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EDUCATION, NEIGHBORHOOD EFFECTS AND GROWTH: AN AGENT-BASED MODEL APPROACH

TANYA ARAÚJO* and MIGUEL ST. AUBYN
ISEG (School of Economics and Management),
Technical University of Lisbon (TULisbon),
Rua do Quelhas, 6 1200-781 Lisboa, Portugal
**tanya@iseg.utl.pt*
Research Unit on Complexity and Economics (UECE)†

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Endogenous, ideas-led growth theory and the literature on agent-based modeling with neighborhood effects are crossed. In an economic overlapping generations framework, it is shown how social interactions and neighborhood effects are of vital importance in the endogenous determination of the long run number of skilled workers and therefore of the growth prospects of an economy. Neighborhood effects interact with the initial distribution of skilled agents across space and play a key role in the long run stabilization of the number of skilled individuals. Our model implies a tendency toward segregation, with a possibly positive influence on growth, if team effects operate. The long run growth rate is also shown to depend on the rate of time preference. Initial circumstances are of vital importance for long run outcomes. A poor initial education endowment will imply a long run reduced number of skilled workers and a mediocre growth rate, so there is no economic convergence tendency. On the contrary, poor societies will grow less, or will even fall into a poverty trap, and will diverge continuously from richer ones.

Keywords: Agent modeling; economic growth; education; human capital; neighborhood effects; poverty trap.

1. Introduction

Recent improvements in multidisciplinary methods and, particularly, the availability of powerful computational tool are giving researchers an increasing opportunity to investigate economies in their true complex nature. The adoption of a complex systems approach allows the modeling of macroeconomic structures from a bottom-up perspective — understood as resulting from the repeated local interaction of economic agents — without disregarding the consequences of the structures themselves to individual behavior, emergence of interaction patterns and social

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*Corresponding author.

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welfare. Agent-based models are at the leading edge of this endeavor. Our paper is a contribution to interweaving two lines of research that have progressed in separate ways. These are the endogenous, ideas-based macroeconomic growth models, where the representative optimizing agent is the device that allows solving different allocative decisions, and the agent-based economic literature, with a strong emphasis on heterogeneity and social interactions (social networks).

Several new, endogenous growth models emphasize the role of "ideas" in driving economic growth.[a] As stated by Jones [15], ideas are nonrival goods — a good idea can be used by anyone without diminishing the possibility of its use by others. Some good ideas, or inventions, are not directly linked to economic production and growth — as in the case of the works of Shakespeare or the symphonies of Beethoven. They are, however, good examples of nonrivalry — the fact that a theater company in New York uses the Hamlet text in a performance does not hinder any other similar event by another company in, say, Paris or Prague. Other nonrival ideas are clearly linked to economic innovation and to growth — such as the discovery in the Neolithic age that plants could be cultivated or that animals could be domesticated, or, more recently, electricity and the computer.

As in Ref. 15, we develop a model where ideas are produced by a fraction of the working population. These ideas, or innovations, are used by the rest of the workers to produce final goods. If ideas are kept constant, then the usual replication argument is valid. That doubling other inputs leads to twice the output is suggested by the possibility of having another, identical, economy, producing exactly the same and in the same way as the one we are considering. However, when ideas are allowed to grow there will be increasing returns to scale. This derives from the nonrival nature of ideas. Doubling all factors including ideas would be the equivalent of having two more advanced economies, sharing a higher stock of concepts, and therefore producing each one more than before.

We depart from the usual solutions in this type of growth models in what concerns an important allocative social decision, regarding the share of the working population that is engaged in producing ideas. In our overlapping generations framework, ideas are invented by skilled workers, who are agents that have studied in a previous period. An agent decision to study, or else to stay unskilled, is taken following a socially conditioned economic reasoning. On one hand, each agent is concerned with his or her lifelong income, so that it may be worthwhile to give up some present income in order to become part of an education elite that is usually better paid. However, other factors may influence this decision, including a possible subjective bias toward education, and, perhaps more importantly, the so-called neighborhood effects.

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[a] This includes the seminal works of Lucas [20] and Romer [23], and, in our view, also the Schumpeterian models where innovation occurs stochastically at a rate that depends on the quantity of labor allocated to R&D [1, 2].

In a strict sense, there is a neighborhood effect in the education decision if this one is not taken independently from the residential area where the agent lives. For example, it is sensible to assume that, *ceteris paribus*, a child's education attainment depends positively on the average human capital stock in his or her neighborhood. In a more general sense, one can conceive a more general "social space" where a distance can be defined.[b] In this case, one can think, for example, that some norms of behavior within a social group make some individuals more prone to follow higher education than others. In modeling terms, we place agents in a space such that a neighborhood is defined. This neighborhood can be interpreted in social or even familiar and/or in territorial terms. The fundamental point is that an agent's behavior will depend on other agents close to him or her, namely on their decisions concerning education and on their outcomes.[c]

It is worth noting that some empirical studies confirm that neighborhood effects are important in what concerns education decisions and outcomes. Reference 11 surveys several empirical papers are these effects are tested considering different outcomes. Education-related outcomes are examined in nine of these papers, and six of them provide evidence for the relevance of neighborhood effects.

