

**15 Jan 2009**

1.
  - a)  $\dim M = 2, \dim N = 2, \dim M \cap N = 1$
  - b)  $p = (x, y, z), T_p(M \cap N) = \text{span}\{(x/y(y+v), v, 1)\}, v = -(2x^2 - 1)y/(2x^2 - 2y^2), T_p(M \cap N)^\perp = \text{span}\{(-1/x, 1/y, 1), (2x, -2y, -1)\}$
2.
  - a)  $(0, 1/2, 0)$
  - b)  $\pi^{1/2}$
  - c) mais perto:  $(2 - 2a, 3 - 3a, 4, 4a)$ , mais longe:  $(2 + 2a, 3 + 3a, 4 + 4a)$ ;  
 $a = 29^{-1/2}$
3.
  - a)  $78\pi$
  - b)  $0$
4. não é aditiva
5.
  - a)  $2$

**30 Jan 2009**

- 1.
- b)  $\phi(\varphi) = (\cos \varphi, \cos \varphi, \sqrt{2} \sin \varphi), \varphi \in ]-\pi/2, \pi/2[$
2.
  - a)  $\sqrt{3}(e^{2\pi} - 1)$
  - b)  $e^{2\pi-1}$
3.  $x = y = z = 1$
4.
  - a)  $0$
  - b)  $0$
5.  $\{\emptyset, \Omega, A_0, A_1, A_0^c, A_1^c, A_0 \cup A_1, (A_0 \cup A_1)^c\}, \mu(B) = \#B/\#\Omega$
6.
  - a)  $0$
  - b)  $2 + f(0)$

**6 Jan 2010**

1.
  - a)  $\dim M = 3$
  - b)  $T_p M = \{(x, y, z, w) : x + w = 0\}, T_p M^\perp = \{(x, 0, 0, w) : x - w = 0\}$
2.
  - a) mais perto  $(2/3, 0, 2/3)$ , mais longe  $(2, 0, -2)$
  - b)  $(0, 1/2, 0)$
  - c)  $1/2$
  - d)  $(5 - \cos 2)/8$
3.
  - a)  $\{\emptyset, \Omega, A_0, A_1, A_0 \cup A_1, A_0^c, A_1^c, (A_0 \cup A_1)^c\}$

**27 Jan 2010**

1.
  - a)  $\theta \neq 0, \dim M_\theta = 2$
  - b)  $T_p M = \text{span}\{(1, 0, 0, -1), (0, \theta, 1, 0)\}, T_p M^\perp = \text{span}\{(0, -1, \theta, 0), (1, 0, 0, 1)\}$
2.
  - b)  $1/(e^\pi - 1) + \pi^3/24 - 1$
- 3.

a)  $]0, 1[ \times ] - 1, 0[ \times ] 0, 2[$

c)  $\pi/2$

4.

a)  $0, m(E)$ , decrescente

b) sse  $m(f^{-1}(a)) = 0$

**5 Jan 2011**

2.

a)  $2\pi(1 - 1/e)$

b)  $45/56$

c)  $2\pi$

3.  $(1 - e^{-1})/16$

4.  $\pi\alpha^2/6$

5.

b)  $1/2$

**26 Jan 2011**

1.

a)  $(0, 3/8, 0)$

b)  $\sqrt{\pi}$

c)  $(-1, 0, 0)$

2.  $\sqrt[4]{2}$

3.  $0$

4.

b)  $e^{-\lambda}(1 + 2\lambda + 3\lambda^2/2)$

c)  $(1 - e^{-\lambda})/\lambda$

6. integrável

**9 Jan 2012**

2.

a)  $T_p M = \text{span}\{(1, 1, \sqrt{2}), (1, -1, 0)\}$ ,  $T_p M^\perp = \text{span}\{(1, 1, -\sqrt{2})\}$

b)  $\sqrt{2}/3$

3.

a)  $2R/3$

b)  $x^2 + y^2 = 1/2, z = 1/2$

c)  $\frac{3}{8} \frac{R^2 - r^2}{R^{3/2} - r^{3/2}}$

4.

a)  $3$

b)  $1 + 1/4 + 1/9$

**25 Jan 2012**

2.

a)  $3R/4$

b)  $(0, 3/4, 0)$

c)  $\sqrt{\pi}$

3.

a)  $(2x, 2y, -1)/\sqrt{4z + 1}$

b)  $0$

4.

