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The geometry of crashes. A measure of the dynamics of stock market crises

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This paper investigates the dynamics of stocks in the S&P 500 index for the last 30 years. Using a stochastic geometry technique, we investigate the evolution of the market space and define a new measure for that purpose that is a robust index of the dynamics of the market structure and provides information on the intensity and the sectoral impact of crises. With this measure, we analyse the effects of extreme phenomena on the geometry of the market. Nine crashes between 1987 and 2001 are compared by looking at the way they modify the shape of the manifold that describes the S&P 500 market space.

Keywords: Agent-based modelling; Applied finance; Artificial economy; Complexity in economics; Complexity in finance; Computational finance; Economic modelling; Evolutionary model of currency crisis

1. Introduction

In 1999, Mantegna defined a distance metric based on the correlation coefficients between the log-price difference of a pair of market securities. This metric allows the determination of the distance between stocks evolving in time in a synchronous fashion. Since the metric was further discussed by Mantegna and Stanley (2000) in the book that coined the term 'Econophysics', it has been applied in a considerable number of research works (Kullmann *et al.* 2000, Bonanno *et al.* 2001a, b, 2003, 2004, Gopikrishnan *et al.* 2001, Marsili 2002, Onnela *et al.* 2003c, Matteo *et al.* 2004). The fact that the metric is a properly defined distance gives a meaning to geometric notions in the study of the market. As Mantegna (1999) did when the distance was first introduced, many papers using the metric follow a topological approach.

Provided that a distance exists between stocks, it is sufficient to form an additional hypothesis on the topological space of the stocks (such as, for example, choosing the subdominant ultrametric space, which is obtained from the minimal-spanning tree that links the stocks (Mantegna and Stanley 2000)) in order to end up with a connectivity pattern for the stocks. In so doing, one can naturally move away from a situation where all the stocks are connected to a network of stocks, in which the connectivity pattern is endogenously determined. From the topological point of view, this opens up a large number of promising possibilities for exploration.

Using Mantegna's metric we followed a different perspective. In a previous contribution (Vilela Mendes *et al.* 2003) we developed a method for the reconstruction of an economic space. By using a stochastic geometry technique, we proved that economic spaces are low-dimensional entities and that this low-dimensionality is caused by the small proportion of systematic information present in the correlations among stocks. Using our reconstruction method we found that part of the correlation contribution is virtually indistinguishable from random and surrogate data (obtained by independent time permutation for each stock).

In the present paper, we investigate the hypothesis that market spaces uniformly contract during crashes along their effective dimensions and conclude that, otherwise, some crashes may act differently in specific directions, causing interesting changes in the shape of the market space. In order to capture that distortion effect, a structure index is used to compute the lack of uniformity among the market effective dimensions. As a consequence, we are able to characterize the structures that emerge in relevant historical periods and to identify the economic sectors that are associated with important changes in the leading directions of the evolving market space.

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It is observed empirically that both during expansion and normal periods the market tends toward randomness, whereas in disturbed periods its structure is reinforced, not only in the topological sense (as revealed by clustering measures) but also in the geometrical sense, considering distortions of form. From this observation we propose a new measure for the dynamics of market structure that captures that distortion effect in the shape of the market space.

Other authors have also reported the existence of a dynamic pattern during market crashes (Drozdz *et al.* 2002, Lillo and Mantegna 2002, Marsili 2002, Onnela *et al.* 2002, 2003a, b, Sornette 2002, Sornette and Helmstetter 2003, Johansen and Sornette 2002, 2004). Sornette and co-workers successfully demonstrated that dynamic patterns can often be found in preceding events. For several extreme phenomena, they found evidence of incoming instabilities in the precursory patterns of time trajectories of market data (such as price, volume and volatility variables). Among their main contributions, there is an issue that appears to be crucial for understanding the behaviour of the market: the identification of a distinct signature for endogenous and exogenous shocks originating crashes. In particular, they proved a systematic association of large events with positive feedback processes. Later in the paper we shall address that issue while applying our structure index to discriminate distinct processes at work in the S&P 500 stock market.

The identification of economic sectors as clusters of stocks with similar economic dynamics was reported by Bonanno *et al.* (2001b), Gopikrishnan *et al.* (2001) and Marsili (2002). Gopikrishnan *et al.* (2001) used techniques that are related to the metric we use, although with a different perspective. Diagonalizing the correlation matrix, they tried to identify particular eigenvectors with traditional industrial sectors. In our analysis, the effective dimensions of a market space may not correspond to economic sectors. We argue that the lack of uniformity among the effective dimensions reveals the existence of a dynamic pattern (which we empirically verify to correspond to crashes). To evaluate the impact of those extreme phenomena in different economic sectors (and the sectoral dynamics among different crashes), we compute the index of market structure for different market spaces, each comprising stocks that belong to a specific economic sector.

The fact that the correlation matrix changes in crash periods has also been shown by Onnela *et al.* (2002, 2003a, b). Using clustering techniques, they examined the occurrence of changes in the stock market from a topological perspective. From the minimal-spanning tree that links the stocks, they showed that, during a market crisis, there may be (as on Black Monday) a topological shrinking of the tree.

However, less important market crashes cannot be observed from the correlation matrix itself. While extreme events such as Black Monday are captured through changes in the correlation matrix, those with a smaller impact on the market synchronous behaviour require a fine-grain approach in order to be identified.

In our method, after using the correlation matrix to prove that economic spaces are low-dimensional entities (see also Vilela Mendes *et al.* (2003)), we focus on the geometrical aspects of those reduced spaces and show the impact of market crashes along the dimensions that carry the systematic information related to the market correlation structure. During a crash, the specific directions corresponding to the effective dimensions of a market space are affected differently, causing changes in the space shape. Experimentally, we observe that, in disturbed periods, the market space increases its structured evolution, not only in the topological sense (as revealed by clustering measures) but also in the geometrical sense, generating distortions of form.

In sections 2 and 3 the method is explained in detail and is applied to a set of companies that are, or have been, in the S&P 500 index. In section 4 we discuss the results obtained for specific sectors and the role of those sectors in important market crashes. Finally, a summary and conclusions are presented.

2. Method

The idea is simply stated in the following terms.

1. Pick a representative set of N stocks and their historical data of returns over some time interval.
2. From the returns data, using an appropriate metric, compute the matrix of distances between the N stocks. The problem is now reduced to an embedding problem in which, given a set of distances between points, one asks what is the smallest manifold that contains the set. Given a graph G and an allowed distortion there are algorithmic techniques (Linial *et al.* 1995) to map the graph vertices to a normed space in such a way that the distances between the vertices of G match the distances between their geometric images, up to the allowed distortion. However, these techniques are not directly applicable to our problem because, in the distances between assets, computed from their return fluctuations, there are systematic and unsystematic contributions. Therefore, to extract factor information from the market, we have to separate these two effects. The following stochastic geometry† technique is used.

†Stochastic Geometry concerns the study of random geometric structures. Kendall *et al.* (1998) and Stoyan *et al.* (1997) provide detailed information on the subject.

