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Physics Letters A 372 (2008) 429–434

PHYSICS LETTERS A

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The seismography of crashes in financial markets

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Received 28 January 2007; received in revised form 10 June 2007; accepted 25 July 2007

Available online 9 August 2007

Communicated by A.R. Bishop

Abstract

This Letter investigates the dynamics of stocks in the S&P500 for the last 33 years, considering the population of all companies present in the index for the whole period. Using a stochastic geometry technique and defining a robust index of the dynamics of the market structure, which is able to provide information about the intensity of the crises, the Letter proposes a seismographic classification of the crashes that occurred during the period. The index is used in order to investigate and to classify the impact of the thirteen crashes between July 1973 and March 2006 and to discuss the available evidence of change of structure after the *fin de siècle*.

JEL classification: C0; G1

Keywords: Financial markets; Stochastic geometry; Complexity; Market spaces; Market structures

1. Introduction

The nature of financial crashes has been intensely discussed for long. Recently some new methods are being used to analyze and describe the dynamics of changes in complex markets, based on different contributions from econophysics. This Letter recurs to such methods and the available information on the trajectories of returns of dominating firms for three decades in order to propose new measures and a classification of the intensity of the crises. As a measure for these perturbations is defined, our investigation follows previous papers comparing them to "economic earthquakes" [1–4] and, in particular, the suggestion that the histogram of price changes for any stock is "the analog of the Gutenberg–Richter histogram of earthquake magnitude" [1], developing a new strategy for the quantification and qualification of these extreme events. Inquiries into the statistical properties of this distribution suggest the existence of a hierarchical organization expressed as scale invariance over the history of the values of a control parameter [1].

The current investigation, dealing with historical dependence in the financial assets, emphasizes how the crises change the shape of the market and lead to dynamic modifications. The market does not return to the previous configuration. Consequently, this is a case of hysteresis. Although the term is commonly applied to physical systems, such as magnetic materials which do not return to the original form after an impulse, it may be extended by analogy to social systems such as the economy.[2] The large perturbations described as market collapses or crises provoke important changes in the behavior of the agents, modify the available information and therefore the strategies and actions of other firms and alter the trajectories of the economies.

Section 2 presents the method and measures, whereas Section 3 summarize the results. As results are computed using actual daily returns data which are compared to surrogate (time permuted) data, the electronic implementation of our approach would imply the development of methods for selecting, retriev-

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1 Supported by FCT Portugal project PDCT/EGE/60193/2004.

2 We thank one anonymous referee for this suggestion.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2007.07.079

ing and updating stock market data, accordingly. The predictive character of our method is to be explored in future work.

2. Method and measures

The stochastic geometry strategy is simply stated in the following terms [5]:

(1) Pick a representative set of N stocks and their historical data of returns over some time interval and, from the returns data, using an appropriate metric, compute the matrix of distances between the N stocks.
The problem is reduced to an embedding problem in which, given a set of distances between points, one asks what is the smallest manifold that contains the set. Given a graph G and an allowed distortion there are algorithmic techniques to map the graph vertices to a normed space in such a way that distances between the vertices of G match the distances between their geometric images, up to the allowed distortion. However, these techniques are not directly applicable to our problem because in the distances between assets, computed from their return fluctuations, there are systematic and unsystematic contributions. Therefore, to extract relevant information from the market, we have somehow to separate these two effects. The following stochastic geometry technique is used:
(2) From the matrix of distances compute the coordinates for the N stocks in an Euclidean space of dimension smaller than N and then apply the standard analysis of reduction of the coordinates to the center of mass and compute the eigenvectors of the inertial tensor.
(3) Apply the same technique to surrogate data, namely to data obtained by independent time permutation for each stock and compare these eigenvalues with those obtained in (2), in order to identify the directions for which the eigenvalues are significantly different as being the market characteristic dimensions.
In so doing, we are attempting to identify the empirically constructed variables that drive the market and the number of surviving eigenvalues is the effective dimension of this economic space.
(4) From the eigenvalues of order smaller than the number of characteristic dimensions, compute the difference between eigenvalues in (2) with those in (3). The normalized sum of those differences is the index S, which measures the evolution of the distortion effect in the shape of the market space.