Agent-based models are increasingly used in different fields of economics [16]. Many of these models fall into the field of finance [17–19, 22, 24] and a very important part of them deals with innovation and diffusion processes [6, 7, 9, 10, 12, 25–27]. Among the latter, some of the models encompass the study of economic growth. Most of them fall into the domain of evolutionary economics, as in the standard book of Nelson and Winter [21]. In their seminal book on evolutionary growth, Nelson and Winter [21] include complete discussions about the weakness of the neoclassical approaches in what concerns economic growth and the representation of technical change. According to these authors, despite the considerable importance of neoclassical theory to economic growth, it provided an inadequate channel for analyzing technical change (production of ideas and innovation). The weakness is due to the impossibility of conciliating technical change at the microeconomic level and analyses of growth undertaken at the aggregate level of the economy. The key challenge of an evolutionary formulation would thus be the fruitful integration of the micro and the aggregate description. To this end, economic models using computer simulations would be the most suitable vehicle. However, these economic growth models do not usually account for both ideas-based macroeconomic growth and the dynamics of interactions among economic agents, in what concerns education decisions and outcomes. They generally emphasize innovation processes where agents are producers (firms). Neighborhood effects may have an important role, as in the case of Dosi and Fagiolo (2003), since the technological behavior of a firm (innovation or imitation) depends on what is happening in its vicinity. As already mentioned, our model will also display neighborhood effects

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[b] Reference 11 surveys the literature on the role of neighborhoods in influencing socioeconomic outcomes. See Ref. 3 for more on the notion of social space.

[c] See Ref. 5 for another modeling strategy considering neighborhood effects in education.

but of a different nature. Although ours is also an agent-based framework, growth derives from ideas produced by the educated portion of the population. Modeling individuals as agents, we are able to emphasize two important local effects — the first is the decision to educate or not, and the second concerns the local interactions that may be at work when ideas are produced.

Thus, our approach differs from other recent agent-based, endogenous growth models that come in the evolutionary economics strand as economic growth is based on human capital formation (education) and ideas produced, and not so much on local innovation processes that spread through the economy. However, and as in both behavioral and evolutionary economics, modeling the formation of economic and social networks is a key issue in this paper. We address the role of network characteristics (such as neighborhood size and spatial distribution) in a structure that encompasses both regular and random contributions, highlighting how economic outcomes can be related to the initial economic structure (network). As a matter of fact, introducing heterogeneous agents and neighborhood effects into a growth model allows us to identify such important phenomena as economic divergence and how some economies can even fall into a poverty trap, with income growth not occurring at all.

The rest of this paper is structured as follows. In Sec. 2 we present the main features of our education and growth model. Results from simulating the model are provided in Sec. 3. Section 4 concludes the paper.

2. An Agent-Based Model of Education and Growth — The Features

2.1. *Population characteristics*

There are N individuals, or agents, in our economy. Each agent lives for two periods. Population size does not change and generations overlap. Consequently, there are at each point in time $N/2$ young persons, or juniors, and $N/2$ seniors.

A young person is either a student or a young worker. In the latter case, he or she becomes immediately part of the unskilled labor force. Otherwise, and if a young person spends the time studying, he or she will become a skilled worker when older. This implies that at each point in time the population comprises four groups:

(i) The young students;
(ii) The young, or junior, unskilled workers;
(iii) The senior unskilled workers, i.e. those that did not study in the previous period;
(iv) The skilled workers, necessarily senior, as they have studied when young.

2.2. *Organization in space and decision to educate*

Agents are disposed on a ring (or a one-dimensional lattice without boundary conditions) with a fixed neighborhood size (ng). As such, the proximity of an individual to each of its $(2ng)$ neighbors decreases smoothly in moving away either from its left or from its right. An agent does not change its location when it becomes older. However, at the end of each period, the junior agents become seniors, all senior agents die and a young agent will be born in each vacant location. Newborn agents will decide whether to start working immediately as an unskilled agent or else to become a student and to postpone its working life as a skilled worker.

The education decision is taken mixing two different perspectives. There is a neighborhood effect — it becomes more probable that an agent becomes educated if its neighbors are skilled workers. Also, the agent takes a comparative income point of view. Studying implies a period of time without earnings, so the agent compares the present value of its income as a skilled worker with the present value of an unskilled workers' income.