3. From the matrix of distances compute the coordinates for the N stocks in a Euclidean space of dimension smaller than N.
4. Apply the standard analysis of reduction of the coordinates to the center of mass† and compute the eigenvectors of the inertial tensor.
5. Apply the same technique to surrogate data, namely to data obtained by independent time permutation for each stock.‡
6. Compare the eigenvalues in point 4 with those in point 5 and identify the directions for which the eigenvalues are significantly different as being the market characteristic dimensions. In so doing, we are attempting to identify the empirically constructed variables that drive the market and the number of surviving eigenvalues is the effective dimension of this economic space.
7. From the eigenvalues of order smaller than the number of characteristic dimensions, compute the difference between the eigenvalues in point 4 and those in point 5. The normalized sum of those differences is the index S, which measures the evolution of the distortion effect in the shape of the market space.

For both surrogate and actual data, the sorted eigenvalues, from large to small, decrease with their order. In the surrogate case, the amount of decrease is linear in the order number, proving that the directions are being extracted from a spherical configuration. The display of a uniform and smooth decrease in the values of the sorted eigenvalues is characteristic of random cases and is also observed experimentally when the market space is built from historical data corresponding to a period of business as usual.

Considering the lack of uniformity among the market effective dimensions we are able to characterize the extent to which crashes act differently on specific directions, causing changes in the shape of the market space. Looking for relevant distortions in the shape of the S&P 500 market space over the last 30 years, we found that some of the highest values of the index are those computed for some important dates, such as 19 October 1987, 27 October 1997 and 11 September 2001.

In addition to the geometrical analysis of the whole S&P 500 market space, our measure is applied to sets of stocks that belong to specific economic sectors. The results show that some crashes act differently on specific sectors and that the deviation from random behaviour may be limited to a few days after the day of the crash and also to a small number of sector-oriented groups of stocks.

3. Measures

From the returns for each stock,

$$r(k) = \log(p_t(k)) - \log(p_{t-1}(k)), \tag{1}$$

a normalized vector,

$$\boldsymbol{\rho}(k) = \frac{\boldsymbol{r}(k) - \langle \boldsymbol{r}(k) \rangle}{\sqrt{n(\langle \boldsymbol{r}^2(k) \rangle - \langle \boldsymbol{r}(k) \rangle^2)}}, \tag{2}$$

is defined, where n is the number of components (number of time labels) in the vector $\boldsymbol{\rho}$. With this vector, one defines the distance between stocks k and l by the Euclidian distance of the normalized vectors,

$$d_{ij} = \sqrt{2(1 - C_{ij})} = \| \boldsymbol{\rho}(k) - \boldsymbol{\rho}(l) \|, \tag{3}$$

as proposed in Mantegna (1999), with C_{ij} being the correlation coefficient of the returns $r(i)$ and $r(j)$.

The fact that this is a properly defined distance gives a meaning to geometric notions and geometric tools in the study of the market.

Given the set of distances between points, the question now is reduced to an embedding problem: one asks what is the smallest manifold that contains the set. If the proportion of systematic information present in the correlations between stocks is small, then the corresponding manifold will be a low-dimensional entity. The following stochastic geometry technique was used for this purpose.

3.1. *The stochastic geometry technique*

After the distances (d_{ij}) are calculated for the set of N stocks, they are embedded in R^D, where $D < n$, with coordinates $\boldsymbol{x}(k)$. The center of mass $\boldsymbol{R}$ is computed and coordinates reduced to the center of mass,

$$\boldsymbol{R} = \frac{\sum_k \boldsymbol{x}(k)}{k}, \tag{4}$$

$$\boldsymbol{y}(k) = \boldsymbol{x}(k) - \boldsymbol{R}, \tag{5}$$

and the inertial tensor,

$$T_{ij} = \sum_k y_i(k) y_j(k), \tag{6}$$

is diagonalized to obtain the set of normalized eigenvectors $\{\lambda_i, \boldsymbol{e}_i\}$. The eigenvectors $\boldsymbol{e}_i$ define the characteristic directions of the set of stocks. The characteristic directions correspond to the eigenvalues (λ_i), which are clearly different from those obtained from surrogate data.

†The concept of the center of mass is that of an average of the masses of the components of a object multiplied by their distances from a reference point. It is also called the centroid or center of gravity, corresponding to the point of a body (or object) at which the force of gravity can be considered to act and which undergoes no internal motion. It is a point at which the object's mass can be assumed to be concentrated.

‡Surrogate data is generated by permuting data (one-day return) of each stock randomly in time. As each stock is independently permuted, time correlations among stocks disappear while the resulting surrogate data preserve the mean and the variance that characterize actual data.

They define a reduced subspace of dimension d, which carries the systematic information related to the market correlation structure (Vilela Mendes *et al.* 2003). In order to improve the decision criterion on how many eigenvalues are clearly different from those obtained from surrogate data, the normalized difference τ is computed,

$$\tau(i) = \lambda(i) + 1 - \lambda'(i), \tag{7}$$

and the significantly different eigenvalues are those for which $\tau(i) > 1/2$.

3.2. Index of market structure

Since the largest d eigenvalues define the effective dimensionality of the economic space, at time t, we compute S as

$$S_t = \sum_{i=1}^{d} \frac{\lambda_t(i) - \lambda'_t(i)}{\lambda'_t(i)} = \sum_{i=1}^{d} \frac{\lambda_t(i)}{\lambda'_t(i)} - 1, \tag{8}$$

where $\lambda_t(1), \lambda_t(2), \ldots, \lambda_t(d)$ are the largest d eigenvalues of the market space and $\lambda'_t(1), \lambda'_t(2), \ldots, \lambda'_t(d)$ are the largest d eigenvalues obtained from surrogate data, namely from data obtained by independent time permutation of each stock. In computing S, at a given time t, both λ_t and λ'_t are obtained over the same time window and for the same set of stocks.

Vilela Mendes (2001) proposed an index that quantifies the effect of structure-generating mechanisms in dynamical models, based on the fact that a structure in a collective system acquires a characteristic length larger than that of the individual components of the system. We develop this strategy for the definition of our structure index S: as the dynamics of systems develop a structure-generating mechanism, the index S measures the normalized difference between the characteristic size of those structures and the characteristic size of the individual components of the system. This is a geometrical approach to define and to measure emergence.

In portfolio optimization models, when the systematic and unsystematic contributions to the portfolio risk are distinguished, the former is associated with the correlation between stocks (collective structure) and the latter with the individual variances of each stock (Vilela Mendes *et al.* 2003). Consequently, when S is applied to the market space, the eigenvalues obtained from surrogate data (λ'_t) may be taken as reference values that represent the characteristic size with which each leading direction contributes to the shape of the market whose components were uncorrelated. These eigenvalues correspond to the characteristic size of the individual (isolated) components of the market. On the other hand, the eigenvalues obtained from actual data (λ_t) represent the characteristic size of each structure emerging from the dynamics of the market, that is, associated with each leading direction of the market space.

