Weeks 1 and 2: Chap. 1 – Vectors

1 Direct applications

1.1. Consider the vectors of $\mathbb{R}^2$: $\vec{u} = (1, 2)$ and $\vec{v} = (-1, 1)$. Sketch them in the plane and determine geometrically: a) $\vec{u} + \vec{v}$ b) $\vec{u} - \vec{v}$ c) $-\vec{u} + 3\vec{v}$ d) $||\vec{u}||$ e) $d(\vec{u}, \vec{v})$.

1.2. Solve analitically the previous exercise and compare the results.

1.3. Consider the vectors of $\mathbb{R}^3$: $\vec{u} = (a, 1 + a, 2a), \vec{v} = (1, 1, 3)$ and $\vec{w} = (2, 1, 0)$. Determine the value of $a \in \mathbb{R}$ so that the vector $\vec{u}$ is a linear combination of $\vec{v}$ and $\vec{w}$.

1.4. The vectors $\vec{u} = (-1, -1, -a, -1)$ and $\vec{v} = (a + 2, a, a, a)$, with $a \in \mathbb{R}$, are orthogonal if and only if:

a) $a = 2$ b) $a = 0$ c) $a = -2$ or $a = -1$ d) $a = 1$.

1.5. Compute the distance $d(\vec{u}, \vec{v})$ for the vectors in exercise 1.4 (with $a \in \mathbb{R}$).

2 Definitions and proofs

2.1. Let $\vec{u}, \vec{v} \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Show the following properties of the inner product:
a) $\vec{u}.\vec{v} = \vec{v}.\vec{u}$
b) $(\lambda\vec{u}).\vec{v} = \vec{u}.(\lambda\vec{v}) = \lambda(\vec{u}.\vec{v})$

2.2. Find the distance $d(\vec{u}, \vec{v})$ between the vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$.

2.3. Proof that $||\vec{u}|| > 0$ for any $\vec{u} \in \mathbb{R}^n \backslash \{\vec{0}\}$.

2.4. Define linear combination of vectors.

2.5. Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ be linearly independent. Show that $\vec{u}+\vec{v}$, $\vec{u}+\vec{w}$ and $\vec{v}+\vec{w}$ are also linearly independent.

3 Problems and modelling

3.1. Assume the following economic data, without units:

Country	Productivity	Competition	Economic growth
Portugal	3	2	-1
Canada	8	5	0
Thayland	1	1	-3

a) Which country is closer to Portugal in all three indices?
b) In this model, the portuguese data depends linearly on the others?

3.2. Assume the following grades and weights:

Course	Weight	João's grades	Leonor's grades
Mathematics	3/10	?	15
Accounting	3/10	18	12
Law	3/10	10	14
English	1/10	16	15

a) Compute the average grade of the above students using the inner product of vectors.
b) What is the grade that João needs to obtain in Maths so that he has the same average grade as Leonor?

4 Additional exercises

4.1. Book (K. Sydsaeter & P.J. Hammond, *Essential Mathematics for Economic Analysis*, Prentice Hall, 2008):
Section 15.7: 1 to 8;
Section 15.8: 1 to 6.

4.2. Let $\vec{u}, \vec{v} \in \mathbb{R}^n$. Show the triangular inequality: $||\vec{u} + \vec{v}\,|| \leq ||\vec{u}\,|| + ||\vec{v}\,||$.
(Hint: Decompose $||\vec{u} + \vec{v}\,||^2$ and use the Cauchy-Schwarz inequality.)