In formal terms, an agent decides to study if

$$ns(t) \cdot rw(t) > nu(t), \tag{1}$$

where $ns(t)$ and $nu(t)$ are, respectively, the number of skilled and of unskilled workers in the neighborhood; rw is the relative weight given to the skilled. Note that when rw equals 1 the agent will decide in favor of education if the number of skilled people in the neighborhood exceeds the number of unskilled workers. In more general terms, though, the agent decides according to a weighted average of the number of skilled and unskilled neighbors, and rw is defined as

$$rw(t) = \alpha \frac{ws(t-1)}{\beta(\rho) \cdot wu(t-1)}, \tag{2}$$

where $ws(t-1)$ and $wu(t-1)$ denote skilled and unskilled labor wages in the previous period, respectively. Although the agent observes a local sample only, the wage setting in our economy, described later, implies that skilled and unskilled labor wages are the same across skilled and unskilled workers. ρ is a discount rate, $\beta(\rho)$ a monotonous function with $\frac{\partial \beta}{\partial \rho} > 0$ and α an exogenous parameter.[d]

The relative importance of skilled neighbors increases with the skilled to unskilled wage ratio and it decreases with the discount rate. When ρ increases, the future is more heavily discounted, implying that the agent is less inclined to study and to wait for future, possibly higher, earnings.

2.3. *Production*

Production of final goods depends on two inputs — the existing stock of ideas and the amount of unskilled labor. We assume a constant intratemporal labor marginal productivity, given the stock of ideas. The final goods production function is

$$Y(t) = A(t)LU(t), \tag{3}$$

[d] The function $\beta(\rho)$ is derived in the appendix.

where $Y(t)$, $A(t)$ and $LU(t)$ denote, respectively, final goods aggregate production, the stock of ideas and the number of unskilled workers, either junior or senior. This formulation is similar to the one in Ref. 15. Given the stock of ideas, production has constant returns to scale in labor, which is the sole rival input.

Ideas are produced by senior skilled workers. We assume that

$$\Delta A(t) = A(t-1) \cdot (\delta LS(t) + \gamma SD(t)), \tag{4}$$

where $LS(t)$ is the number of skilled workers and δ is a parameter directly related to skilled labor marginal productivity. When $SD(t)$ is equal to zero, Eq. (4) implies that the stock-of-ideas rate of growth, $\Delta A(t)/A(t-1)$, depends on the number of skilled workers. This feature is present in a number of seminal endogenous growth papers, namely Refs. 1, 13 and 23. Following Ref. 15, the ideas production function is said to exhibit a strong form of scale effects, in the sense that an increased number of skilled workers or researchers lead not only to a more-than-proportional increase in production, but also to an augmented rate of growth.[e]

$SD(t)$ is a distance measure defined as

$$SD(t) = \frac{1}{LS(t)} \sum_{\substack{i,j=1 \\ i \neq j}}^{LS} \frac{1}{|i-j|}, \tag{5}$$

with $i \neq j$ and being thus smaller when skilled workers are located far from each other and larger in the opposite situation. The parameter γ allows for specifying the strength of team effects.

2.4. *Wages and income distribution*

We assume that there is a social pact in our economy. We follow Avdagic, Rhodes and Visser's [4] definition: "Social pacts generally arise between representatives of government and organized interests, who negotiate and coordinate policies across a number of formally independent, but actually related and interconnected policy areas (incomes, labor market, fiscal, and social policies) and levels (national, sectoral, regional, local)." Social pacts have been particularly important and prevalent in European countries in what concerns income distribution (see Refs. 8 and 14 for recent analysis of social pacts).

Note that the income share that corresponds to what would have been produced in time t if the stock of ideas remained constant is equal to $A(t-1)LU(t)$. On the other hand, the income surplus generated from new ideas invented at time t equals $(A(t)-A(t-1))LU(t)$. In our artificial society, the two types of workers are organized in a similar manner to capital and labor in industrial economies, and they reach an

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[e] As in our model the population is constant, the number of skilled workers is naturally bounded. In a steady state, this number will be a fixed proportion of all labor, and A will grow at a constant rate. In more general terms, models of this type will usually imply that economies with a growing population will tend to grow faster. See Ref. 15, especially Sec. 5, for a full discussion of scale effects on the production of ideas.

agreement such that unskilled workers' income at time t is

$$YU(t) = A(t-1)LU(t) + \sigma(t)(A(t) - A(t-1))LU(t), \tag{6}$$

and skilled workers' income is equal to

$$YS(t) = (1 - \sigma(t))(A(t) - A(t-1))LU(t). \tag{7}$$

Note that when $\sigma(t) = 0$ unskilled workers get their exact contribution to production, in the sense that they earn what would have been produced if there were no skilled workers in society. And, still with $\sigma(t) = 0$, skilled workers earn exactly their input as well, the additional income generated from new ideas they had at time t.