Although the dynamical structure-generating mechanism in market spaces is not related to positive Lyapunov exponents as proposed in Vilela Mendes (2001), the dynamical features we attempt to capture are those associated with changes occurring in the leading directions of the market space. As the eigenvalues obtained from actual data describe the structure emerging from the dynamics of the correlations between stocks, they may be taken as a measure of the collective structure of the market, this being the structure generated by the dynamics of the market (i.e. by the synchronous behaviour of stocks).

4. Results and discussion

Results were computed in relation to actual daily returns data as well as to surrogate data that were generated by permuting each stock (one-day return data) randomly in time. As each stock is independently permuted, time correlations among stocks disappear, while the resulting surrogate data preserve the mean and the variance that characterize the actual data.

4.1. The S&P 500 effective dimensions

The first set of actual data consists of 249 stocks present in S&P 500 from July 1973 to March 2003, considering all the surviving firms for the whole period. Part of the ordered eigenvalue distributions obtained from actual data and surrogate data is shown in figure 1. The plots represent the largest 25 eigenvalues obtained for the first set of actual data. The largest 25 eigenvalues are compared with the largest 25 eigenvalues obtained from surrogate data. In the lower plot, the comparison between actual and surrogate data is emphasized by computing their normalized difference τ (equation (7)).

Given the decrease obtained from the seventh eigenvalue, we conclude that the market structure is essentially confined to a six-dimensional subspace. This demonstrates that this subspace captures the structure of the deterministic correlations that are driving the market and that the remainder of the market space may be considered, for the current purpose, as being generated by random fluctuations.

To test the robustness of this conclusion, we divided the data into two chronologically successive batches (the first consisting of daily data from July 1973 to March 1988, and the second batch including data from March 1988 to March 2003) and performed the same operations. In spite of the changes in the market through time, in both cases the behaviour of the eigenvalues distribution is very much the same.

Apart from statistical fluctuations, the reconstructed spaces exhibit a reasonable degree of stability, confirming that the number of characteristic dimensions of the S&P 500 market space is six. Considering this result, our analysis of the S&P 500 market shape is based on six-dimensional subspaces. The question now is to assess the extent to which the occurrence of extreme phenomena modifies the shape of this subspace and the pattern of behaviour of firms and sectors.

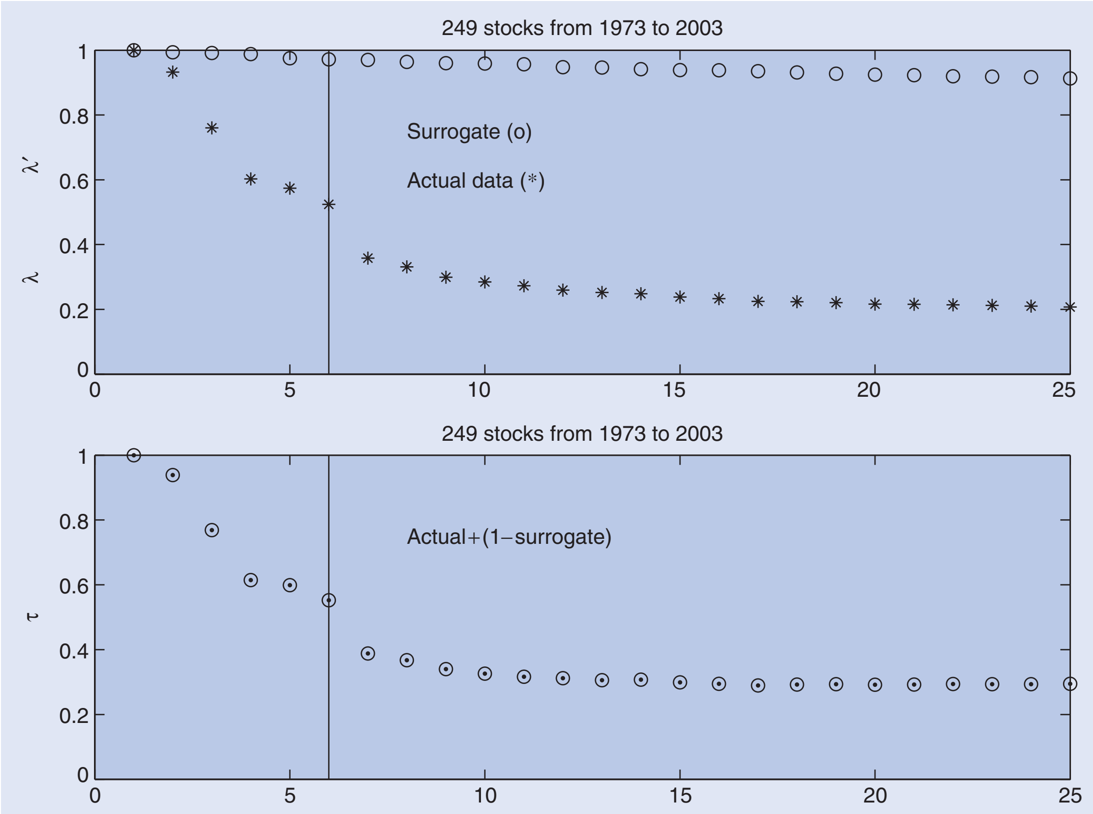

Figure 1. S&P 500 249 stocks: decrease of the largest 25 eigenvalues.