For both surrogate and actual data, the sorted eigenvalues, from large to small, decrease with their order. In the surrogate case, the amount of decrease is linear in the order number, proving that the directions are being extracted from a spherical configuration. The display of a uniform and smooth decrease in the values of the sorted eigenvalues is characteristic of random cases and is also experimentally observed when the market space is built from historical data corresponding to a period of business as usual.

Considering the lack of uniformity among the market effective dimensions we are able to characterize the extent to which crashes act differently on specific directions, causing changes in the shape of the market space. Looking for relevant distortions in the shape of the S&P500 market space through the last 33 years, we found that amongst the highest values of the index are those computed for some important dates, as 19th October 1987, 27th October 1997 and 11th September 2001.

From the returns for each stock

$$r(k) = \log\bigl(p_t(k)\bigr) - \log\bigl(p_{t-1}(k)\bigr) \tag{1}$$

a normalized vector

$$\vec{\rho}(k) = \frac{\vec{r}(k) - \langle \vec{r}(k) \rangle}{\sqrt{n(\langle r^2(k) \rangle - \langle r(k) \rangle^2)}} \tag{2}$$

is defined, where n is the number of components (number of time labels) in the vector $\vec{\rho}$. With this vector one defines the distance between the stocks k and l by the Euclidean distance of the normalized vectors

$$d_{ij} = \sqrt{2(1 - C_{ij})} = \bigl\| \vec{\rho}(k) - \vec{\rho}(l) \bigr\| \tag{3}$$

as proposed in [6], with C_{ij} being the correlation coefficient of the returns $r(i)$, $r(j)$.

The fact that is a properly defined distance gives a meaning to geometric notions and geometric tools in the study of the market. Given that set of distances between points, the question now is reduced to an embedding problem: one asks what is the smallest manifold that contains the set. If the proportion of systematic information present in correlations between stocks is small, then the corresponding manifold will be a low-dimensional entity. The following stochastic geometry technique was used for this purpose.

2.1. *The stochastic geometry technique*

After the distances (d_{ij}) are calculated for the set of N stocks, they are embedded in R^D, where $D < n$, with coordinates $\vec{x}(k)$. The center of mass $\vec{R}$ is computed and coordinates reduced to the center of mass.

$$\vec{R} = \frac{\sum_k \vec{x}(k)}{k}, \tag{4}$$

$$\vec{y}(k) = \vec{x}(k) - \vec{R} \tag{5}$$

and the inertial tensor

$$T_{ij} = \sum_k \vec{y}_i(k)\vec{y}_j(k) \tag{6}$$

is diagonalized to obtain the set of normalized eigenvectors $\{\lambda_i, \vec{e}_i\}$. The eigenvectors $\vec{e}_i$ define the characteristic directions of the set of stocks. The characteristic directions correspond to the eigenvalues (λ_i) that are clearly different from those obtained from surrogate data. They define a reduced subspace of dimension f, which carries the systematic information related to the market correlation structure. In order to improve the decision criterion on how many eigenvalues are clearly different from those obtained from surrogate data, a normalized difference τ is computed:

$$\tau(i) = \lambda(i) + 1 - \lambda'(i) \tag{7}$$

and the significantly different eigenvalues are those to which $\tau(i) > 1/2$.

2.2. Index of the market structure

As market spaces can be described as low dimension objects, the geometric analysis is able to provide crucial information about their dynamics. In previous papers, we developed different applications of this technique, namely for the identification of periods of stasis and of mutation or crashes. Indeed, market spaces tend to contract during crises along their effective dimensions, but each crisis may act differently on specific dimensions. It is in order to capture that distortion, namely the lack of uniformity along the market effective dimensions, that we defined the market structure index, S [5]. Since the largest f eigenvalues define the effective dimensionality of the economic space, at time t, we compute S as:

$$S_t = \sum_{i=1}^{f} \frac{\lambda_t(i) - \lambda'_t(i)}{\lambda'_t(i)} = \sum_{i=1}^{f} \frac{\lambda_t(i)}{\lambda'_t(i)} - 1 \tag{8}$$

where $\lambda_t(1), \lambda_t(2), \ldots, \lambda_t(f)$ are the largest f eigenvalues of the market space and $\lambda'_t(1), \lambda'_t(2), \ldots, \lambda'_t(f)$ are the largest f eigenvalues obtained from surrogate data, namely from data obtained by independent time permutation of each stock. In computing S, at a given time t, both λ_t and λ'_t are obtained over the same time window and for the same set of stocks.

