Maths I

Week 11: Chap. 8 – Extremes and concavities

1 Direct applications

1.1. Book:
8.6: 4, 5;
8.7: 5, 6.

1.2. Let $f(x) = x^3 - 4x^2 + 4x + 12$.

a) Determine the stationary points of f.
b) Determine the extreme points of f using the second derivative.
c) Find out if the extreme points are local or global.

1.3. Let $f\colon I \to \mathbb{R}$ such that $f(x) = \sin(x^2)$, with $I = [-\sqrt{\pi}, \sqrt{\pi}]$.

a) Determine the stationary points of f.
b) Determine the extreme points of f using the second derivative.
c) Find out if the extreme points are local or global.

1.4. Let $f(x) = x^4$, $g(x) = -x^4$ and $h(x) = x^3$.

a) Determine the stationary points of each function.
b) Using the derivatives of order 2 or higher, determine if those points are minima, maxima or inflection points.
c) Determine the concavities of each function.

1.5. Is an inflection point always a stationary point?

2 Definitions and proofs

2.1. State the definition of increasing and decreasing functions.

2.2. State the definition of stationary point.

2.3. Let $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ have a second derivative continuous on I, and a an interior point of I.
a) State the definition of inflection point of f.
b) Prove that if a is an inflection point of f, then $f''(a) = 0$.

3 Problems and modelling

3.1. A faulty freezer operates between -3ºC and +2ºC, and it has an energy consumption that varies with the temperature t as: $t^3 + \frac{3}{2}t^2 - 6t + 10$.

a) Determine the temperatures for which the energy consumption is maximum and minimum.
b) Does the function energy consumption have an inflection point?

3.2. Let $f(x) = \begin{cases} (x+2)^2, & x < -1 \\ |x|, & -1 \leq x \leq +1 \\ e^{-x+1}, & x > +1 \end{cases}$.

a) What is the domain of f?
b) Discuss the continuity and differentiability of f in its domain.
c) Determine the stationaty points of f.
d) Determine the extreme points of f, indicating if local or global.
e) Determine the extreme points of f in $[-4, -1]$.

3.3. Consider $f(x) = x \sin x$.
a) Find the Taylor polynomial of second degree of f around 0.
b) The function f has a unique stationary point in $]-1, 1[$. Determine it.
c) Classify this stationary point using the second derivative.
d) Is there any extreme points of f in $]-1, 1[$?

4 Additional exercises

4.1. Let f be the function and I the interval in exercise 1.3. Show that f has at least two inflection points in I.

4.2. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $a \in \mathbb{R}$ such that $f'(a) = 0$ and $f''(a) < 0$. Prove that a is a local maximum of f.

4.3. Book:

8.6: 1, 3, 6;
8.7: 2 to 4.