dynamic games with complete information

part 1

roadmap

extensive form games

backwards induction

- backwards induction with perfect information
- vs. Nash equilibria
- vs. IEDS

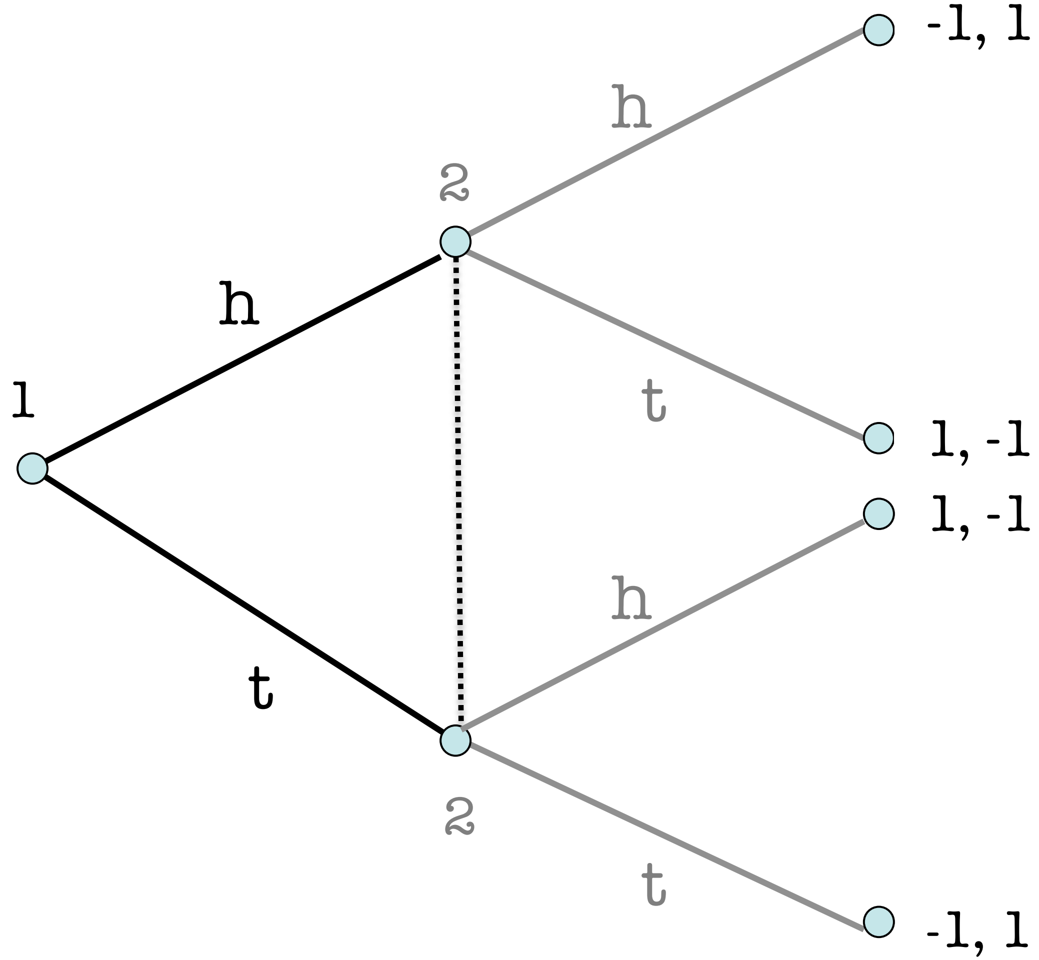
references

sec. 2.1 and 2.4 of Gibbons

Ch 2.2 and 11 of Dutta

extensive form games

example: (sequential-move) matching pennies



extensive form games

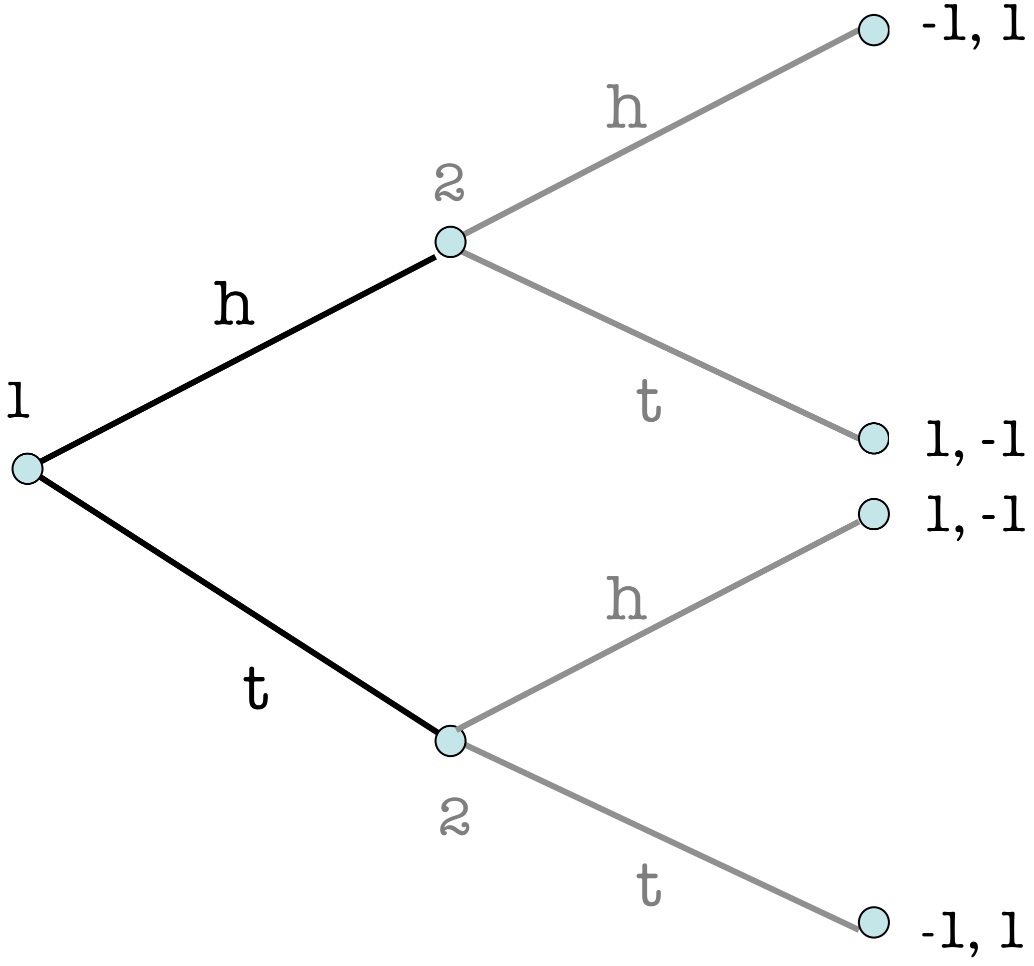
definition of a strategy

a **strategy** is a complete, conditional plan of actions

- **conditional** because it tells each player which branch to follow if arriving at an information set
- **complete** because it tells her what to choose at every information set

extensive form games

example: sequential-move matching pennies



extensive form games

strategies in sequential matching pennies

2 strategies for player 1: {h, t}

2^2 strategies for player 2: {hh, ht, th, tt}

extensive form games

mixed strategies

mixed strategies have same definition:
probability distribution over pure strategies!