Function $\sigma(t)$ is an exponential function defined as

$$\sigma(t) = \exp(-\varepsilon \cdot LU(t)), \tag{8}$$

with $\varepsilon = 1$. In normal times $LU(t)$ is sufficiently large so that $\sigma(t)$ becomes very close to 0. Conversely, where the number of unskilled workers declines and approaches 0, $\sigma(t)$ tends to 1. In the extreme case where there is a tendency for unskilled workers to disappear, i.e. when everyone is becoming educated, the bargaining power of unskilled workers increases, as workers available for more menial jobs become scarce. This implies that unskilled workers start to earn a slice of the income share previously allocated to the skilled workers. Their earnings rise accordingly, and becoming educated looks less interesting from an economic point of view, as given by the relative wage component of Eq. (2). This introduces an automatic adjustment mechanism that avoids what could amount to a collapse of the economy caused by the total disappearance of unskilled workers.[f]

The fairness idea the underlies the social pact is consistent with the property that factor payments exhaust output, the two shares adding up to total income: $YS(t) + YU(t) = A(t)LU(t) = Y(t)$. All income is distributed as wages. Note that there is no physical capital in the model, and therefore no physical investment. However, the decision to educate can be regarded as a choice to invest in human capital, as individuals are trading present income, which could be earned by entering the labor force without delay, for future increased income, received as skilled labor wages. The corresponding unskilled and skilled labor wages, are, respectively,

$$wu(t) = \frac{YU(t)}{LU(t)} = (1 - \sigma(t))A(t-1) + \sigma(t))A(t), \tag{9}$$

$$ws(t) = \frac{YS(t)}{LS(t)} = (1 - \sigma(t))\,(A(t) - A(t-1))\,\frac{LU(t)}{LS(t)}. \tag{10}$$

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[f] This effect is stronger the higher the value of ε. In all model simulations to be discussed later, ε was set to 1. A sensitivity analysis showed that this calibration ensured both that the effect was sufficiently strong to avoid the disappearance of unskilled workers in some extreme cases and that it only really mattered for very low numbers of LU. We thank an anonymous referee for having suggested the consideration of an endogenous mechanism to circumvent unskilled workers' disappearance.

2.5. *Steady state and poverty trap*

Suppose that the composition of the labor force is stable, so that the number of skilled and unskilled workers stays constant. Moreover, assume that the disposition of skilled and unskilled agents in the ring does not change. In that case, the economy is in a steady state characterized by a constant growth rate of final goods production. To show this, note from Eq. (4) that the steady state stock-of-ideas growth rate is equal to

$$g(A_{ss}) = \frac{\Delta A_{ss}}{A_{ss}} = \delta LS_{ss} + \gamma SD_{ss}, \tag{11}$$

where $g(\cdot)$ denotes the growth rate and the subscript ss refers to a steady state variable. LS_{ss} and SD_{ss} are constant by assumption, and this implies that $g(A_{ss})$ is constant.

As LU_{ss} is constant, Eq. (3) implies that steady state final goods production grows at the same rate as A, so that long term growth results necessarily from innovation (new ideas):

$$g(Y_{ss}) = g(A_{ss}). \tag{12}$$

In addition, note from Eq. (9) that unskilled labor wages also grow at the same rate as A. Besides, from Eq. (10), and given that the growth rate of A is constant and that the ratio of skilled to unskilled labor does not change, skilled labor wages will also grow at that rate. Steady state income distribution shares are therefore constant.

Having so defined a steady state, the question of whether this economy will tend to such a long run growth path arises. If the answer is positive, one would like to know how this steady state changes if initial conditions differ. However, and as will be shown in the next section, it is not always the case that the economy converges to a positive growth steady state. In fact, and depending on initial conditions, the economy can be driven toward a pathological condition, the "poverty trap," to be defined next.

Suppose that, for some reason, there are no skilled workers at all in the economy. In this case, no agent will ever decide to go into further education and become a skilled worker. In Eq. (1), $ns(t)$ will always take the value 0, as there are no skilled senior agents in the neighborhood of any junior agent being born. Once the number of skilled agents reaches 0, it will remain 0 forever, as no example of the success of education will push others to the same option. Consequently, A, the stock of ideas, will remain constant once all workers are unskilled. From Eq. (3), it is apparent that Y, the production of final goods, stagnates. We will call this unhappy ending the "poverty trap."

The converse situation where all workers are skilled, is potentially more dramatic but less relevant from an empirical point of view. *Mutatis mutandis*, and for the very same reasons, no unskilled worker would ever be born after the last nonskilled agent died. However, one has to notice that the economy would immediately collapse if there were no unskilled workers. Even if the stock of ideas has been progressing at a good rate, it would not be possible to produce final goods once LU is zero in Eq. (3). As explained previously, this overeducation collapse is avoided in our model because unskilled labor wages start to grow faster when uneducated employees become scarce. A nonvanishing small number of unskilled but well-paid workers will guarantee that some newborns will choose not to become educated, so that some vital and more menial jobs are always assured.