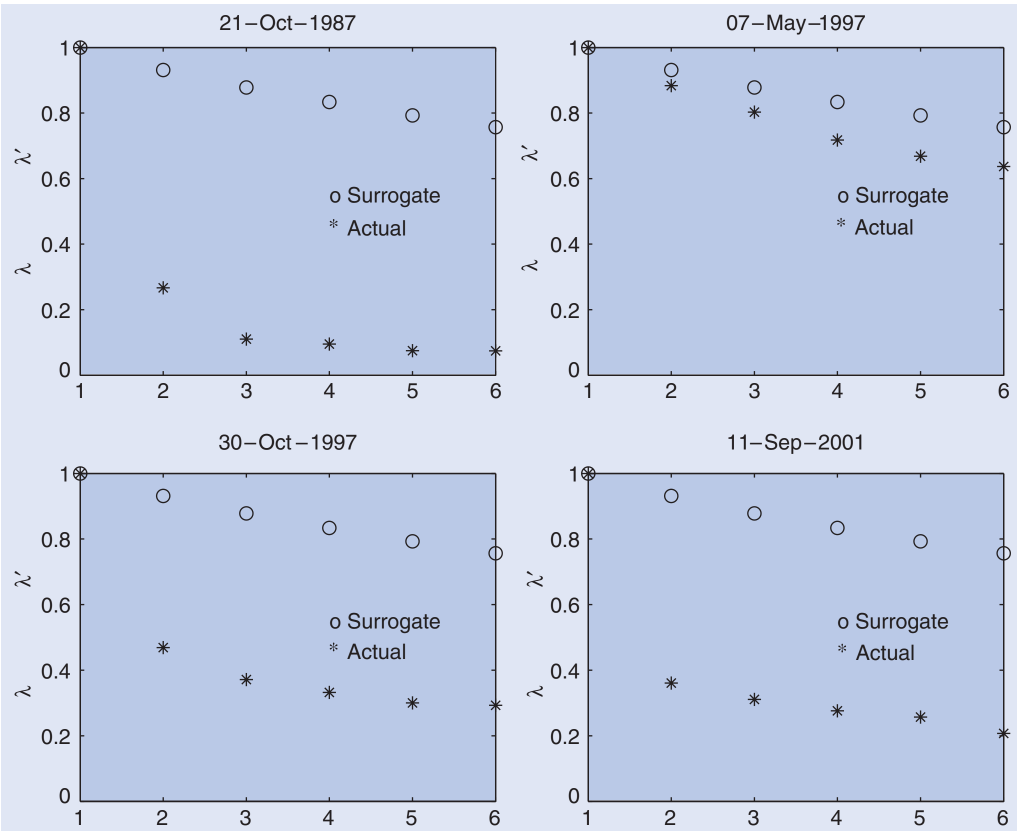

Figure 2. S&P 500 deviation from randomness on different dates, comparing crises and a day of business-as-usual (6 May 1997).

4.2. The dynamics of crashes

As extreme phenomena are dated events and as we look for their consequences in the distributions of the six leading directions, the geometry of the historical data is defined considering short periods. In this sense, instead of the large time intervals that defined the reconstruction of the S&P 500 space as in Vilela Mendes *et al.* (2003), we adopted a 16-day window as the chosen time interval and computed the structure index with the time window centered at several different dates.

The plots in figure 2 show some of these dates, namely the crashes of 19 October 1987, Black Monday, 11 September 2001 and 27 October 1997, the second Black Monday. The second plot in this figure shows an unimportant date: May 6, 1997, as suggested in Bonanno *et al.* (2001b), a typical normal day on the US stock market. The plots in figure 2 show $\lambda(i)$ (with $i = 1, \ldots, 6$) obtained from the S&P 500 market space on four different dates. It is clear that the values of S obtained for the first and second Black Mondays and for 11 September 2001 are high, as there is a large difference in the decrease of the first six eigenvalues computed from actual and surrogate data.

On the contrary, when the same calculation is performed around a typical normal date, the results show that, comparing actual data with surrogate data, there is quite a small difference in the decrease of the first six eigenvalues, which is a further piece of evidence for the robustness of our method.

The geometrical changes in the shape of the market space describe the structural evolution of the characteristic dimensions. As previously indicated, normal periods qualitatively tend to randomness, whereas disturbed periods will tend away from randomness. The null hypothesis for calculating S would state that, independent of the period (normal or disturbed) around which the index is computed, the decrease of the first six eigenvalues is equivalent to that obtained from uncorrelated data (with the same mean, distribution and variance of the actual data and S calculated using the same time window in both situations).

A less detailed but more extensive result is presented in figure 3, where the plot shows the daily values of S for the 30 year period. We used a time moving window of 16 days on a market space including 249 stocks, i.e. all firms surviving throughout the whole period. The eight highest values of S are marked on the plot. The highest peaks are identified and correspond to the following crashes:

1. October 1987	5. April 1999
2. October 1989	6. Dec. 2000/Jan. 2001
3. October 1997	7. April 2001
4. October 1998	8. September 2001

The ranking of the crashes according to the measure of S and its explanation is as follows.

1. October 1987: Black Monday.
2. December 2000 to January 2001: Argentinean financial crisis (Argentina and Turkey bond market sell-off).
3. October 1989: the US stock market falls almost 7%.
4. September 2001: attack on the Twin Towers.
5. April 1999: Nikkei crash (Japan).

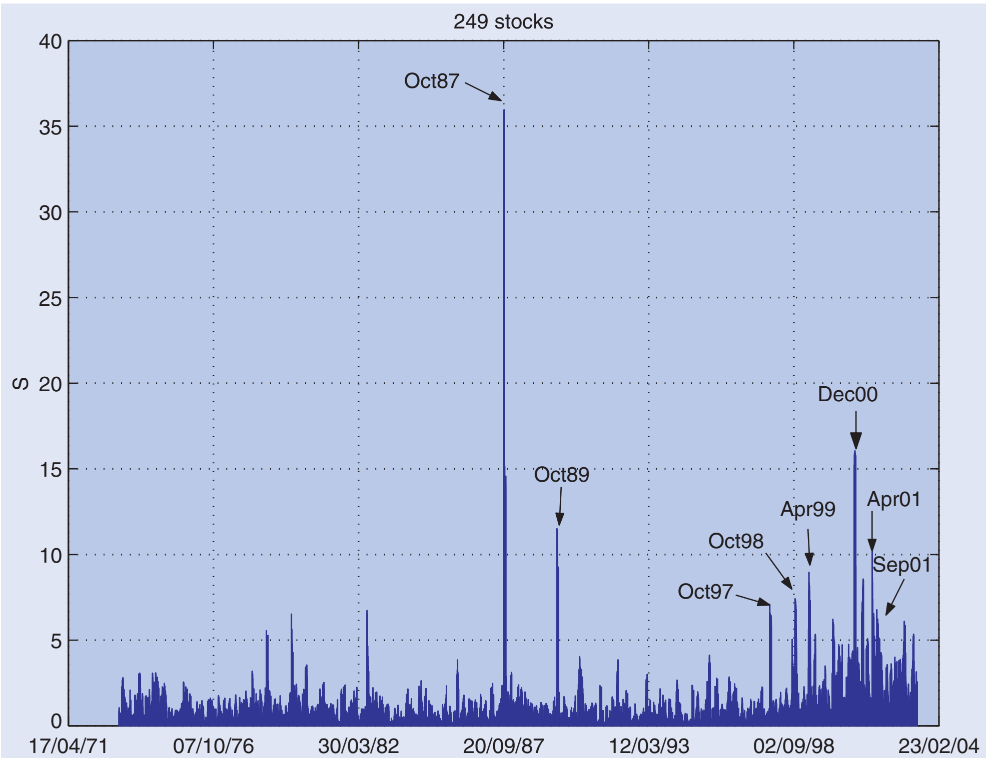

Figure 3. Evolution of the index S, measuring the evolution of the S&P 500 structure.