in the sequential matching pennies, mixed strategies for player 1 are given by a number σ_1, whereas mixed strategies for player 2 are given by 3 numbers σ_2, α_2, β_2

example: normal form

sequential-move matching pennies

2 \ 1	H	T
HH	1,-1	-1, 1
HT	1,-1	1,-1
TH	-1, 1	-1, 1
TT	-1, 1	1,-1

extensive form games

NE

the set of NE in a dynamic game of complete information is the set of NE of its normal form

extensive form games

NE

So, to find NE of an extensive-form game,

1. First, write down its normal-form
2. Second, compute its NE

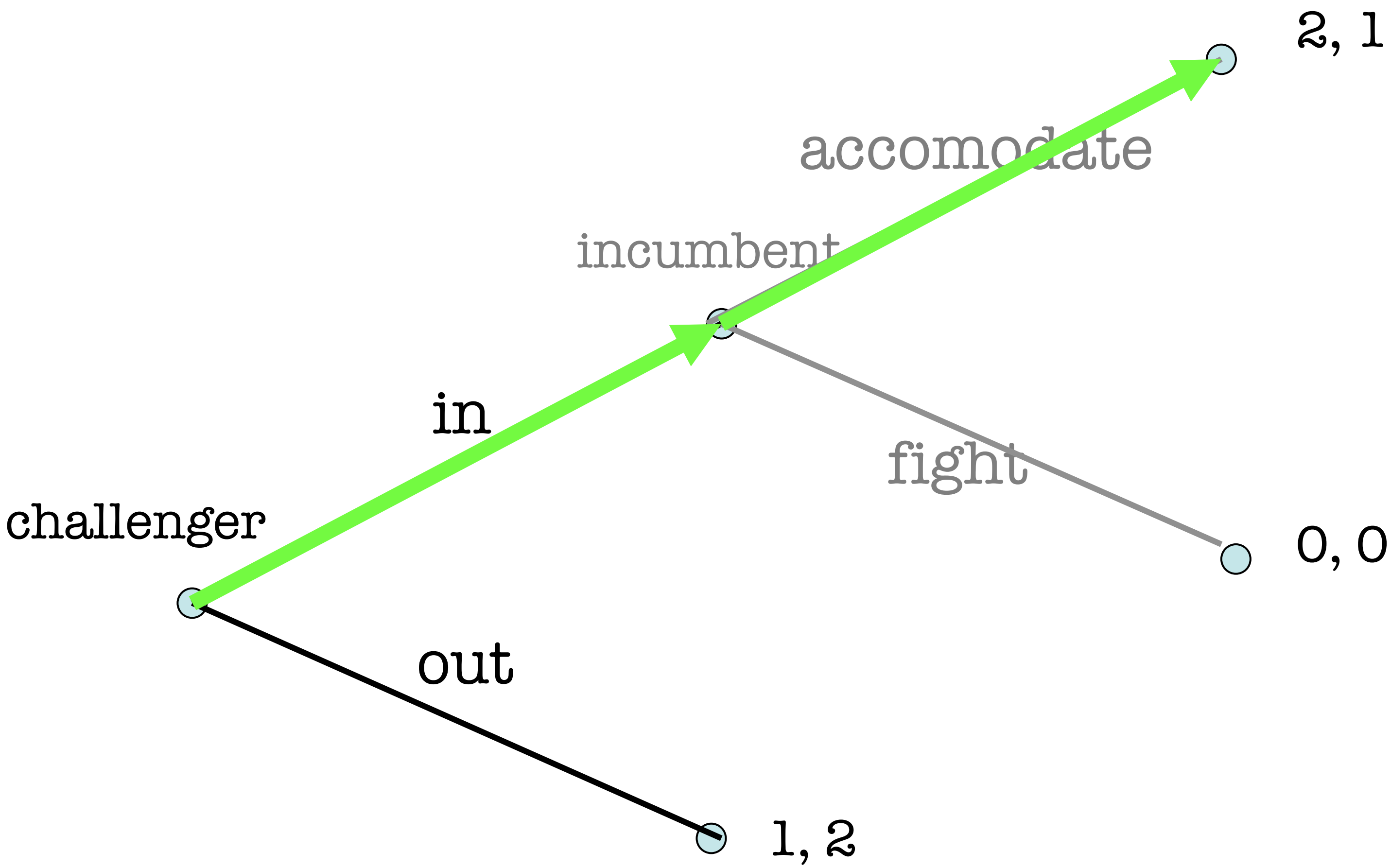
example:
entry game

two NE: (in, accommodate), (out, fight)

challenger \ incumbent	accommodate	fight
in	2, 1	0, 0
out	1, 2	1, 2

does the second NE make sense?

backward induction and NE



backward induction: sequential rationality rules out unreasonable NE or non-credible threats

extensive form games

backward induction

Kuhn's (and Zermelo's) Theorem: every game of perfect information with a finite number of nodes has a solution to backward induction.