a)  $55$

b)  $10$

**11 Jan 2013**

- 1.
- b)  $2\pi^2$
- 2.
- b)  $\frac{\pi}{2} \frac{e-1}{e^2}$
- 3.
- a)  $3R/4$
- b)  $(1, 1, 1, 0)$
- 4.
- a)  $\dim=2$
- b)  $(0, 0, \pm 1)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$  e  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 1\}$ ,  $\pm(x, y, 0)$  em  $S$ ,  $(0, \pm 1, 0)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$
- c)  $\pi/2$

**31 Jan 2013**

- 1.
- a)  $\gamma(t) = (\sin(t), \sin(2t), 0)$ ,  $t \in [0, 2\pi]$ ,  $\Gamma = \gamma([0, 2\pi])$  não é uma variedade
- b)  $p = \gamma(\pi/2) = (1, 0, 0)$ ,  $T_p\Gamma = \text{span}\{(0, 1, 0)\}$ ,  $T_p\Gamma^\perp = \text{span}\{(1, 0, 0), (0, 0, 1)\}$
- 2.
- b)  $(\pi/2)^3$
- 3.
- a)  $(3/2, 3/2)$
- b)  $2\sqrt{R}/3$
- 4.
- a)  $\dim=2$
- b)  $(0, 0, \pm 1)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$  e  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 1/2\}$ ,  $(0, \pm 1, 0)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$ ,  $\pm(x, y, 1-z)/[\sqrt{2}(1-z)]$  em  $S$
- c)  $\pi/4$

**13 Jan 2014**

- 1.
- a)  $\dim=1$
- b)  $T_{(1,1)}M^\perp = \text{span}\{(1, 0)\}$ ,  $T_{(1,1)}M = \text{span}\{(0, 1)\}$
- 2.
- a)  $(1 - \sqrt{30}/30)(1, 2, 3, 4)$
- b)  $\sqrt{\pi}$
- 3.
- a) A componente com  $x > 0$
- b) 1
- c) sim
- d)  $3/16$

**27 Jan 2014**

- 1.
- a)  $\dim=2$
- b)  $T_{(1,1,1)}H^\perp = \text{span}\{(1, 1, -1)\}$ ,  $T_{(1,1,1)}H = \text{span}\{(1, 0, 1), (0, 1, 1)\}$
- c)  $(\sqrt{10}/5, 2\sqrt{10}/5, 1)$
- 2.
- a)  $1/2$
- b)  $(0, 0)$
- 3.

a) Em  $\{(x_1, x_2, x_3) \in \mathbb{R}^3: x_1 = \pm 1, \max_{i=2,3} |x_i| \leq 1\}$ ,  $\nu(x) = (\pm 1, 0, 0)$ ;  
 em  $\{(x_1, x_2, x_3) \in \mathbb{R}^3: x_2 = \pm 1, \max_{i=1,3} |x_i| \leq 1\}$ ,  $\nu(x) = (0, \pm 1, 0)$ ; em  
 $\{(x_1, x_2, x_3) \in \mathbb{R}^3: x_3 = \pm 1, \max_{i=1,2} |x_i| \leq 1\}$ ,  $\nu(x) = (0, 0, \pm 1)$

b) 0

4.

b) 0

5.  $2^{\#\Omega}$

**12 Jan 2015**

1.

a)  $\dim=2$ ,  $T_{(x,y,z)}M^\perp = \text{span}\{(x, y, z)\}$ ,  $T_{(x,y,z)}M = \text{span}\{(-y, x, 0), (-z, 0, x)\}$

b)  $\max_M f = \frac{1}{4}$ ,  $\min_M f = \frac{1}{12}$

c)  $3\pi/5$

2.

a)  $\phi^{-1}(u, v) = (\sqrt{uv}, \log \sqrt{\frac{u}{v}})$

c)  $\pi/2$

3.

a)  $\{\emptyset, \mathbb{R}, \{\alpha\}, \{\alpha\}^c\}$

b)  $\{\emptyset, \mathbb{R}\}$

4.

a)  $\mu(D)$

**26 Jan 2015**

1.

a)  $\text{div} \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ ,  $\nu(x, y, z) = (x, y, z)$

b)  $2\pi$

c)  $(1, 0)$  min local,  $(-1, 0)$  min global,  $(1/2, \pm\sqrt{3}/2)$  max global

2.