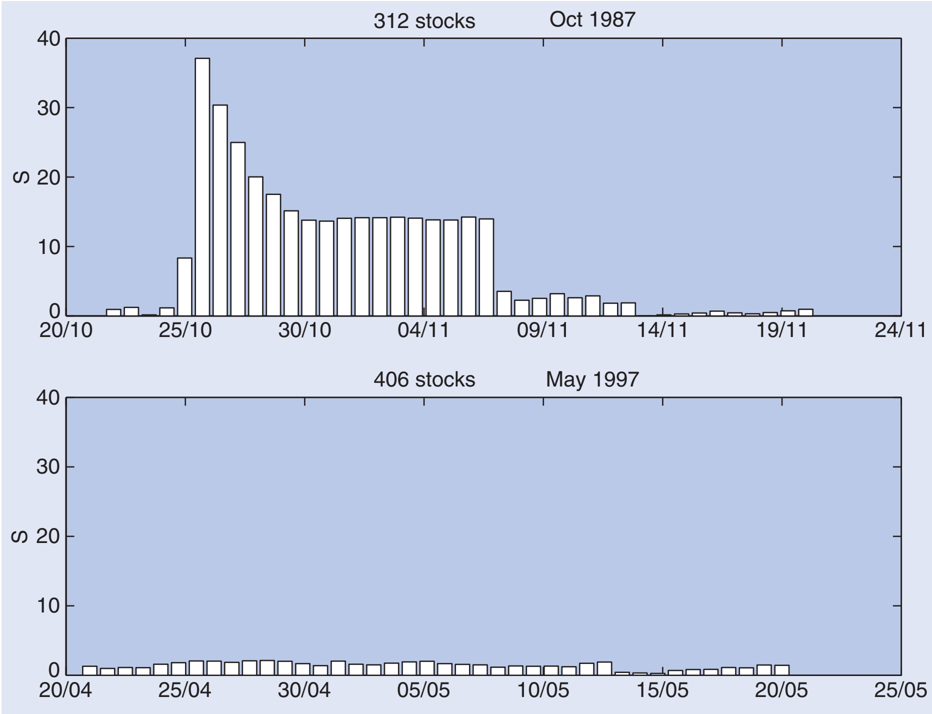

Figure 4. *Black Monday* and a day of business-as-usual.

6. March/April 2001: according to the NBER a recession began in the US in March 2001.
7. October 1998: Russian crash.
8. October 1997: Asian crash, the second Black Monday.

It is quite obvious from figure 3 that we have two periods of crises, clustering in 1987–1989 and in 1997–2001: the nature of these periods is discussed below. It should also be noted that some of the events in the list refer to crises in emergent market countries, with considerable effects on the dynamics of the world economy; others refer to the effect of different factors. Indeed, the nature of the triggering factors varies widely. The 1987 crash is well researched and corresponds to a major malfunctioning of the financial system. As Wright (2002) points out, the Dow Jones suffered a major loss of 22.61% on 19 October 1987, whereas the losses were 12.82% on 28 October 1929 and 11.73% on 29 October. Considering the 55 days around the trough, the accumulated loss was 39.6% in 1929 and 36.1% in 1987.

Having identified the events corresponding to the eight highest values (peaks) of S in the last 30 years (figure 3), we reconsidered our data, investigating the periods around each peak. In addition to providing a more accurate picture of the evolution, it allows for a better measurement, since, at each window, we consider a larger number of companies in the S&P 500. For the purpose of comparison, the first plot in figure 4 shows the behaviour of S in the region of the highest peak compared with the values of S around a typical normal day on the US stock market (table 1).

Unsurprisingly, the highest peak corresponds to Black Monday, which is not only the largest peak, but also the longest-lasting crisis. The most interesting change in the ranking of crashes concerns the appearance of the NASDAQ collapse in April 2000, which was hidden by the fact that some emerging firms in the nineties were not considered in our previous data set since they did not exist for the whole (30 year) period. Yet, when they are considered, the real picture of a turbulent market appears very clearly: it was in the Information Technology and Telecommunication sector that most speculation and stock activity concentrated in the late nineties, during the Internet bubble, and the NASDAQ crash marks its end. This crash demonstrates the dimension of this speculative process. The NASDAQ attained its highest peak by early March 2000, and then its all-time largest loss by April (35% of the loss in relation to the peak the previous month) (figures 5 and 6).

In order to check the predictive character of our structure index, a smaller time interval (10 day window) was chosen for the computation of S. The idea was to better observe the behaviour of S before the window hit

Table 1. Ranking of crises according to the value of S_{max}.

Ranking	Date	S_{max}	Stocks included
1	October 1987	37.7	312
2	Dec. 2000/Jan. 2001	16.2	426
3	October 1987	11.3	330
4	Mar/April 2001	8.5	426
5	April 2000	8.6	424
6	April 1999	8.1	417
7	October 1997	7.1	408
8	October 1998	6.4	414
9	September 2001	6.3	426