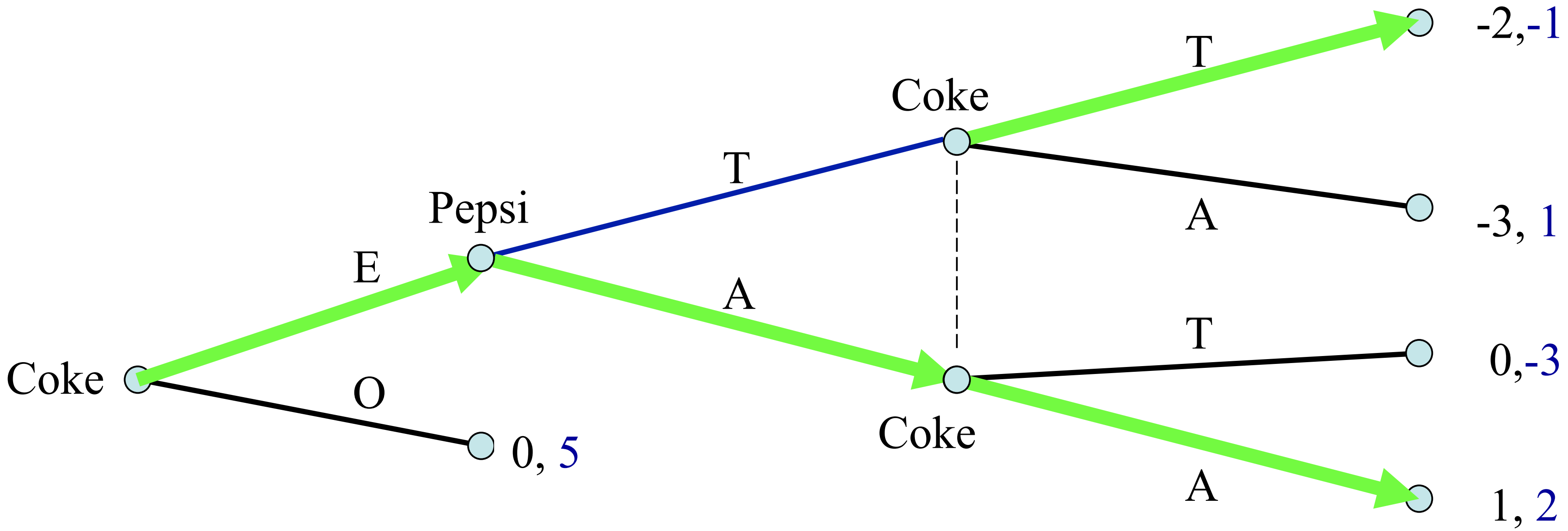
extensive form games

backward induction and IEDS

backward induction (in the extensive form) is the same as solving the game by IEDS (in the strategic form)