a)  $\phi^{-1}(u, v) = (\sqrt{uv}, \log \sqrt{\frac{u}{v}})$

c)  $1/\pi$

3.

b) 0

4.  $\mu = \sum_{n=1}^{10} \delta_{1/n}$

**15 Jan 2016**

1.

a)  $\dim M = 2$ ,  $T_{(x,y,z)}M = \text{span}\{(z, 0, x), (0, z, y)\}$ ,  $T_{(x,y,z)}M^\perp = \text{span}\{(x, y, -z)\}$

b) Não existem

c) 0

2.

a)  $(5 - \cos 2)/8$

b)  $\sqrt{\pi}$

3.

a)  $\sqrt{2}\pi e^{-\pi^2/4}$

b) 0

4.

b) 0

5. 2

**2 Fev 2016**

1.

a)  $S: (x, y, 0)$ ;  $V_1: (0, 0, 1)$ ;  $V_2: (0, 0, -1)$

- b) 0  
 c)  $S$ : não existem;  $C$ :  $(\sqrt{2}/2, \sqrt{2}/2, 0)$  max,  $(-\sqrt{2}/2, -\sqrt{2}/2, 0)$  min  
 d)  $1/2$   
 e)  $S$ :  $1/2$ ;  $C$ : 0

2.  
 a)  $\sqrt{2}\pi$   
 b)  $e - 1/e$   
 3. 0  
 4. constante  
 5. 0

**9 Jan 2017**

1.  
 a)  $\dim M_2 = 1$ ,  $T_{(x,y)}M_2 = \text{span}\{(-4y, x)\}$   
 b) Para  $(0, 1)$

$$\phi(t) = \begin{cases} (2 \cos(t/2 + \pi/4), \sin(t/2 + \pi/4)), & 0 < t \leq \pi/2 \\ (\cos t, \sin t), & \pi/2 < t < 3\pi/4 \end{cases}$$

Mesma ideia para  $(0, -1)$

- c)  $\dim M = 1$ ,

$$T_{(x,y)}M = \begin{cases} T_{(x,y)}M_2, & (x, y) \in M_2 \\ \text{span}\{(-y, x)\}, & (x, y) \in M_1 \end{cases}$$

- d)  $(4\sqrt{5}/5, \sqrt{5}/5)$

- e)  $3\pi/2$

2.

- a)  $1/2 + \sin^2 1$

- b)  $\sqrt{\pi}$

3.

- a)  $3/2$

- b)  $\log 2$

- c) 1

**31 Jan 2017**

1.

- a) 1-variedade

- b)  $2(x, y)$

- c) 0

- d)  $\pi$

2.  $4\pi$

3.

- a)  $(37/6\pi, 37/6\pi, 0)$

- b) 0

4.

- a)  $\mathcal{X}_B$

- b)  $\{\emptyset, \Omega, A \cup B, (A \cup B)^c\}$

5.  $6\pi$

**8 Jan 2018**

1.

- a)  $\dim M_2 = 1$ ,  $T_{(x,y)}M_2 = \text{span}\{(yb^2, -xa^2)\}$

b) Para  $(0, a)$

$$\phi(t) = \begin{cases} (b \cos(at/b + \pi/2(1 - a/b)), a \sin(t)), & 0 < t \leq \pi/2 \\ a(\cos t, \sin t), & \pi/2 < t < \pi \end{cases}$$

Mesma ideia para  $(0, -a)$

c)  $\dim M = 1$ ,

$$T_{(x,y)}M = \begin{cases} T_{(x,y)}M_2, & (x, y) \in M_2 \\ \text{span}\{(-y, x)\}, & (x, y) \in M_1 \end{cases}$$

d) 4

e)  $3\pi/2$

2.

a) (1,2,3)

b)  $(\sin^2(1) + 1)/2$

c)  $\sqrt{\pi}$

d)  $\sum_{n \geq 1} 1/n^2$

**30 Jan 2018**

1.

a) Não

b) Não

2.

a)  $(0, 0, 4/(3\pi))$ ,  $\text{vol} = 3\pi^2$

b) 0

3.

a) Não

b) 2

4.

a)  $\sqrt{\pi}$

b)  $-\sum_{n=1}^{+\infty} n^{-2} = -\pi^2/6$

5.

a)  $2x + 10y = 1$

b)  $\varphi(1/4, 1/20) = 1/80$