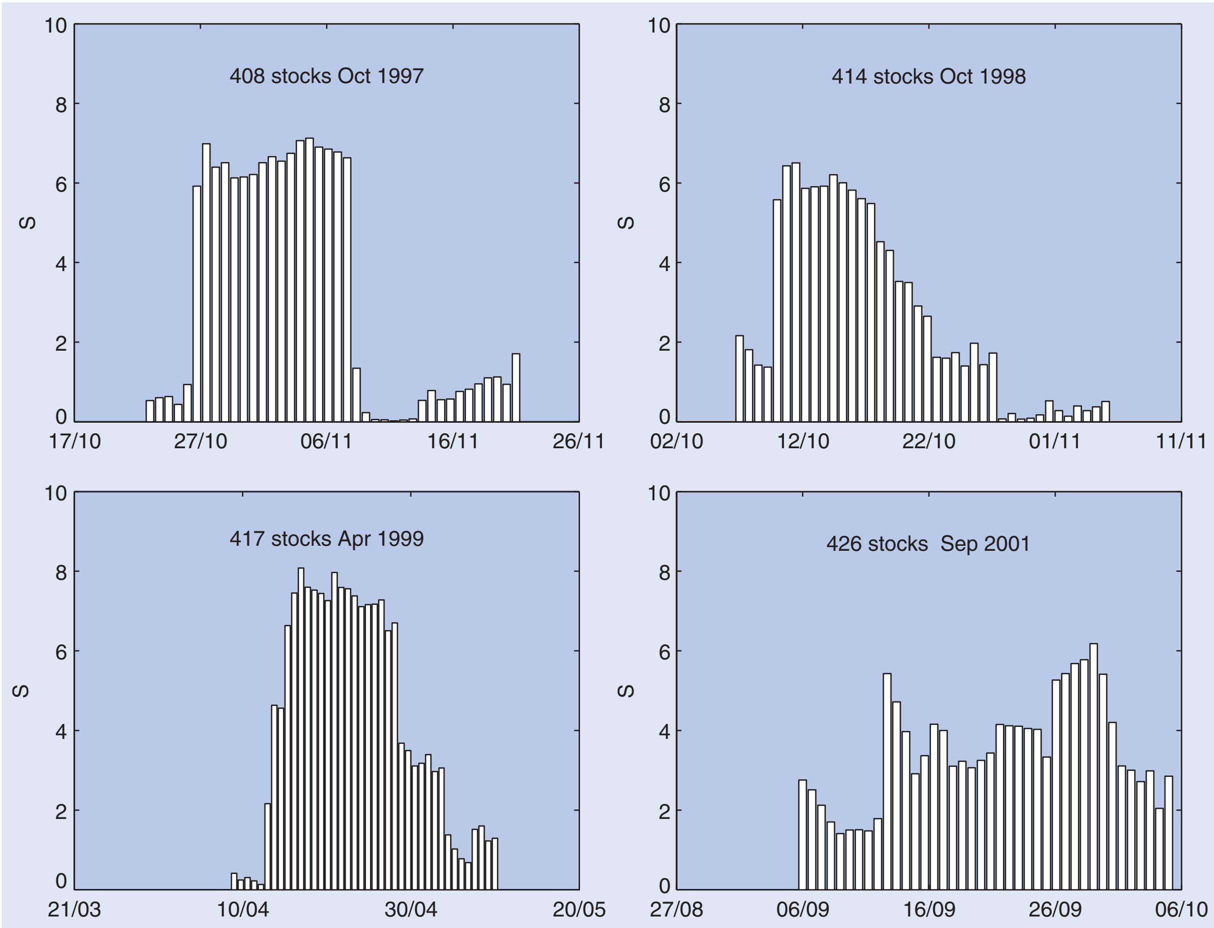

Figure 5. Crises of 1997, 1998, 1999 and September 2001.

the specific day of the crash, mainly on 19 October 1987 and 27 October 1997. It is remarkable that the behaviour of S remains almost unchanged before the 10 day window hits the day of the crash and after leaving the region comprising that time interval.

In the next section, sectoral dynamics is taken into account, showing that some crises tend to concentrate in specific sectors, while other crises tend to exhibit a pattern of perturbation in all sectors.

4.3. *Comparison of sectoral dynamics*

When, instead of the whole set of stocks, we consider sub-sets including the stocks of firms belonging to the same economic sector† and compute the index of market structure for each of these sub-sets, evidence of some interesting properties emerges.

In a previous paper and using several topological indexes (Vilela Mendes *et al*. 2003), we verified that, in periods of expansion, sector-oriented sub-sets are characterized by a smaller average distance between stocks. The average behaviour of companies belonging to the same economic sector is more synchronous than the behaviour of the overall market taken as a whole: in the jungle of the crisis, tribes of firms act together.

We now analyse sectoral dynamics by considering the consequences of crashes on the leading directions of nine sector-oriented market spaces, each restricted to stocks in one of the following sectors: Energy, Materials, Industry, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology and Utilities.

The histograms in figures 7–9 show the value of S_{max} obtained from the nine different market spaces, all for the same time period, which is indicated at the top of the plot. The results show the remarkable impact of the Asian crisis on the Financial sector and the strong effect of the attack on the Twin Towers on the Materials and Industrial sectors.

From the plots in figures 7–9 we can see that some crises tend to concentrate in a few specific sectors (financial companies for the Asian crash, industrial materials and financial companies for the case of the reaction to 11 September). In contrast, the Black Monday crisis exhibits a pattern of perturbation in all

†Detailed structures of sectors and other information from Global Industry Classification Standard (GICS®), available at http://www.standardpoors.com/, referenced in June, 2005.

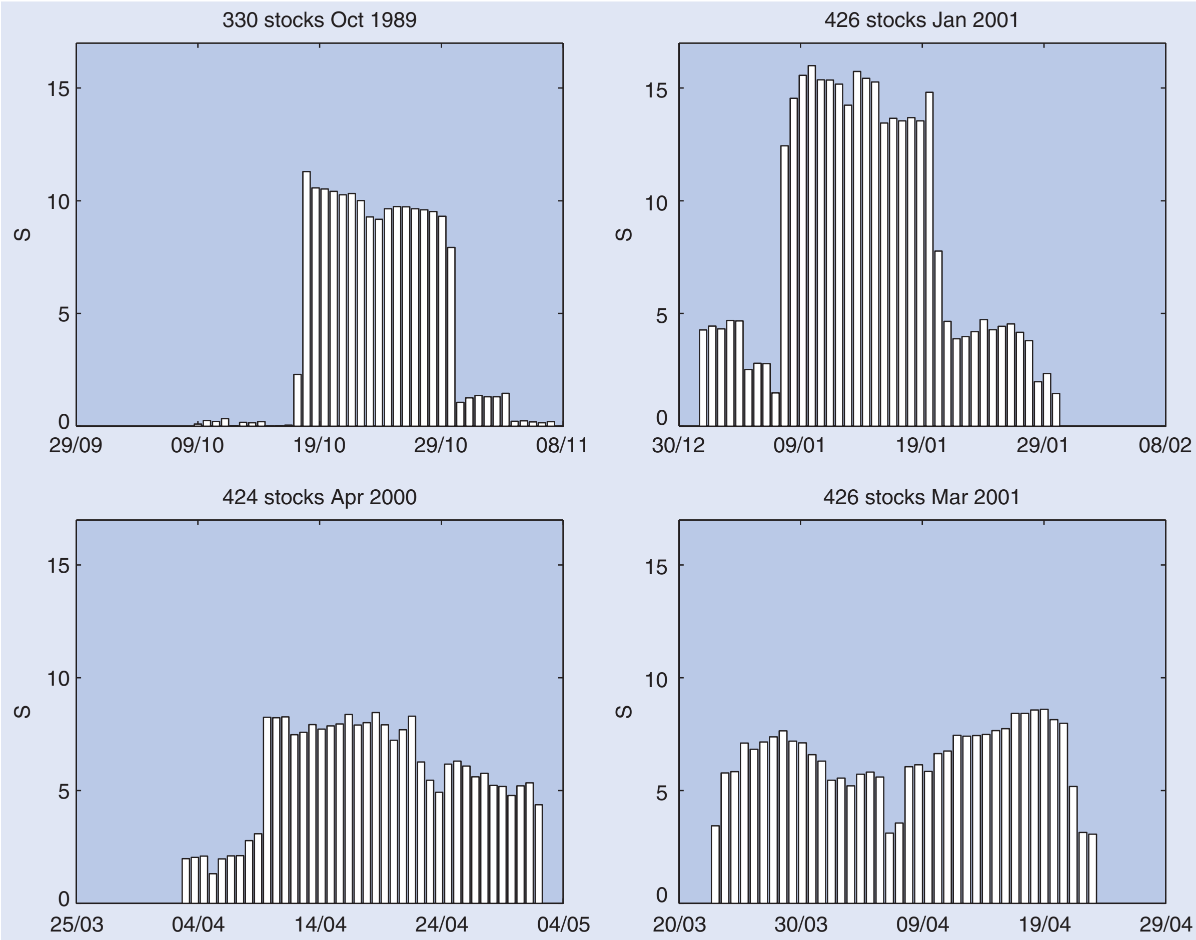

Figure 6. Crises of 1989, 2000 and 2001.