3. Results and discussion

Results were computed using actual daily returns data and comparing them to surrogate data that are generated by permuting each stock (one-day return data) randomly in time. As each stock is independently permuted, time correlations among stocks disappear while the resulting surrogate data preserve the mean and the variance that characterize actual data.

The set of actual data consists in 230 stocks present in S&P500 from July 1973 to March 2006, considering all the surviving firms for the whole period. Although we acknowledge that this population does not necessarily represent the behavior of the whole economy, we consider the information useful enough to provide information on trends of the dynamics of market, as it includes a large part of the winners after a long period of competition. The next section proposes a classification of these crashes according to the value of S and using an inspiration from seismography.

3.1. A seismographic classification

An approximate value of the index S for the 1987 Crash, the Black Monday (*BM*), is taken as the higher value of a scale from 8*BM* down to 1*BM*, or S from 40 to 5. Market perturbations measuring less than 1*BM* are not considered to qualify as market crashes. Although this does not suggest any comparability or similarity of causes between earthquakes and financial crises, this procedure for classification and quantification in seismography [7] suggests a way of describing perturbations in speculative markets, since both kinds of shocks tend to occur in all magnitudes and may be described according to power laws.

Table 1
Ranking of the crises according to S_{max} and *BM*

Ranking	Date	S_{max}	*BM*
1	October 1987	38.6	8
2	December 2000/January 2001	18.3	3.8
3	October 1989	10.6	2.2
4	March/April 2001	10.3	2.13
5	July 2003	9.53	2
6	October 1998	9.2	1.95
7	April 1999	9	1.9
8	October 1997	8.36	1.73
9	September 2001	7.95	1.65
10	April 2000	7.55	1.6
11	August 1982	7.5	1.55
12	October 1979	7.22	1.50
13	October 1978	6.60	1.37

3.2. The dynamics of crashes

The results are presented in Fig. 1, where the plot shows the daily values of S for the 33 years period and a time interval (moving window) of 16 days on a market space including 230 stocks.

The highest peaks are identified and correspond to the following crashes (chronological order):

1. October 1978, Iranian crisis;
2. September 1979, Iranian crisis and second oil shock;
3. August 1982, general recession;
4. October 1987, the Black Monday;
5. October 1989, a US crash;
6. October 1997, the Asian crash;
7. October 1998, the Russian crash;
8. April 1999, the Japan crash;
9. April 2000, Nasdaq;
10. December 2000/January 2001, crashes in Argentina and Turkey;
11. April 2001, the start of a world recession;
12. September 2001, the terrorist attack against New York;
13. July 2003, general recession.

Once the highest values of S are identified, S_{max} is defined as the higher value of S in each time interval. Table 1 shows the same 13 peaks and the corresponding values of S_{max} and *BM*. The chronological order was replaced and the events were ordered according to the values of S_{max} and *BM*.

Along the whole period (33 years), the accumulated value of S (*AS*) is computed and the successive cumulative histories are presented in Table 2, the curve being drawn in Fig. 2. In that figure, we measure the slope of the curve that better fits *AS* for

