

## Soluções da ficha de exercícios 7

1. a)  $z = -i \vee z = \frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \vee z = -\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2}$

b)  $z = \pm 2 \vee z = \pm 2i \vee z = \frac{1}{2} \pm i \frac{\sqrt{3}}{2} \vee z = -1$

4. a)  $u(x, y) = \frac{(x+2)(x-1) + y^2}{(x-1)^2 + y^2} \quad v(x, y) = \frac{y(x-1) - y(x+2)}{(x-1)^2 + y^2}$

b)  $u(x, y) = 4x + 3y \quad v(x, y) = 3x + 4y + 4$

5. a)  $z = \ln \sqrt{2} + i \frac{\pi}{4} + 2k\pi i \quad k \in \mathbb{Z}$

b)  $z = (2k+1)\pi i \quad k \in \mathbb{Z}$

c)  $z = 2k\pi - \ln(2 \pm \sqrt{3})i \quad k \in \mathbb{Z}$

d)  $z = -k\pi + k\pi i \quad k \in \mathbb{Z}$

6. a)  $-6 + 9i$    b)  $\frac{7}{2} + \frac{3}{2}i$    c)  $-2$    d)  $\frac{1}{2}$

8. a), b) Não

10.  $z_0 = -1 - i$  e  $f'(z_0) = -2 - 2i$

11.  $f(z) = \left(1 + \frac{i}{2}\right)z^2 + ic \quad c \in \mathfrak{R}$  e  $f'(z) = (2+i)z$

13. 
$$\begin{cases} \frac{\partial u}{\partial \rho} = \frac{1}{\rho} \frac{\partial v}{\partial \theta} \\ \frac{\partial u}{\partial \theta} = -\rho \frac{\partial v}{\partial \rho} \end{cases}$$

14. a)  $\mathbb{C}$    b)  $\mathbb{C} \setminus \{0\}$

15.  $u(x, y) = 3xy^2 - x^3 - 3x^2y + y^3$

$k = 0, 1, 2$

16.  $f_0 = ic \quad \text{com } c \in \mathfrak{R}$   
 $f_1 = x - y + i(x + y + c) \quad \text{com } c \in \mathfrak{R}$   
 $f_2 = x^2 - y^2 + i(2xy + c) \quad \text{com } c \in \mathfrak{R}$

17.  $v(x, y) = 2xy - 3y + c$  com  $c \in \mathfrak{R}$

18.  $f(x + iy) = (-3x^2y + y^3) - i(3xy^2 - x^3 - 2)$  ou  $f(z) = (z^3 + 2)i$

19. b)  $v(x, y) = e^{-x}(x \cos y + y \sin y) + c$  com  $c \in \mathfrak{R}$

c)  $f(z) = iz e^{-z} + ic$   $f'(2 + i) = (1 - i)e^{-2-i}$