static games with incomplete information

part 1

roadmap

Bayes-Nash equilibrium (BNE)

- examples
- Harsanyi's proposal
- definition

references

Chap. 20 of Dutta

example:

prisoners' dilemma – 2 is tough

1 \ 2 type 1	Not to confess	Confess
Not to confess	-1,-1	-6,<u>0</u>
Confess	<u>0</u>,-6	<u>-3</u>,<u>-3</u>

example:
prisoners' dilemma – 2 is accommodating

1 \ 2 (type 2)	Not to confess	Confess
Not to confess	-1,1	-6,-2
Confess	0,-4	-3,-5

example:

coordination game: 2 is matched

1 \ 2 type 1	Book launch	Movie
Book launch	2,1	0,0
Movie	0,0	1,2

example:

coordination game: 2 is mismatched

1 \ 2 (type 2)	Book launch	Movie
Book launch	2,0	0,1
Movie	0,2	1,0

static games with incomplete information

definition

a **game of incomplete information** is one in which players do not know some relevant characteristic of their opponents, which may include their payoffs, their available options, and even their beliefs

static games with incomplete information

examples

prisoners' dilemma

player 1 always plays Confess

player 2, type tough plays Confess

player 2, type accommodating plays Not Confess

coordination game

player 1 plays mixed strategy λ

player 2, type matched plays mixed strategy μ_1

player 2, type mismatched plays mixed strategy μ_2

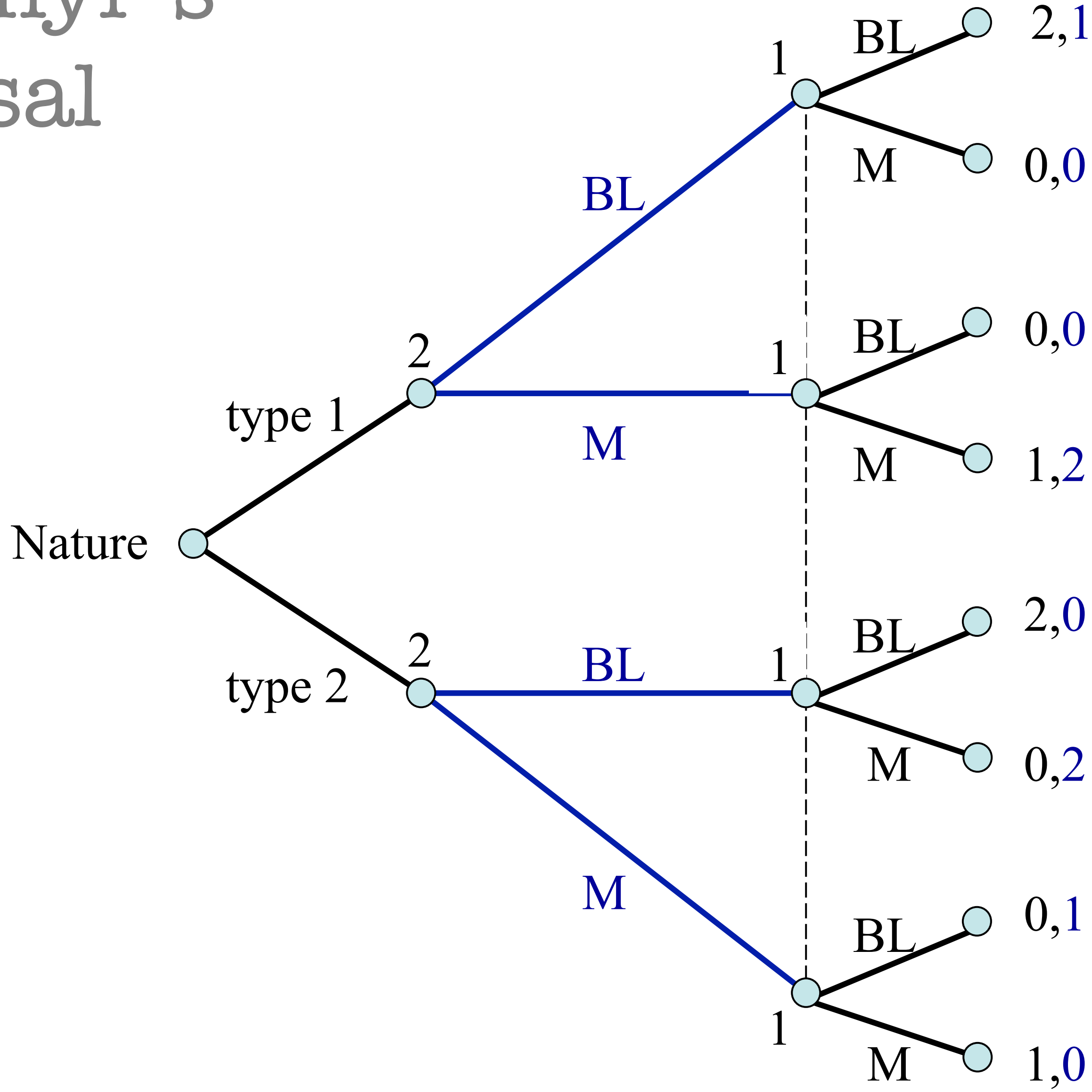
static games with incomplete information