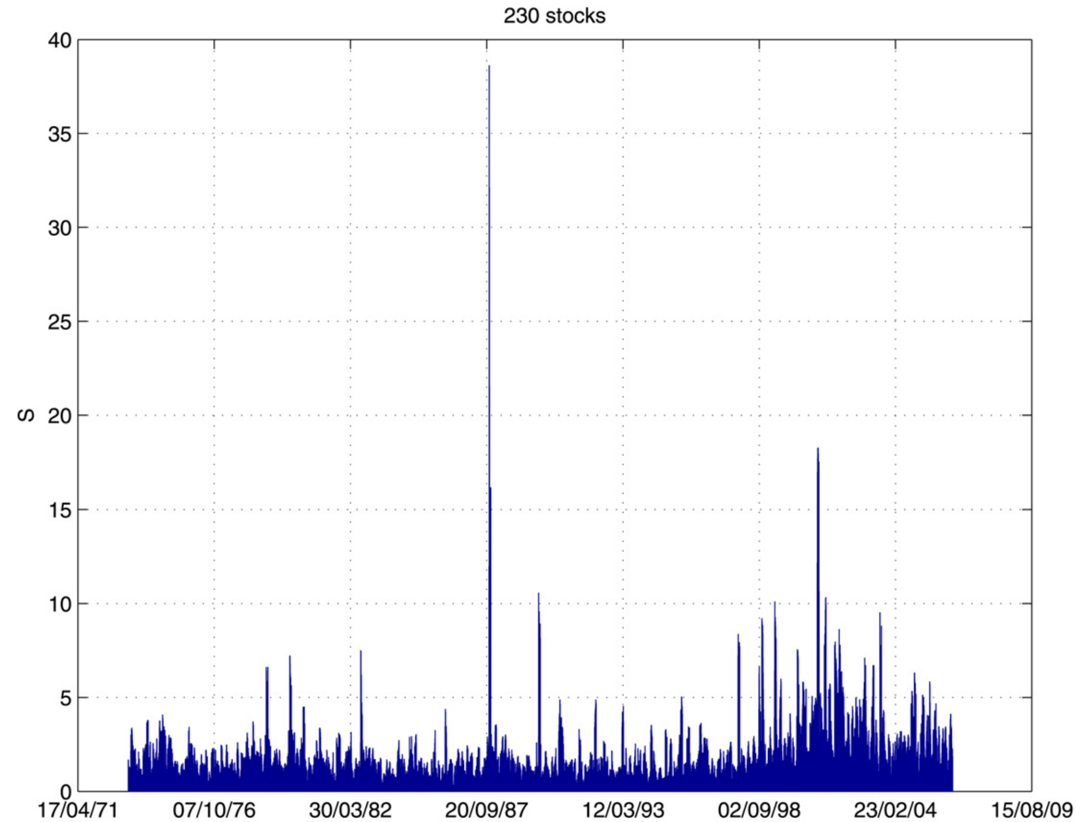

Fig. 1. The evolution of the index S measuring the evolution of the S&P500 structure for the surviving firms for 1973–2006.

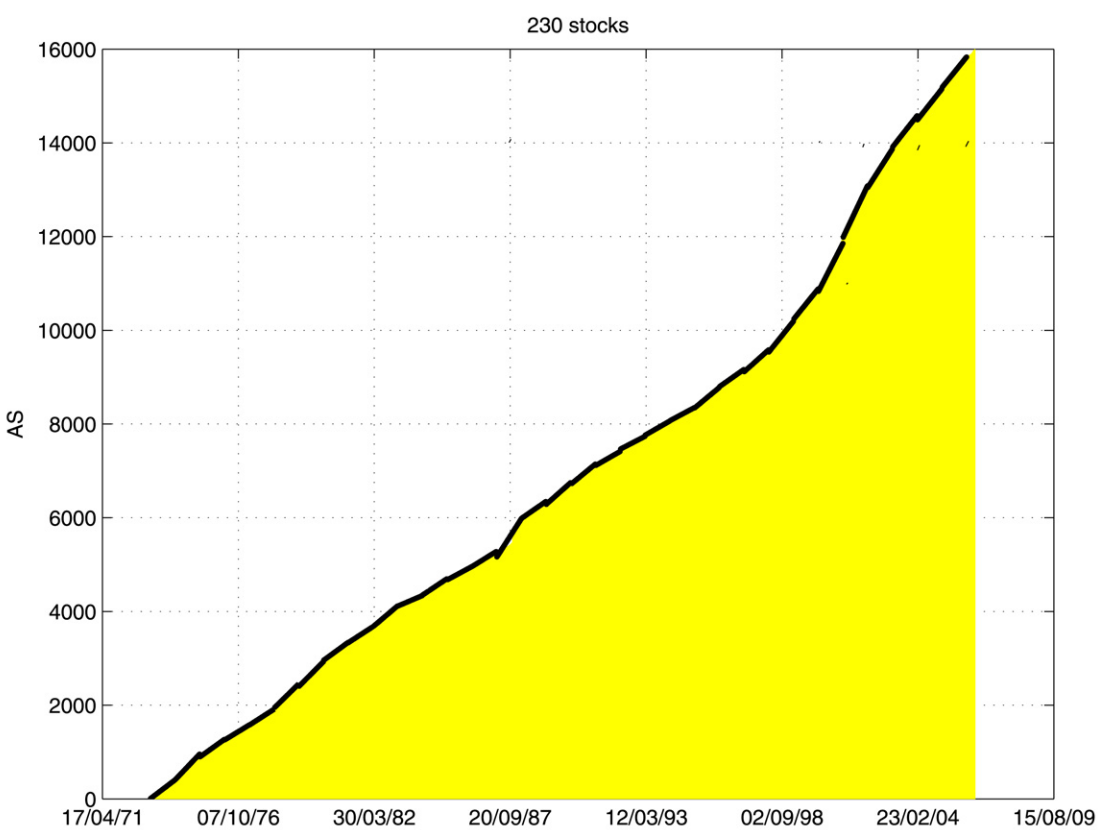

Fig. 2. Accumulated values of S and their yearly fit.