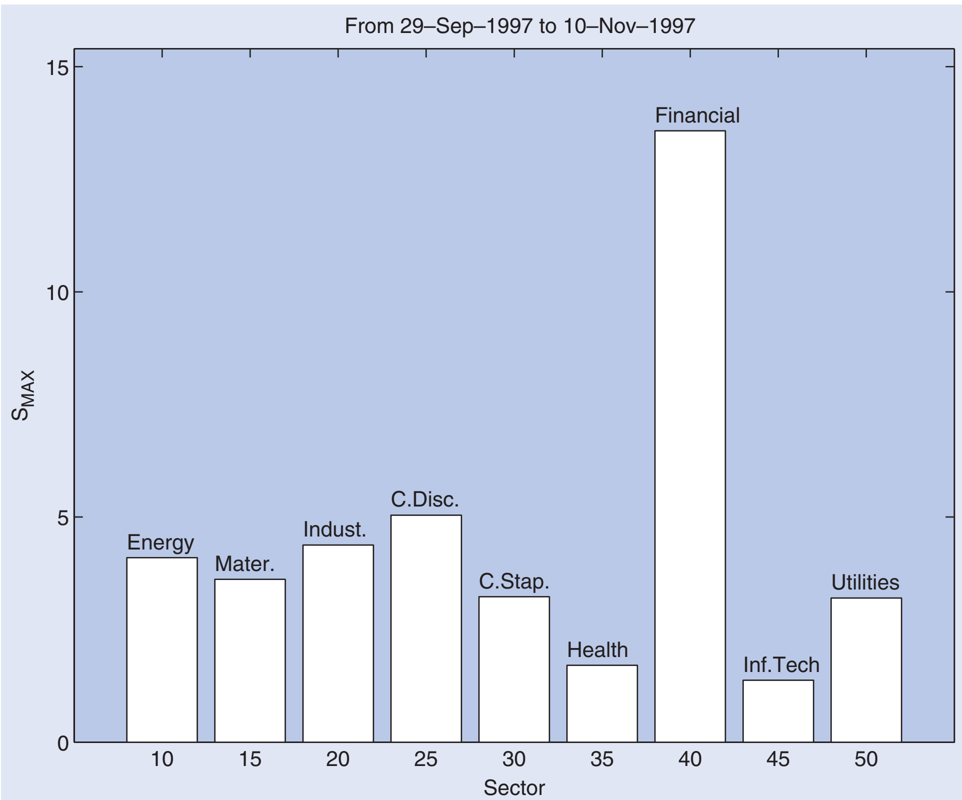

Figure 7. Asian crash.

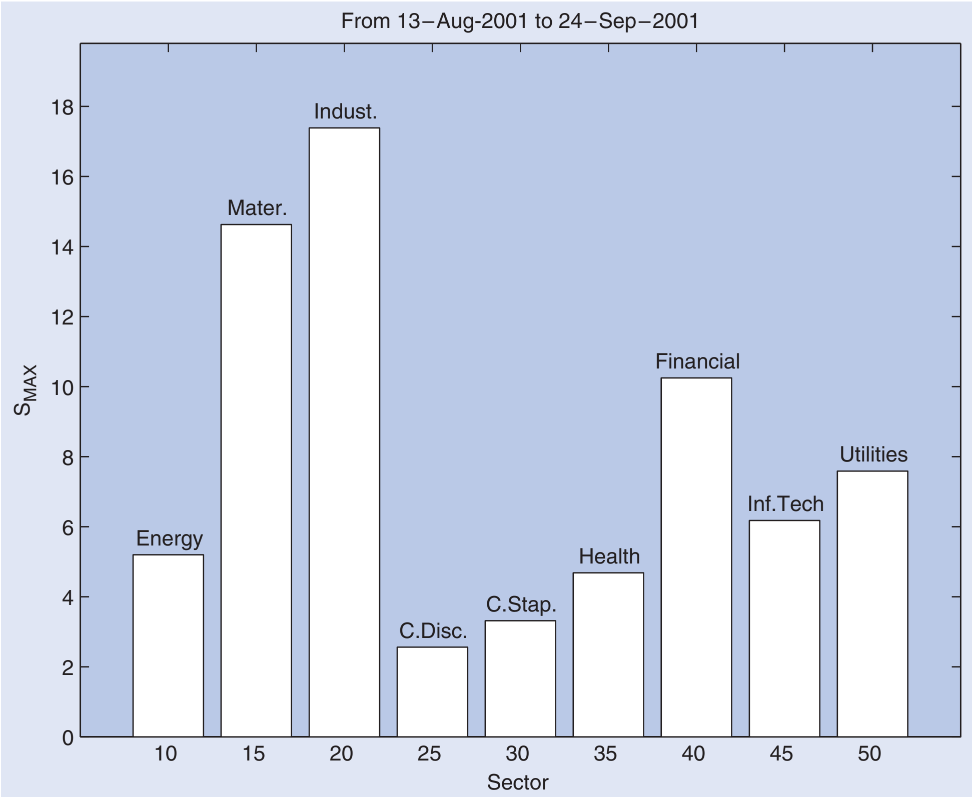

Figure 8. September 11.

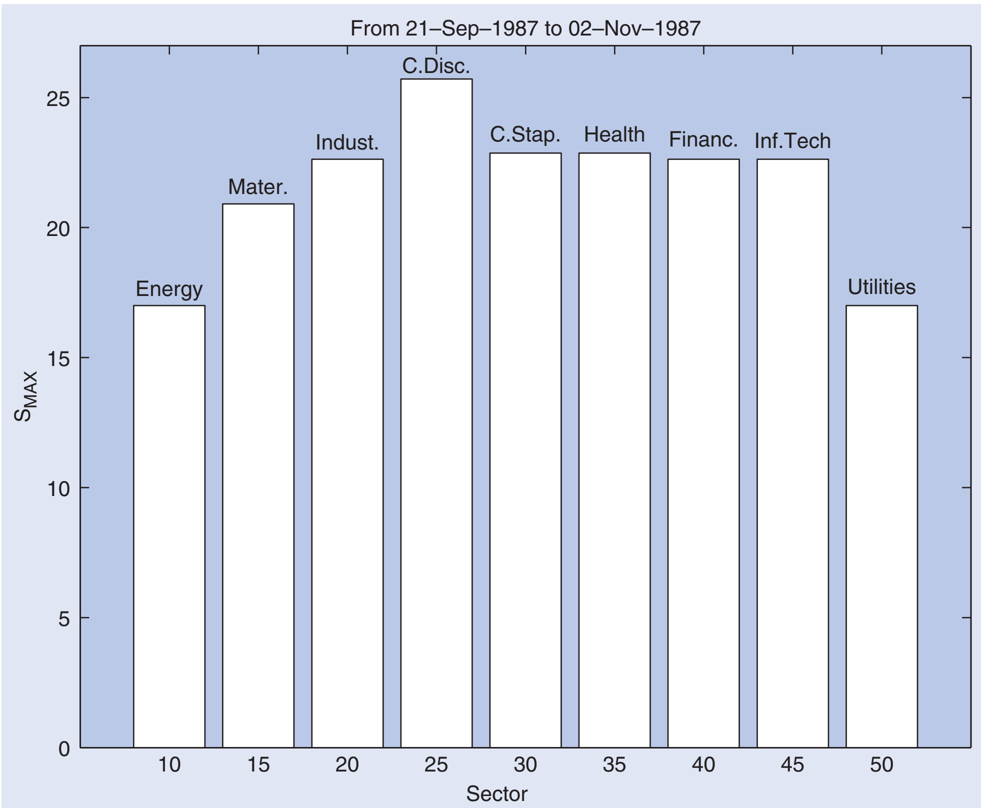

Figure 9. Black Monday.

sectors (figure 9). The plot in figure 9 shows the extraordinarily unique character of Black Monday 1987: this is the only case of a crash provoking similar dynamics in all major sectors, whereas in all other crises the dynamics and time pattern of the main sectors is clearly divergent. Table 2 summarizes the sectoral pattern of the crashes, indicating the sectors leading the structural change.

