LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Degree:	2nd	**Field of Study:**	Master in Actuarial Sciences				
Code:	PPE	**Course name:**	Probability and Stochastic Processes			**Credits ECTS:**	8
Scientific field:	Statistics and Actuarial Science			**Department:**	Mathematics		
Curricular year:	1st	**Semester:**	1st: X / 2nd:	**Type:**	Obligatory: X / Elective:		
Responsible lecturer:	Alexandra Bugalho Moura						

Contact hours:	Lectures	Practicals	Lectures/Practicals	Total	Total workload
			58.5	58.5	224

Aims and scope

- It is our aim with this Curricular Unit that students acquire the necessary background to precede to the study of other more advanced stochastic phenomena that arise in insurance business, in more advanced Curricular Units.
- The first part of the Curricular Unit is intended to introduce important concepts of probability distributions and their characteristics. In addition to a more advanced study of topics already taught in the first cycle, new concepts are introduced, with actuarial science applications, as is the case of measures for evaluating the tails of the distributions.
- In the second part some of the most relevant stochastic processes used for modelling actuarial phenomena are introduced.

Summary

- Distributions and basic distributional quantities: random variable, distribution and survival functions, multivariate random variables, moments, quantiles, generating functions, sums of random variables, residual life, censored random variables, limited random variables, tails of distributions
- Characteristics of actuarial models: the role of the parameters, the exponential and the linear exponential family
- Continuous models: creating new distributions; identification of some distributions; extreme value distributions
- Introduction to copulas
- General notions of stochastic processes and their classification
- Discrete time Markov chains
- Continuous time homogeneous Markov chains
- Time Inhomogeneous Markov Chains
- Actuarial Applications

Main bibliography

- Klugman, S.A.; Panjer, H.H. & Willmot, G.E. (2008), *Loss Models, From Data to Decisions*, (3rd edition), John Wiley & Sons.
- Core Reading 2011, CT4 Models, The Actuarial Profession.
- Dickson, D., Hardy, M., and Waters, H., (2009) Actuarial Mathematics for Life Contingent Risks, Cambridge University Press.
- Ross. S. M. (2010), *Introduction to Probability Models*, (Tenth Edition), Academic Press, New York.
- Ross. S. M. (1996), *Stochastic Processes*, 2nd ed. John Wiley & Sons, New York.
- Taylor, H. M. & Karlin, S. (1998), *An Introduction to Stochastic Modelling*, (3rd edition), Academic Press, New York.

Teaching and assessment methodologies

Sessions are of a theoretical-practical nature, based on oral presentations, accompanied by the projection of slides containing the main results, which will be derived, explained and exemplified.

Students must solve the recommended exercises, as assigned homework, so that proposed solutions may be discussed in the class. The final grade, on the scale of 0 to 20, is assigned on the basis of a written exam.

COURSE CONTENTS

1. DISTRIBUTIONS AND BASIC DISTRIBUTIONAL QUANTITIES

1.1. Overview of some concepts: Random variable; distribution function; continuous, discrete and mixed random variables; decomposition of a distribution function.
1.2. Some well-known discrete random variables: uniform on a finite set, Bernoulli, binomial, Poisson, geometric, negative binomial and hypergeometric.
1.3. Some well-known continuous random variables: uniform, normal, lognormal, Pareto, exponential, gamma, chi-square, t, F and beta.
1.4. Multivariate random variables; independent random variables.
1.5. Moments and related quantities.
1.6. Residual life; left censored and shifted random variable; limit loss variable.
1.7. Moment generating function, probability generating function and cumulant generating function.
1.8. Sum of independent random variables; central limit theorem.
1.9. Tails of distributions: comparison of the tail based on moments, on the limiting tail behaviour, on the hazard rate function and on the mean excess loss function; the equilibrium distribution and the tail behaviour.

2. CHARACTERISTICS OF ACTUARIAL MODELS

2.1. Parametric and scale distributions: scale distribution and scale parameter; location and shape parameters; the exponential family and the linear exponential family and the overdispersed linear exponential family.
2.2. Mixed distributions: discrete and continuous mixtures.

3. SEVERITY MODELS (CONTINUOUS MODELS)

3.1. Creating new distributions: sums of distributions; transformation of random variables; mixing of distributions; frailty models; spliced distributions.
3.2. Recognition of families of distributions and their relations.
3.3. Extreme value distributions: distribution of the maximum; stability of the maximum of the extreme value distribution; generalised Pareto distribution.

4. INTRODUCTION TO COPULAS

4.1. Introduction
4.2. Skalar's theorem and copulas
4.3. Measures of dependency
4.4. Tail dependence
4.5. Archimedean copulas
4.6. Elliptical copulas
4.7. Extreme value copulas

5. GENERAL NOTIONS OF STOCHASTIC PROCESSES

5.1. Some definitions.
5.2. Specification of a stochastic process.
5.3. Classification of a stochastic process.

6. DISCRETE TIME MARKOV CHAINS

6.1. Definitions.
6.2. Transition probability matrices.
6.3. First step analysis.
6.4. Classification of states.
6.5. Limit behaviour.
6.6. Applications to no claim discount and *bonus-malus* systems.

7. CONTINUOUS TIME HOMOGENEOUS MARKOV CHAINS

7.1. Introduction: time homogeneous Markov process; Chapman-Kolmogorov equations.
7.2. The transition probability matrix.
7.3. The forward and backward differential equations.
7.4. The embedded Markov chain.
7.5. Stationary and limiting distributions.

8. TIME INHOMOGENEOUS MARKOV CHAINS

8.1. Introduction; Chapman-Kolmogorov equations.
8.2. Kolmogorov's forward differential equations.
8.3. Probabilities of remaining in states for given time periods.
8.4. Kolmogorov's backward differential equations.
8.5. Applications in insurance.