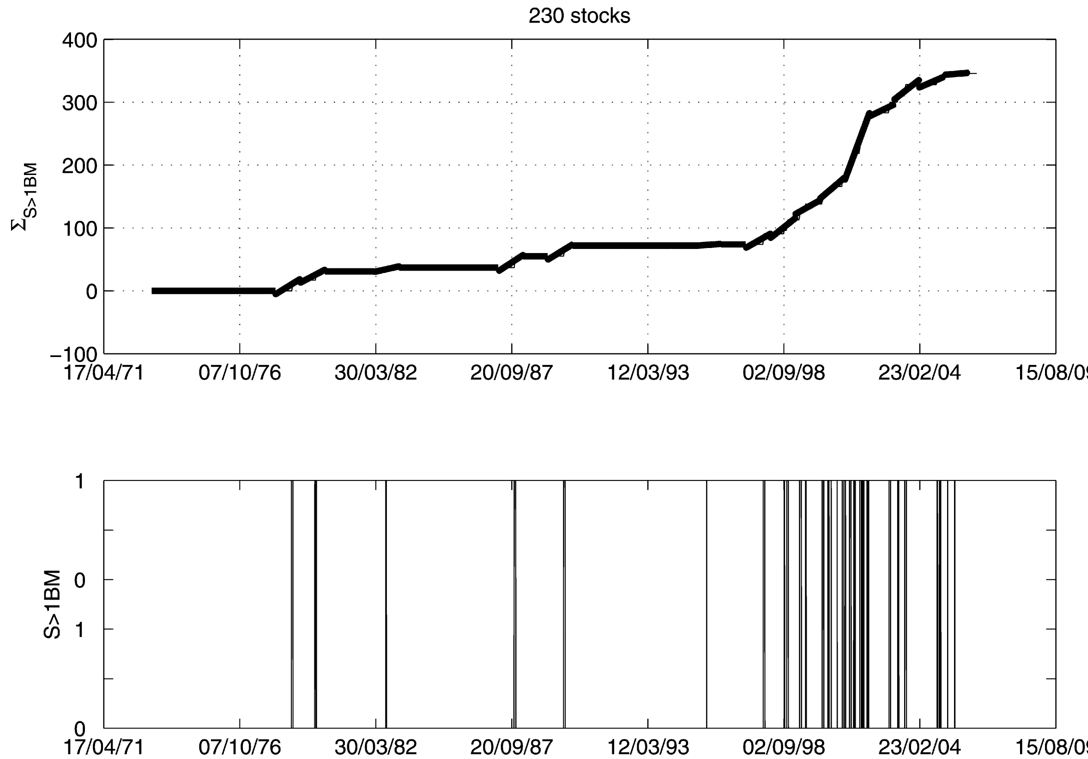

Fig. 3. Yearly fit of the slope of S (above) and concentration of crashes through the period (below).