backward induction and IEDS

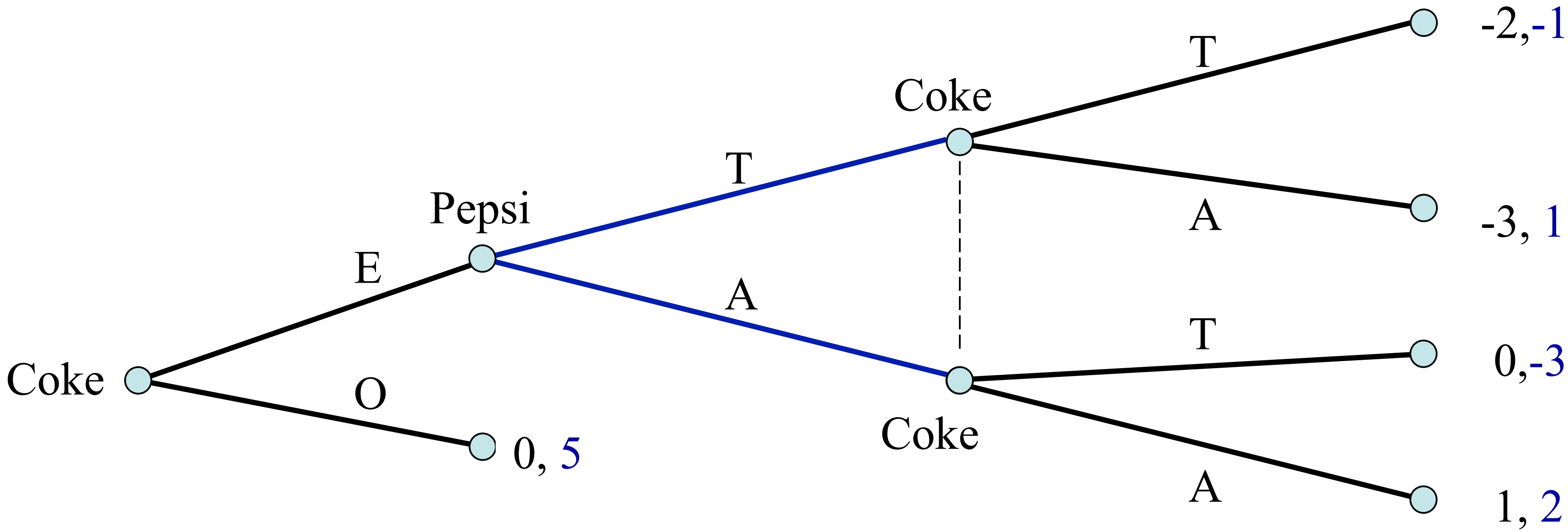
example:



backward induction and IEDS

Coke \ Pepsi	T	A
ETT	-2,-1	0,-3
ETA	-2,-1	1,2
EAT	-3,1	0,-3
EAA	-3,1	1,2
OTT	0,5	0,5
OTA	0,5	0,5
OAT	0,5	0,5
OAA	0,5	0,5

SPNE: example
dynamic game without perfect information



SPNE: example

dynamic game without perfect information

Post-entry payoffs

Coke \ Pepsi	T	A
T	-2,-1	0,-3
A	-3,1	1,2

SPNE: example

dynamic game without perfect information

Nash equilibria

Coke \ Pepsi	T	A
ET	-2,-1	0,-3
EA	-3,1	1,2
OT	0,5	0,5
OA	0,5	0,5

Not credible!