From the above results, one can observe that the Financials sector is the sector that most frequently appears as a leading sector. Its appearance as the leading sector in both the Argentinean and Asian crises is in accordance with the appropriate expectations, since each of these crises corresponds to a major malfunctioning of the financial system. Another encouraging result refers to the Information Technology leadership in the NASDAQ crisis, settling the end of the Internet Bubble in the second half of the nineties.

Table 2. Description of the sectors dominating each crash.

Date	Leading sectors
October 1987, Black Monday	All
January 2001, Argentinean crisis	Financials
October 1989, US stock market	Consumer staples/ Financials
September 2001, Twin Towers	Industrials/Materials/ Financials
April 2000, NASDAQ	Information Technology (IT)
October 1998, Russian crash	Energy/Utilities
April 1999, Nikkei crash	Consumer discretionary
April 2001, US recession	Energy/IT
October 1997, Asian crash	Financials

Finally, we compare the sectoral dynamics among different crashes, taking the examples of Materials and Financials. Because in the Black Monday crisis the index S reaches very high values in all sectors, this crisis was intentionally excluded from the plots in figure 10.

Returning to the geometrical tail of our index, a three-dimensional look at the market space that evolves from October 1989 to September 2001 and comprises, on average, 80 Financial stocks (the lower plot in figure 10) reveals that: (i) it starts from an elliptical form (in 1989), (ii) it acquires prominence in a particular direction in the 1997 Asian Crash, and (iii) it returns to a close-to-spherical form until the Argentinean Financial crisis in December 2000. After a partial shape recovery, a new relevant distortion appears in September, 2001.

A smoother dynamics characterizes the market space built from stocks in the Materials sector along the same time period (1989 to 2001). According to the results presented above (the upper plot of figure 10), the only relevant shape distortion of that market space is that taking place on 11 September 2001, when the structure index S reaches a value three times higher than the highest value obtained so far for the Materials market space.

5. Conclusions

A stochastic geometry technique has proven to be useful for the purpose of describing and interpreting the evolution and changes in the dynamics of a market.

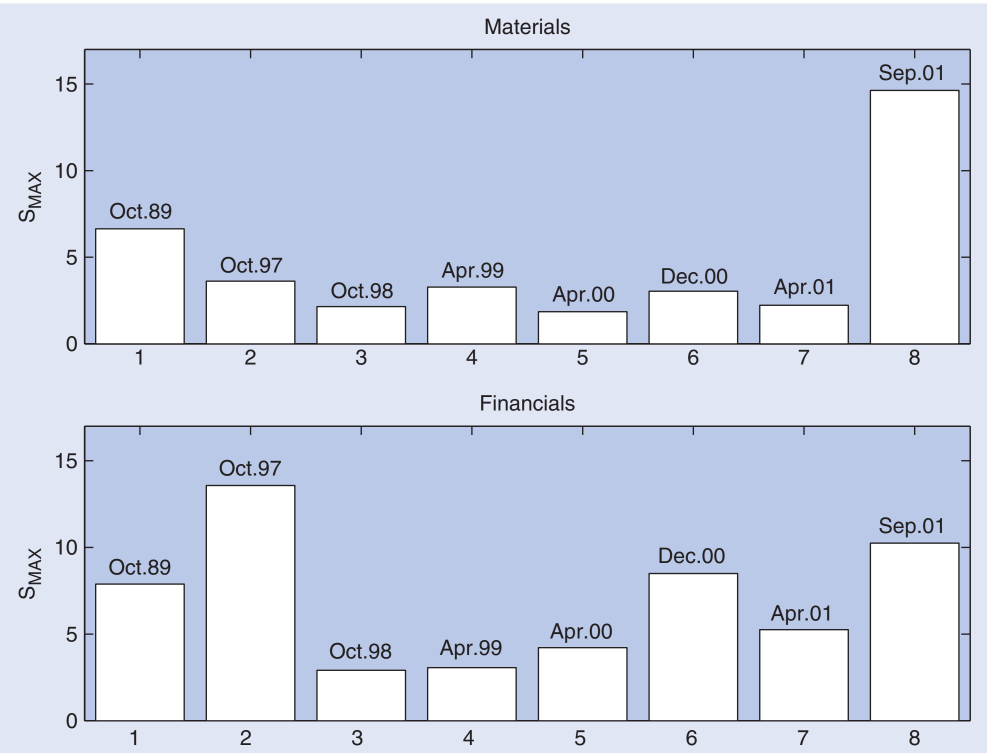

Figure 10. Materials and Financials dynamics.

Furthermore, the index *S*, as defined in this paper, allows for a useful taxonomy of the nine major stock market crises occurring in the last 30 years. The measure *S* proved to be useful and capable of discriminating among the distinct processes at work in the stock market.

As the index *S* captures the lack of uniformity among the market effective dimensions, we are able to characterize the extent to which crashes act differently in specific directions, causing changes in the shape of the market space. Looking for relevant distortions in the shape of the S&P 500 market space over the last 30 years, we identified the events corresponding to crises in emergent market countries, with considerable effects on the dynamics of the world economy. Others events that were also identified refer to the effect of different factors, showing that the nature of the triggering factors varies widely.

The identification of the characteristics of each crisis allows for their differentiation. Some crises were caused either by disarrangements of national stock markets from emerging economies (Russia, Asia) and global players (Japan) or by purely exogenous factors (the 11 September attack). The crash provoked by exogenous factors is less consequential and is rapidly superseded. Instead, the Black Monday crisis followed another pattern: it is deeper, longer and involved a large number of sectors. The Argentinean crises (December 2000 to January 2001) and the following NASDAQ crisis (April 2000) and the US recession (April 2001) initiated or followed the end of the Internet Bubble of the second half of the nineties.

Black Monday (1987) was the deepest and longest of all the crashes, as well as the more general, since it involved all economic sectors. The data suggest that another structural crisis may be at work in the clustering of six crashes between April 1997 and September 2001.

Finally, we have provided evidence of the existence of structure in financial market dynamics and, furthermore, of relevant changes in structure, mostly in periods of crises and crashes. Considering this evidence, the predictive character of our structure index is to be explored in future work.

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