated

   One-shot: actions more unscrupulous, less cooperative, information limited; secrecy valuable

   Repeated: can build up relationships and reputations

   can obtain and convey information

   can harness selfishness to achieve coop outcomes

what is game theory?

DIMENSIONS OF ANALYSIS OF GAMES

4. Information: complete or limited/asymmetric
   - Knowing other players' skills, motives problematic
   - Real game becomes that of obtaining, or conveying or concealing information

5. "Cooperative" or "non-cooperative"
   - Cooperative: actions agreed and jointly implemented
     - Problem is that of splitting a pie
   - Non-coop: actions taken separately by each player
     - Still, outcome can show cooperation if in private interest (for example in repeated interactions)

what is game theory?

DIMENSIONS OF ANALYSIS OF GAMES

4. Information: complete or limited/asymmetric
    - Knowing other players' skills, motives problematic
    - Real game becomes that of obtaining, or conveying or concealing information

5. Rules fixed or manipulable
    - If latter, then real game is that of manipulating the rules
    - These are "strategic moves" - threats, promises

6. "Cooperative" or "non-cooperative"
    - Cooperative: actions agreed and jointly implemented
        - Problem is that of splitting a pie
    - Non-coop: actions taken separately by each player
        - Still, outcome can show cooperation if in private interest (for example in repeated interactions)

what is game theory?

TERMINOLOGY

strategies

Choices available to each of the players, i.e., complete plans of action

payoffs

Numerical representation of the objectives of each player

Can take into account fairness, reputation, etc. (players are not necessarily selfish)

Probabilistic average when there is uncertainty

what is game theory?
STANDARD ASSUMPTIONS

rationality

Players can perfectly calculate and implement best strategy

common knowledge of rules

All players know the game (strategies, payoffs,...) being played

A knows that B knows that...

equilibrium

Players play strategies that are mutual best responses

Does not automatically mean "good"

what is game theory?

WEBSITES

A website for students and for geeks with lecture notes, demonstrations, applications,...
http://www.gametheory.net/

Online course at Stanford
http://www.game-theory-class.org/

Ariel Rubinstein's website (where you can freely download a book!) http://arielrubinstein.tau.ac.il/

David Levine's webpage http://dklevine.com/

Alvin Roth's webpage and market design blog
http://kuznets.fas.harvard.edu/~aroth/alroth.html
http://marketdesigner.blogspot.pt/

normal form games with complete information

part 1

roadmap

definition of a normal form game

dominated and dominant strategies

equilibrium in strictly dominant strategies

references

- sec 1.1.A and1.1.B of Gibbons
- ch of 3 and 4 of Dutta
- sec 2.1-2.5, 2.9.1and 2.9.2 of Osborne

prisoners' dilemma

	C	D
C	1, 1	-1, 2
D	2,-1	0, 0

battle of sexes

At their **separate** workplaces, Chris and Pat must choose to attend either opera or a prize fight in the evening

Both Chris and Pat know the following:

- Both would like to spend the evening together
- But Chris prefers the opera and Pat prefers the prize fight

	opera	prize fight
opera	2, 1	0, 0
prize fight	0, 0	1, 2

matching pennies

Each of the two players has a penny

They both have to simultaneously choose whether to show head or tail

Both players know the following:

- If the two pennies match, then player 2 wins player 1's penny
- Otherwise, player 1 wins player 2's penny

	head	tail
head	-1, 1	1, -1
tail	1, -1	-1, 1

definition of a (normal form) game

set of players {1, 2, 3, ..., n}

set of (pure) strategies /actions for each player i, S_i (i=1,...,n)

payoff function for each player i: $u_i(s_1,s_{2,\dots}\ s_n)$ (i = 1, ..., n) where

$$u_i: S_1 \times S_2 \times \dots \times S_n \longrightarrow R$$

prisoners' dilemma

	C	D
C	1, 1	-1, 2
D	2,-1	0, 0

battle of sexes

	opera	prize fight
opera	2, 1	0, 0
prize fight	0, 0	1, 2

matching pennies

	heads	tails
heads	-1, 1	1, -1
tails	1, -1	-1, 1

more on a (normal form) game

simultaneous-move

Each player chooses his/her strategy without knowledge of others' choices

complete information

Each player's strategies and payoff function are common knowledge among all the players

assumptions on the players

Rationality

- Players aim to maximize their payoffs
- Players are perfect calculators

Each player knows that other players are rational

And knows the others know he is rational

Cournot duopoly

A product is produced by only two firms: firm 1 and firm 2. The quantities are denoted by q_1 and q_2, respectively. Each firm chooses the quantity without knowing what the other firm has chosen

The market price is $P(Q) = a - Q$, where $Q = q_1+q_2$

The cost to firm i of producing quantity q_i is $C_i(q_i) = cq_i$

strictly dominant and dominated strategies (strong definition)

s_i is a **strictly dominant strategy** for player i or strictly dominates all other strategies for player i

if, for all $s_i' \in S_i$ such that $s_i' \neq s_i$, and for all $s_{-i} \in S_{-i}$,

$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

(and s_i' is a **strictly dominated strategy** for player i) example:

prisoners' dilemma

	C	D
C	1, 1	-1, 2
D	2,-1	0, 0

dominant and dominated strategies (weak definition)

s_i is a **(weakly) dominant strategy** for player i or weakly dominates all other strategies for player i if,

for all $s_i' \in S_i$ such that $s_i' \neq s_i$,

for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$

and for some $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

(and s_i' is a **(weakly) dominated strategy** for i) example:

(weakly) dominant strategy

1 \ 2	L	R
U	7,3	5,3
D	7,0	3,-1

equilibrium in dominant strategies

dominant strategy solution

a game has a (strictly) dominant strategy solution if every player has a (strictly) dominant strategy

$(s_1,\ldots,s_n)$ is an equilibrium in (strictly) dominant strategies if for all $i=1,\ldots,n$, s_i is a (strictly) dominant strategy