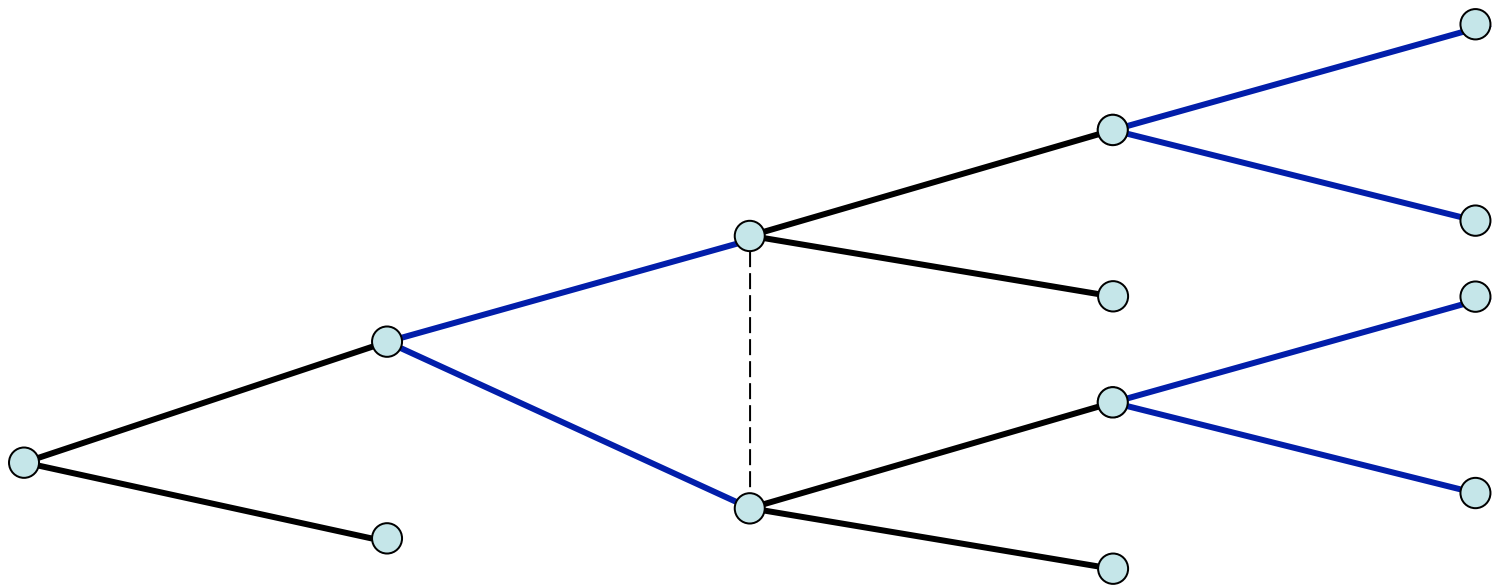
SPNE
definition of a subgame

a **subgame** is a collection of nodes and branches such that

1. it starts at a single decision node
2. it contains every successor to this node
3. if it contains any part of an information set, then it contains all nodes in that information set

SPNE

subgames: examples and counterexamples



SPNE
definition

a subgame perfect Nash equilibrium is a vector of strategies that, when confined to any subgame of the original game, have the players playing a Nash equilibrium within that subgame.

in a game of perfect information, the SPNE coincides with the backward induction solution

(and every finite dynamic game of complete and perfect information has a SPNE)

SPNE

example: sequential bargaining

player 1 and 2 bargain over 1 USD; timing is as follows:

per. 1: player 1 proposes to take a share s1, leaving 1 - s1 to player 2; player 2 accepts or rejects (in which case play continues to per. 2)

