Weeks 2 e 3: Chap. 2 – Matrices

1 Direct applications

1.1. Book (K. Sydsaeter & P.J. Hammond, *Essential Mathematics for Economic Analysis*, Prentice Hall, 2008):

Section 15.2: 1, 2 and 4;
Section 15.3: 1, 3 and 4;
Secção 15.4: Exercícios 1, 2 e 4;
Secção 15.5: Exercícios 1 a 4.

1.2. Find an example of a diagonal matrix D and a vector $\vec{v}$ with the same dimension, and compute $D\vec{v}$.

1.3. Give an example of an upper triangular matrix S and find its transpose.

1.4. Give two vectors $\vec{u}$ and $\vec{v}$ with the same dimension, and a linear combination of them.

1.5. Determine the rank of the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 8 & 16 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & -2 \end{bmatrix}.$$

2 Definitions and proofs

2.1. Let A and B be matrices with dimension $k \times p$, and $\alpha, \beta \in \mathbb{R}$. Show that:
a) $A + B = B + A$ b) $(\alpha + \beta)A = \alpha A + \beta A$ c) $\alpha(A + B) = \alpha A + \alpha B$.

2.2. Let I be the identity matrix of dimension n and $k \in \mathbb{N}$. Prove that $I^k = I$.

2.3. Consider a matrix A with dimension $k \times p$, a matrix B with dimension $p \times \ell$ and $\lambda \in \mathbb{R}$. Prove that:
a) $(\lambda A)' = \lambda A'$ b) $(AB)' = B'A'$.

2.4. Let a matrix A with dimension $m \times n$. Show that if $n = 1$, $A'A = 0 \Rightarrow A = \mathbf{0}$.

2.5. Consider two commutable matrices A and B (i.e. $AB = BA$) and C a matrix such that $C = 3A^2 - 5A - I$, where I is the identity. Show that C and B commute.

3 Problems and modelling

3.1. Consider the matrix $R(\theta) = \begin{bmatrix} \cos\theta & \text{-}\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and the vector $\vec{e_x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.
a) Represent on the unit circle and find the values of $\sin\theta$ and $\cos\theta$ for the angles $\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2, \pi, 3\pi/2$.
b) Compute $R(\theta)\vec{e_x}$ and sketch the result, concluding that $R(\theta)$ represents the rotation of $\vec{e_x}$ by θ around the origin.
c) Verify that $[R(\theta)]^2 = R(2\theta)$ using the identities: $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$.
d) Interpret geometrically the previous result.

3.2. Three companies presented the following results (in million euros) in 2008:

Company	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	5	2	-1	2
2	2	8	0	5
3	1	3	-1	2

In 2009 the results were:

Company	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	2	7	3	5
2	4	4	6	6
3	-1	-1	-1	0

a) Determine, for each company, in each quarter, the changes between 2008 and 2009.
b) Determine, for each company, in each quarter, the average result of the two years.

3.3. Consider the set of vectors $\{(1,0,0),(0,1,0),(-4,2,-8)\}$.
a) Determine by the definition if it is a set of linearly independent vectors.
b) Define rank of a matrix and determine the previous result by studying the rank.

4 Additional exercises

4.1. Determine the rank of the matrices:

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 2 & 4 & 0 & -1 \\ 1 & -1 & 2 & 2 \end{bmatrix},\ B = \begin{bmatrix} 2 & 1 & 3 & 7 \\ -1 & 4 & 3 & 1 \\ 3 & 2 & 5 & 11 \end{bmatrix},\ C = \begin{bmatrix} 1 & -2 & -1 & 1 \\ 2 & 1 & 1 & 2 \\ -1 & 1 & -1 & -3 \\ -2 & -5 & -2 & 0 \end{bmatrix}.$$

4.2. Book (K. Sydsaeter & P.J. Hammond, *Essential Mathematics for Economic Analysis*, Prentice Hall, 2008):

15.2: 3;
15.3: 2, 5;
15.4: 3, 6, 7;
15.5: 5, 7.