3. Simulation Results

To simulate our model, the exogenous parameters are set to the values shown in Table 1, in which a baseline and six alternative scenarios are defined. In the following, besides analyzing the baseline and the different scenarios, we study with special care the possibility for an economy to fall into a poverty trap.

3.1. *The baseline*

When the baseline economy starts, there are 50 unskilled agents, junior or senior. The other 50 individuals are either students, if they are junior, or skilled employees, if they are senior. In a typical, average, baseline simulation, the number of employees equals 75, 25 being skilled. The location (on the ring) of each agent, junior or senior and skilled or unskilled, is randomly determined.

Figure 1 displays the initial setting in a typical baseline simulation. Each agent is represented by a small circle: unskilled agents are colored black, the white being students or skilled agents. As their initial location is randomly chosen, there are groups or neighborhoods of skilled or unskilled agents of different size. An unaccompanied agent is an agent whose closest neighbor in the clockwise sense has a different color.[g]

In the baseline, we set $\alpha = 1$, so that agents are not biased toward education, in the sense that they do not give any special importance to it *de per se*. They will choose to acquire skills if and only if there are some skilled neighbors that earn sufficiently more than those that remained unskilled. The skilled labor productivity parameter, δ, was set to 0.03. As no team effects were considered ($\gamma = 0$) and once $LS = 25$ in Eq. (1), it gives an implied initial growth rate of 75% per half generation, which seems a sensible value.[h]

The neighborhood range was set to 3, meaning that when an agent decides whether to pursue his or her studies by considering the status and remuneration of his or her six closest senior neighbors (three to the left and three to the right). According to Eq. (1), if there is no skilled agent in the neighborhood, the agent will not chose to become skilled, and, symmetrically, if all neighbors are skilled,

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[g] Later in the paper we shall use the concept of "unaccompanied agent" in order to account for segregation effects.

[h] If half a generation lasts for, say, 25 years, this would imply a 1.9% annual growth rate.

Table 1. Exogenous parameters.

Parameters	Baseline	Scenarios 1 Education is important	Scenarios 2 Future less valued	Scenarios 3 Higher initial number of unskilled agents	Scenarios 4 Lower initial number of unskilled agents	Scenarios 5 Team effect in producing ideas	Scenarios 6 Neighborhood range increases
Number of agents	100						
Skilled labor productivity, δ	0.03						
Number of periods	30						
Relative importance of education, α	1	*2*	1	1	1	1	1
Discount rate, ρ	0.05	0.05	*0.1*	0.05	0.05	0.05	0.05
Initial number of unskilled agents	50	50	50	*80*	*20*	50	50
Team effect in producing ideas, γ	0	0	0	0	0	*0.2*	0
Neighborhood range, ng	3	3	3	3	3	3	*5*

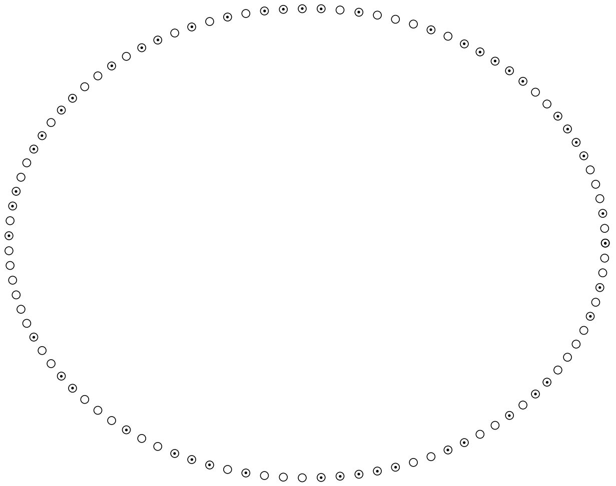

Fig. 1. Baseline, initial setting.

the agent will become a student. If the neighborhood comprises three skilled and three unskilled agents, the education decision will be taken if the skilled labor future wage exceeds sufficiently the unskilled one so that it is worth spending some years without a wage in present value terms. If the number of skilled agents in the neighborhood is positive but smaller than the number of unskilled individuals, then an even greater skilled labor wage will be required in order to convince the agent to remain at school.

The baseline discount rate equals 5%, a value not very different from empirically observed real interest rates. Convergence to the steady state was ensured after 22–25 periods, so the number of time periods for each simulation, 30, was chosen to make certain the economy reached a steady state where the number of skilled workers remains constant.