common priors

assumption of a common prior:

probabilities of types must become part of the game and are known by all players

Harsanyi's proposal



static games with incomplete information

Bayes-Nash equilibrium

Harsanyi's proposal

- turn it into a game of complete but imperfect information
- use Nash equilibrium as the solution concept

a Bayes-Nash equilibrium of the game is a triple (λ, μ_1, μ_2) in which each player – and each player type - plays a best response, as follows:

(1) μ_i maximizes the type i of player 2's payoff when λ is 1's strategy;

(2) λ maximizes player 1's payoff when the type i of player 2 is playing μ_i and the probabilities of types 1 and 2 are respectively p and 1-p

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Bayes-Nash equilibrium in the coordination game

two pure-strategy Bayes-Nash equilibria whenever $p \geq 2/3$:
1. player 1 plays BL while player 2 plays (BL,M)
2. player 1 plays M and player 2 plays (M,BL)

if $2/3 > p \geq 1/3$, there is only one pure-strategy Bayes-Nash equilibrium, namely the first one

if $p < 1/3$, there is no pure-strategy Bayes-Nash equilibrium

there is always a mixed-strategy Bayes-Nash equilibrium:
$\lambda = 2/3$, $\mu_1 = 1/3$, $\mu_2 = 1/3$

there may be additional Bayes-Nash equilibria, characterized by $\lambda = 2/3$ and $p(3\mu_1 - 1) = (1-p)(1-3\mu_2)$

example

Cournot duopoly with incomplete information

A homogeneous product is produced by only two firms, 1 and 2. The quantities are denoted by $q1$ and $q2$

They choose the quantities simultaneously

The market price is given by $P(Q) = a - Q$, where $Q = q1 + q2$

Firm 1's cost function: $C1(q1) = c.q1$

All the above is common knowledge

example

Cournot duopoly with incomplete information

Firm 2's cost depends on some factor (e.g. technology) that only 2 knows. Its costs can be:

- High: $C2(q2) = cHq2$
- Low: $C2(q2) = cLq2$

Before production, firm 2 can observe the factor and know its marginal cost; however, firm 1 cannot observe firm 2's cost. She believes 2's cost function is

- High: $C2(q2) = cHq2$ with probability p
- Low: $C2(q2) = cLq2$ with probability 1-p

example

Cournot duopoly with incomplete information

The BNE is $(q^*_2(c_H), q^*_2(c_L), q^*_1)$ such that:

$$q^*_2(c_H) = \frac{1}{3}(a - 2c_H + c) + \frac{1-p}{6}(c_H - c_L)$$

$$q^*_2(c_L) = \frac{1}{3}(a - 2c_L + c) + \frac{p}{6}(c_H - c_L)$$

$$q^*_1 = \frac{a - 2c + pc_H + (1-p)c_L}{3}$$

example:
coordination game

1 \ 2	Book launch	Movie
Book launch	$2+t_1$,1	0,0
Movie	0,0	1,$2+t_2$

example: coordination game - BNE

t_i follows U [0,x]

Bayes-Nash equilibrium:
player 1 plays BL if t_1 above c and M otherwise
player 2 plays M if t_2 above p and BL otherwise

player 1 plays BL with probability (x-c)/x
player 2 plays M with probability (x-p)/x

but $\lim_{x\to 0} (x-c)/x = \lim_{x\to 0} (x-p)/x = 2/3$