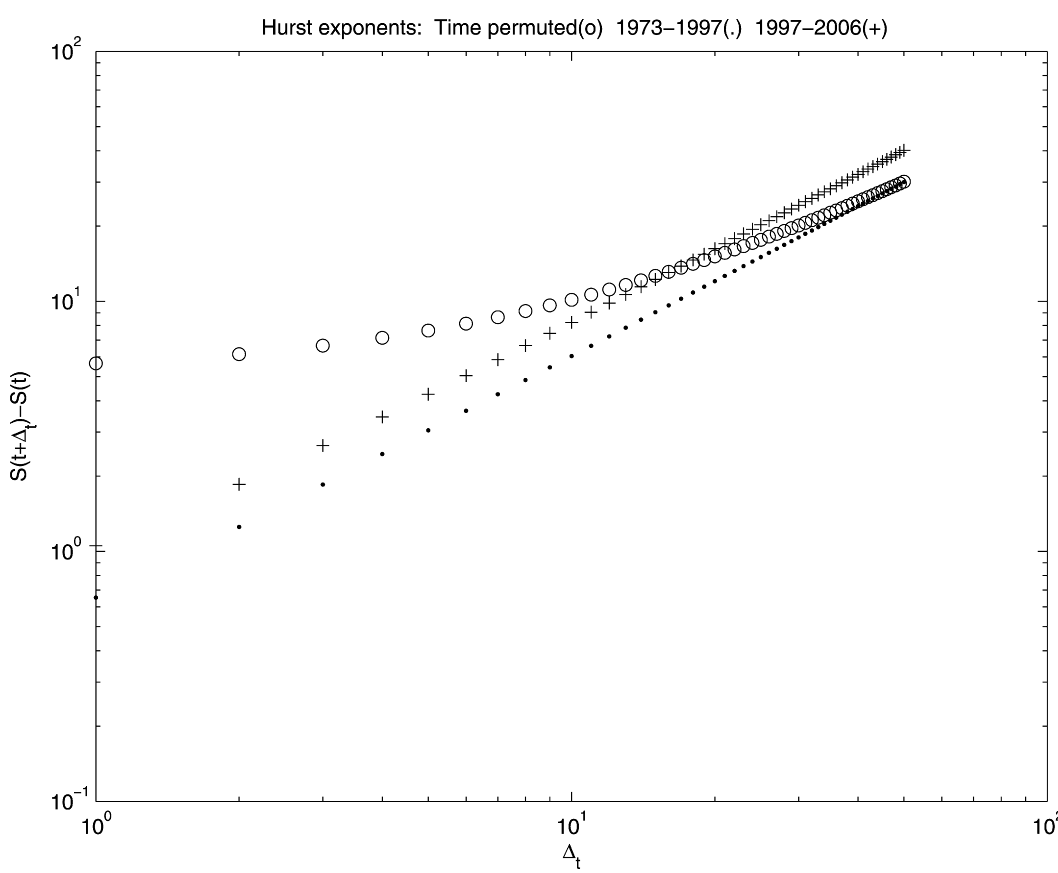

Fig. 4. Hurst exponents.

Table 2
Slopes of the *AS* yearly fit

Year	Slope	Year	Slope	Year	Slope
1973	1.5	1974	2.1	1975	1.5
1976	1.2	1977	1.3	1978	2
1979	2	1980	1.5	1981	1.3
1982	1.7	1983	0.87	1884	1.4
1985	1.1	1986	1.3	1987	3.2
1988	1.4	1989	1.8	1990	1.7
1991	1.2	1992	1.1	1993	1.2
1994	1.1	1995	1.7	1996	1.4
1997	1.8	1998	2.5	1999	2.5
2000	4	2001	4.3	2002	3.2
2003	2.5	2004	2.6	2005	2.5

each year period, the slope quantifying each successively longer history. The following table shows the values of the slope for the histories obtained for each year.

3.3. *Empirical evidence for the change of structure*

Studying the cumulative history of the index of market structure (Fig. 2), it is intuitive that a major change is occurring since around 1997, imposing a new dynamic structure. This intuition is now investigated by empirical means.

Fig. 3 presents the empirical evidence for the concentration of seisms in the period after 1997.

Any of these crashes ever compares to the one of 1987. Indeed, there are some sound reasons to suspect the existence of different dynamics through time in the evolution of financial markets. The 1987 crash singles out as the deepest general crisis, incomparable to the following ones. The subsequent years witnessed the response to that shock through the construction of new methods of regulation. For a period, not only no large seisms occurred, but also only smaller fluctuations are detectable. Yet, since the second half of the nineties, we obtain higher average values of the index S and a concentration of a number of crashes and their replica. This difference in the empirically described evolution suggests that the Clinton period of the "Internet boom" corresponded to a new structure of the market or to emergence of a new phase of turbulence in the financial markets.

Complementary evidence is provided by the computation of the Hurst exponent for the period under consideration: whereas for 1973–1997 the exponent is $H = 0.7$, for the time interval 1997–2006 it is $H = 0.87$, indicating stronger evidence for long term memory.

Fig. 4 shows the value of the Hurst exponents for both periods, compared to the values obtained from time permuted data, suggesting the presence and the evolution of some structure. For the recent period, this structure is not under the impact of larger seisms (such as the Black Monday) although it is defined by frequent and important seismic activity.

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