The results reported are the average results over 1000 simulations. As can be read from Table 2, the average steady state number of skilled and unskilled workers, respectively 19.8 and 59.2, is close to initial settings. In this steady state, and from Eq. (10), the number of ideas grows at a rate equal to 0.60, as can be read from the first line.

As the number of unskilled workers is constant in the steady state, this is also the output growth rate [recall equality (12)]. Also, both skilled and unskilled labor wages will grow at that very same rate. Note that the steady state skilled labor wage exceeds the unskilled wage by 78%, as can be read from the line "steady state relative skilled labor wage."

Figure 2 displays the steady state setting of agents on the ring. As time goes by, there is a segregation mechanism going on. Agents that are born close to skilled individuals tend to go into further education and persons close to unskilled agents have a higher probability of not pursuing their studies. The long run effect is the one

Table 2. Simulation results. Average results over 1000 simulations.

		Scenarios					
		1	2	3	4	5	6
	Baseline	Education is important	Future less valued	Higher initial number of unskilled agents	Lower initial number of unskilled agents	Team effect in producing ideas	Neighborhood range increases
Steady state Y growth rate	0.60	1.07	0.27	0.17	0.85	1.5	0.52
Steady state number of unskilled employees	59.2	29.8	82.1	88.1	43.1	40.2	65.3
Steady state number of skilled employees	19.8	35.0	8.8	5.8	28.8	29.7	17.3
Steady state relative skilled labor wage	1.78	0.88	2.46	2.0	1.29	2.1	1.9
Initial number of partitions	50.5	50.9	51.4	32.2	32.1	50.1	50.5
Steady state number of partitions	6.0	5.6	3.9	2.3	4.9	5.1	3.9

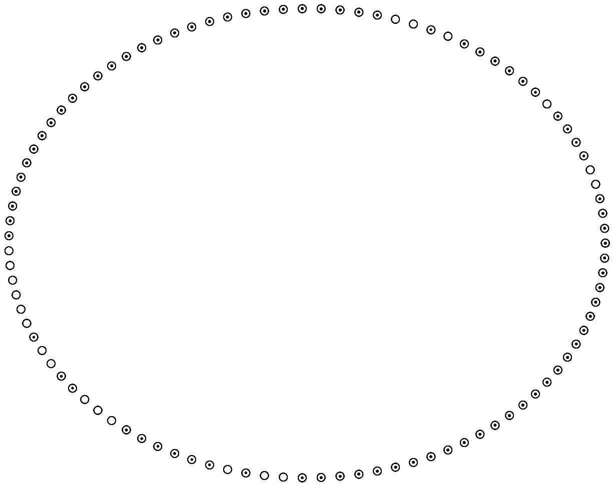

Fig. 2. Baseline, final setting.

described in Fig. 2: skilled agents are clustered in a small number of neighborhoods, a very different pattern when compared to the random initial distribution depicted in Fig. 1.

It is possible to summarize the degree of clustering by counting the number of observed partitions, which corresponds to the number of unaccompanied agents existing in the ring. The initial and steady state number of partitions are presented in Table 2. The initial average number of partitions is 50.5, which decreases drastically to 6.0.

Clustering of skilled agents will be present as well in every scenario described below — the final setting of agents is always, in qualitative terms, similar to the one presented in Fig. 2. This is a spatial aspect of the model that suggests a reason for high tech firms to be clumped in location terms.

3.2. *Scenario 1 — Education is important*

In this variant the parameter α takes the value 2 in Eq. (2). This implies that, *ceteris paribus*, the weight put on education doubles. In more precise terms, this means that one skilled wage unit is valued as two unskilled wage units. Agents value education *de per se*, in the sense that they dislike less the time working in an activity that requires their acquired skills. This change in preferences implies that the economy settles in a steady state that is significantly different from the baseline. The number of skilled workers is higher (35.0), and, consequently, the steady state output growth rate is also higher (1.07).

However, and due to the relative abundance of skilled agents, the skilled labor remuneration is now smaller than the unqualified labor wage (in Table 2, the long run ratio of skilled to unskilled labor wage equals 0.88). Unskilled labor earns a premium, as it is a relatively undesirable activity. Again, and even more than in the baseline, skilled agents tend to cluster in neighborhoods — the average final number of partitions equals 5.6.

3.3. *Scenario 2 — The future is less valued*

The discount rate increases from 5% to 10% in this scenario. Future wages are more heavily discounted. Consequently, investing is education becomes less attractive, as one of its motivations is the higher future earnings that compensate for present thrift, as students abstain from wage income when studying. In comparison with the baseline, a smaller number of agents will choose education, and the steady state number of skilled workers is lower (8.8, compared to 19.8 in the baseline). This affects the long run output growth rate, which declines from 0.6 to 0.27. As in the baseline, skilled workers cluster in a limited number of neighborhoods, the partition number being even lower and equal to 3.9. Finally, it is worth noting that the relative wage of skilled workers is now higher, a logical consequence of their relative scarcity.