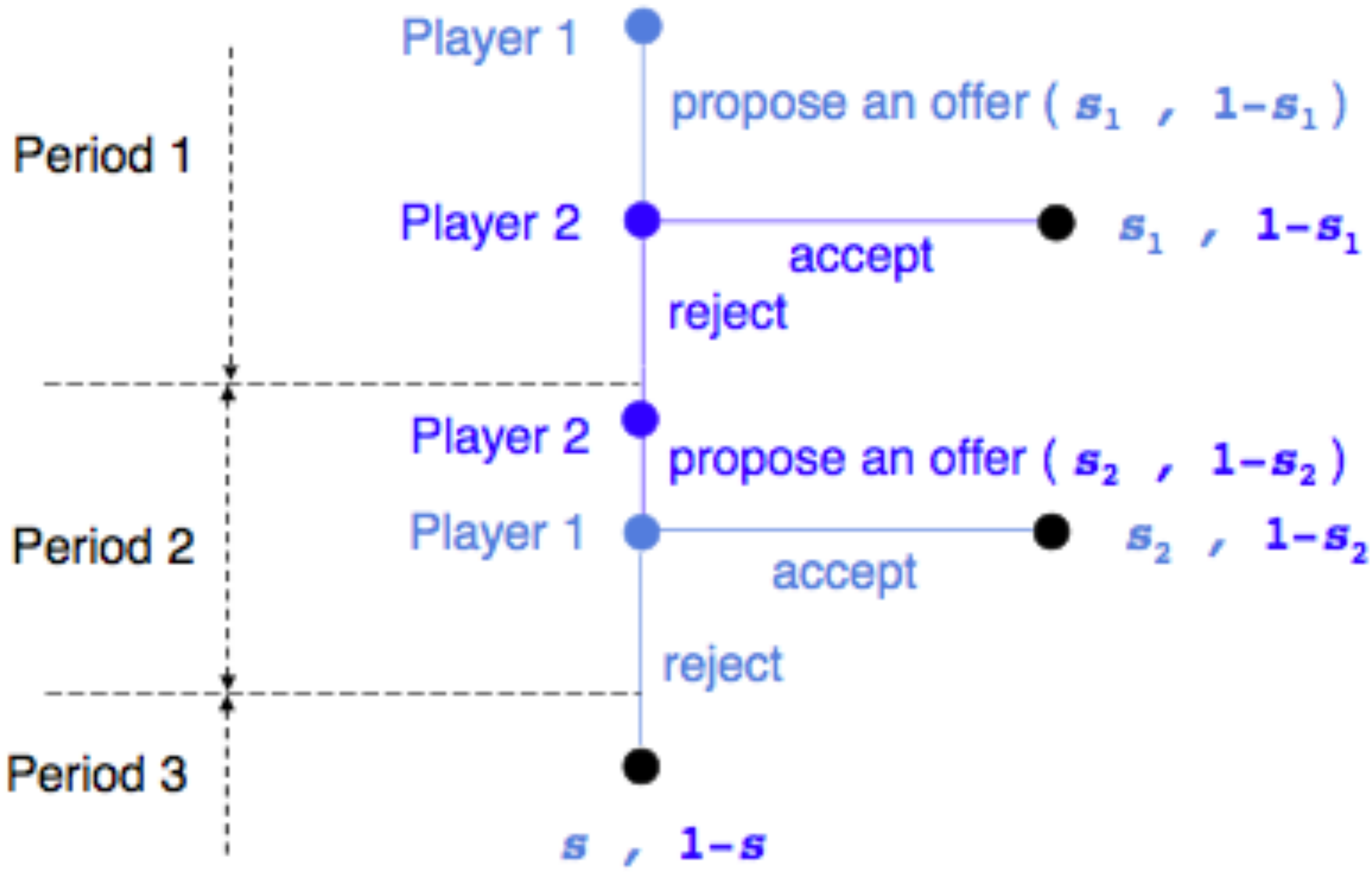
per. 2: player 2 proposes that player 1 takes a share s2, leaving 1 - s2 to player 2; player 1 accepts or rejects (in which case play continues to per. 3)

per. 3: player 1 receives s and leaves 1-s to player 2, $0 < s < 1$

players discount payoffs by a factor t, $0 < t < 1$

SPNE

example: sequential bargaining



SPNE

example: solving sequential bargaining

per. 2:

player 1 accepts s2 iff s2 ≥ ts;

player 2 faces the following:

1. offers s2 = ts or
2. offers s2 < ts (to be rejected) and receives 1 –s next period (with discounted value t(1-s))

since t(1-s) < 1-ts, player 2 should propose an offer (s2*, 1-s2*), where s2*= ts (to be accepted by player 1)

SPNE

example: solving sequential bargaining

per. 1:

player 2 accepts 1 – s1 iff 1 – s1 ≥ t(1 – s2*) = t(1- ts) or s1 ≤ 1- t(1 – ts)

player 1 faces the following:

1. Offers 1 – s1 = t(1- ts) to player 2, leaving 1- t + tts for herself or
2. Offers 1- s1 < t(1-s2*) (to be rejected) and receives s2* = ts next period, with discounted value tts

since tts < 1- t + tts, player 1 should propose (s1*,1-s1*), where s1* = 1- t + tts