

**15 Jan 2009**

1.
  - a)  $\dim M = 2, \dim N = 2, \dim M \cap N = 1$
  - b)  $p = (x, y, z), T_p(M \cap N) = \text{span}\{(x/y(y+v), v, 1)\}, v = -(2x^2 - 1)y/(2x^2 - 2y^2), T_p(M \cap N)^\perp = \text{span}\{(-1/x, 1/y, 1), (2x, -2y, -1)\}$
2.
  - a)  $(0, 1/2, 0)$
  - b)  $\pi^{1/2}$
  - c) mais perto:  $(2 - 2a, 3 - 3a, 4, 4a)$ , mais longe:  $(2 + 2a, 3 + 3a, 4 + 4a)$ ;
- $a = 29^{-1/2}$
3.
  - a)  $78\pi$
  - b)  $0$
4. não é aditiva
5.
  - a)  $2$

**30 Jan 2009**

- 1.
- b)  $\varphi(\theta) = (\cos \theta, \cos \theta, \sin \theta), \theta \in ] - \pi/2, \pi/2[$
2.
  - a)  $\sqrt{3}(e^{2\pi} - 1)$
  - b)  $e^{2\pi-1}$
3.  $x = y = z = 1$
4.
  - a)  $0$
  - b)  $0$
5.  $\{\emptyset, \Omega, A_0, A_1, A_0^c, A_1^c, A_0 \cup A_1, (A_0 \cup A_1)^c\}, \mu(B) = \#B/\#\Omega$
6.
  - a)  $0$
  - b)  $2 + f(0)$

**6 Jan 2010**

1.
  - a)  $\dim M = 3$
  - b)  $T_p M = \{(x, y, z, w) : x + w = 0\}, T_p M^\perp = \{(x, 0, 0, w) : x - w = 0\}$
2.
  - a) mais perto  $(2/3, 0, 2/3)$ , mais longe  $(2, 0, -2)$
  - b)  $(0, 1/2, 0)$
  - c)  $1/2$
  - d)  $(5 - \cos 2)/8$
3.
  - a)  $\{\emptyset, \Omega, A_0, A_1, A_0 \cup A_1, A_0^c, A_1^c, (A_0 \cup A_1)^c\}$

**27 Jan 2010**

1.
  - a)  $\theta \neq 0, \dim M_\theta = 2$
  - b)  $T_p M = \text{span}\{(1, 0, 0, -1), (0, \theta, 1, 0)\}, T_p M^\perp = \text{span}\{(0, -1, \theta, 0), (1, 0, 0, 1)\}$
2.
  - b)  $1/(e^\pi - 1) + \pi^3/24 - 1$
- 3.

a)  $]0, 1[ \times ] - 1, 0[ \times ] 0, 2[$

c)  $\pi/2$

4.

a) 0,  $m(E)$ , decrescente

b) sse  $m(f^{-1}(a)) = 0$

**5 Jan 2011**

2.

a)  $2\pi(1 - 1/e)$

b)  $45/56$

c)  $2\pi$

3.  $(1 - e^{-1})/16$

4.  $\pi\alpha^2/6$

5.

b)  $1/2$