3.4. *Scenario 3 — The initial number of unskilled agents is higher*

Initial conditions matter. As will be shown in more detail later, when the initial number of skilled people decreases, the probability of extinction becomes nonnegligible and the economy may fall into stagnation. For an initial number of unskilled workers of 80, this probability is equal to 25%. Even when this extreme outcome is avoided, this economy, which starts unskilled, tends to stick to a mediocre growth path. The steady state number of unskilled workers averages 88.1, a much higher value than the baseline (59.2). Note that the corresponding growth rate is smaller (0.17), and that the skilled agents' wage equals double the unskilled labor one. In this relatively undeveloped economy, skilled agents are a small elite. Even if they are well paid, a relatively small number of young people enter this small social group that does not show up in spatial terms.

3.5. *Scenario 4 — The initial number of unskilled agents is lower*

Scenario 4 is the converse of scenario 3. Here, the initial number of unskilled agents, 20, is lower than in the baseline. The economy starts as a more skilled one. The fact that skilled people predominate implies that more young people tend to imitate their social environment, resulting in a steady state where the number of skilled employees, 28.8, is high as compared to the baseline. The fact that skilled workers abound implies that their steady state earnings come near to unskilled labor values, the relative wage being 1.29 only. As opposed to scenario 3, this is a relatively developed economy where education is widespread, social differences are less important, new ideas are regularly produced and the growth rate, 0.85, is stronger.

3.6. *Scenario 5 — Team effect in producing ideas*

The baseline does not incorporate team effects — the same skilled workers produce the same number of ideas, irrespective of their dispersion. In scenario 5, we include a team effect, such that production of ideas increases when skilled workers are close to each other. In terms of Eq. (4), we make $\gamma = 0.2$ so that when $SD(t)$ is higher, skilled agents work together and share their thoughts, giving rise to more new ideas. In comparison with to the baseline, increased skilled labor productivity leads to higher relative skilled labor wages (2.1, compared to 1.78). Higher wages induce more education choices from agents, and the steady state average number of skilled agents, 29.7, is therefore higher. As in scenario 1 or 4, more skilled agents means an increased steady state rate of growth. Here, this outcome is reinforced by the prevalence of team effects.

3.7. *Scenario 6 — The neighborhood range increases*

In this scenario the neighborhood range increases from 3 to 5, i.e. ng takes the value 5 in Eq. (1). When taking an education decision, the agent increases his or her observation range, and looks at the five, and not three, neighbors to the left and to the right. Results are not very different from the baseline, except for one outcome — segregation is stronger, the average number of partitions becoming inferior (3.9, compared to 6.0). As agents see further, smaller groups of skilled or unskilled workers "resist" less what is going on around them. Greater increases in the neighborhood range, till $ng = 18$, not presented in the table but computed by the authors, produced results qualitatively similar to $ng = 5$.

3.8. *The initial number of unskilled workers and the poverty trap*

Table 3 contains average results over 1000 simulations for different initial conditions in what concerns the number of unskilled agents. All other parameters are equal to baseline values. The number of initial unskilled agents was made equal to 20, then to 40, etc., till 90, and the percentage of stagnation cases was retained. Moreover, in the remaining cases of convergence to the steady state, the long run number of skilled workers and the implied growth rate are also presented.

When the population is essentially unskilled, there is a considerable chance for skilled workers to become extinct, as they do not show up sufficiently in any neighborhood, and therefore do not reproduce. Recall that when skilled workers stop existing, new ideas are not produced anymore and the economy ceases to grow. The economy falls into a poverty trap. From Table 3, it is apparent that the probability of stagnation becomes significant, or higher than 8%, when initial values for unskilled workers exceed 70.

The two last columns of Table 3 contain the average steady state number of skilled workers excluding stagnation events, and the implied growth rate. The number of steady state skilled workers, and therefore the long run growth rate, depends

Table 3. Initial number of unskilled workers and the poverty trap.

Initial number of unskilled workers	Percentage of stagnation events (poverty trap)	Average number of steady state skilled workers (excluding stagnation)	Implied growth rate
20	0.0	28.8	0.85
40	0.0	23.2	0.69
60	1.4	14.7	0.44
70	8	9.4	0.28
75	15	7.6	0.22
80	25	5.8	0.17
85	28	3.2	0.09
90	29.2	1.1	0.03

strongly on initial conditions. An economy that starts poor, with an unskilled population, will not converge to the growth path of an economy that starts with an educated population. The long run *growth rate*, and not only income level, depends negatively on the initial number of unskilled workers.

4. Conclusions

This paper has shown that is it possible and fruitful to cross distinct lines of research — endogenous, ideas-led growth theory and agent-based modeling with neighborhood effects. On the one hand, it is appealing and convincing to consider that growth, in the long run, depends essentially on innovation, and that innovation comes from new ideas, produced by skilled agents. On the other hand, several empirical studies have shown that people tend to become more educated if they are born in a favorable environment, where most people are already educated, and where education is a profitable investment, in the sense that it entitles individuals to a higher income in the future.

Our economic model takes these different hypotheses seriously and in a complementary way. In an overlapping generations framework, we have shown how social interactions are of paramount importance in the endogenous determination of the long run number of skilled workers and therefore of the growth prospects of an economy. Neighborhood effects interact with the initial distribution of skilled agents across space and play a key role in the long run stabilization of the number of skilled individuals. Our model implies a tendency toward segregation — similar agents tend to live close to each other. This segregation may have a positive influence on growth, if team effects operate and ideas are best generated when people have close contact. In a real world explanation, this provides a reason why high growth economies tend to have a higher proportion of high tech firms that are usually located close to each other. Moreover, and as in other endogenous growth models, the long run growth rate depends on parameters describing agents' preferences. Namely, if they heavily discount future income or do not love education, the steady state growth rate tends to be lower.

We have also shown how initial circumstances are of vital importance for long run outcomes. A poor education endowment may mean that skills are not reproduced, all workers become unskilled, no new ideas are produced, and the economy stagnates in the long run. Even when stagnation does not occur, it is usually the case that the long run outcome will be characterized by a socially divided society, where a small number of relatively well-paid skilled workers only assures a mediocre growth rate. If starting conditions are such that skilled workers already abound, the long run will be such that skilled workers will be much less scarce, their wage will be lower than that of unskilled labor, and the stock of ideas and final goods production will grow at a higher pace. Therefore, our model does not display any kind of economic convergence tendency. On the contrary, poor societies will grow less and will diverge continuously from richer ones.

Appendix

At the beginning of period t, an agent has perfect knowledge of period $t-1$ wages, namely $ws(t-1)$ and $wu(t-1)$, the skilled and unskilled labor wage, respectively. Assume that agents take these values as the ones that will prevail in the future, and, for the sake of simplicity, denote them by ws and wu.

Suppose that skilled agents spend nine more years at school than unskilled agents. For example, one can think that they spend two more years at secondary school, four additional years to take a first degree, and finally three more years in some form of graduate studies. PVE, the present value of future wages for an agent that is starting his or her education to become skilled, is then

$$PVE = ws[(1+\rho)^{-9} + (1+\rho)^{-10} + \cdots + (1+\rho)^{-\tau}], \tag{A.1}$$

where ρ is a rate of time preference or discount rate, and τ is the number of years to the end of active life, likely to be between 45 and 50.

At the same time that a future skilled agent starts his or her education, unskilled agents start working. With the hypothesis above, this means that they work nine more years when compared to skilled workers. Let PVU be the present value of all wages earned by unskilled workers:

$$PVU = wu[1 + (1+\rho)^{-1} + \cdots + (1+\rho)^{-\tau}]. \tag{A.2}$$

Comparing Eqs. (A.1) and (A.2), it is apparent that ws must be greater than wu for there to be any chance of PVE being greater than PVU. In this case, and from a pure income perspective, i.e. putting aside any subjective preference for education, the agent would choose to proceed into further education and not to remain unskilled. Let β be the ratio between ws and wu that makes the present value of skilled labor wages equal to the present value of unskilled labor wages:

$$\frac{ws}{wu} = \beta \Rightarrow PVE = PVU. \tag{A.3}$$

From Eqs. (A.1)–(A.3), it gives

$$\beta = \frac{1 + (1+\rho)^{-1} + \cdots + (1+\rho)^{-\tau}}{(1+\rho)^{-9} + \cdots + (1+\rho)^{-\tau}} = \frac{A}{B}, \tag{A.4}$$

with $A = 1 + (1+\rho)^{-1} + \cdots + (1+\rho)^{-\tau}$ and $B = (1+\rho)^{-9} + \cdots + (1+\rho)^{-\tau}$. It is easy to show that $A = \frac{1+\rho-(1+\rho)^{-\tau}}{\rho}$ and that $B = \frac{(1+\rho)^{-8}-(1+\rho)^{-\tau}}{\rho}$. Replacing A and B in expression (A.4) and simplifying, it gives

$$\beta = \frac{(1+\rho)^9 - (1+\rho)^{8-\tau}}{1 - (1+\rho)^{8-\tau}}. \tag{A.5}$$

Note that β approaches $(1+\rho)^9$ when τ tends to infinity, and that β is an increasing function of ρ. When the discount rate is higher, the future is less valued, and therefore the skilled labor wage has to be higher for agents to become indifferent between acquiring skills through education and remaining unskilled. In our simulations, we set $\tau = 48$.

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