How should we measure the return on public investment in a VAR?[a]

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Abstract:

Pereira's (2000) method of computing the return on public investment in a VAR is extended. A new return measure which accounts for public and private costs is proposed. An application to US data shows non-trivial differences between alternative return rates.

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1. Introduction

There has been considerable interest in measuring the effects of public investment on aggregate economic activity since Aschauer's (1989a, 1989b) contributions[1]. However, Aschauer's reliance on static OLS regressions has long been superseded by more sophisticated techniques. VAR models have been widely used: apart from handling matters of stationarity and cointegration, they make it possible to address the issue of reverse causality between output and public capital, and, more generally, to consider the dynamic effects among those two variables and other production factors, such as private capital. The impact of public investment on its private counterpart is a major concern when analyzing the merits of the former.

Dynamic feedbacks, however, make it difficult to quantify the macroeconomic return on public capital. The microeconomic rate of return draws on the standard marginal product[2], which holds other inputs constant; it is therefore unable to account for effects of crowding in or out. Pereira (2000) proposed an alternative rate, anchored on a VAR estimate of the "total marginal product" of public investment: this marginal product is based on the long-term response of output to a shock to public investment, and therefore incorporates the dynamic behaviour of the remaining inputs in the system. Though a major improvement on the microeconomic rate of return, we argue that Pereira's method fails to account for all the relevant investment costs in some circumstances.

This paper complements the work of Pereira (2000) and proposes a new definition of rate of return on public investment that tackles the above shortcomings. In section 2 we summarize

[1] See Batina (2001) for a survey of this literature.

[2] Recall that the rate of return equals the marginal productivity minus the rate of depreciation.

Pereira's method, discuss its limitations and present our preferred measure. Section 3 illustrates these different rates of return with a US dataset. Section 4 concludes.

2. Alternative definitions of rates of return

2.1 The approach of Pereira (2000)

We start by restating the methodology of Pereira (2000). Consider a VAR model with four endogenous variables, all in logs and first differenced: real public investment (*G*), real private investment (*I*), private employment and real private GDP (*Y*). Following an orthogonal impulse to public investment, the long-term accumulated elasticity of *Y* with respect to *G* (ε_G) is obtained from the accumulated impulse-response functions (IRFs) of the VAR:

$$\varepsilon_G = \frac{\Delta \log Y}{\Delta \log G}, \tag{1}$$

where Δ denotes a long-term response[3]. Pereira then proceeds to compute the marginal productivity of public investment,

$$MPG \equiv \frac{\Delta Y}{\Delta G} = \varepsilon_G \frac{Y}{G}, \tag{2}$$

where *Y/G* is set equal to an end-of-sample ten-year average of the output to public investment ratio[4]. This marginal productivity differs from its microeconomic counterpart, as it drops the *ceteris paribus* assumption and includes the indirect effects of public investment on GDP through the dynamic responses of private inputs.

[3] *Long-term* refers to the time horizon over which the IRFs converge.
[4] The use of a ten-year average minimizes contamination by cyclical fluctuations.

Finally, assuming that capital goods last for 20 years, Pereira computes the annual rate of return on public investment as the value of r that solves $(1+r)^{20} = MPG$. For reasons explained below, we will call this rate the *partial-cost dynamic feedbacks rate of return*.

2.2 A critique and a suggested alternative

We agree with Pereira (2000) on the importance of including in the analysis the indirect output effects of public investment: taking on board the induced response of private inputs – and especially of private investment – boils down to accounting for possible crowding in or crowding out effects. Under some circumstances, however, we disagree on the way investment costs are measured.

From a fiscal viewpoint, only the costs incurred by the public sector matter. Whatever the response of private investment, one should compare the cost of public investment *alone* to the *total* output gains (both direct and indirect), which generate further tax revenues. This is what Pereira (2000) does, and he discusses (p. 517) whether public investment pays for itself in budgetary terms.

From a macroeconomic viewpoint, however, it is important to account for the full cost of investment, both public and private. If crowding in (out) takes place, the previous approach is underestimating (overestimating) the total investment effort to achieve a given output change, and is therefore overestimating (underestimating) the return on public investment. To address this shortcoming, we can easily adapt the methodology of section 2.1.

Following a structural shock to public investment, we compute a long-term accumulated elasticity of Y with respect to private investment (ε_I) and a long-term accumulated marginal productivity of private investment (*MPI*). Thus:

$$\varepsilon_I = \frac{\Delta \log Y}{\Delta \log I}; \tag{3}$$

$$MPI \equiv \frac{\Delta Y}{\Delta I} = \varepsilon_I \frac{Y}{I}. \tag{4}$$

The marginal productivity of *total investment* is given by

$$MPTI = \frac{\Delta Y}{\Delta G + \Delta I} = \frac{1}{MPG^{-1} + MPI^{-1}}, \tag{5}$$

and we determine the return on public investment as the value of r that solves $(1+r)^{20} = MPTI$, terming it the *full-cost dynamic feedbacks rate of return*.

For completeness, and as a benchmark for comparisons, one can also measure the return to public investment when private investment stays constant by estimating a system where private investment is an exogenous variable. The crowding in (out) mechanism is shutted down – private investment contemporaneous and lagged changes, even if they impinge on the system endogenous variables, do not depend on the latter. The computational details are then similar to section 2.1. We call the ensuing rate the *ceteris paribus rate of return*.

3. An application to the US economy[5]

Our annual dataset is an update of Pereira's (2000) and is obtained from the US Bureau of Economic Analysis internet site. We measure private GDP, private investment and public

[5] Econometric results were obtained using GiveWin and PcGive 10. See Doornik and Hendry (2001) for a complete software description. More detailed results are available from the authors on request.

investment excluding defense in constant 2000 dollars, and private employment in full time equivalent employees. The sample runs from 1956 to 2001.

Augmented Dickey Fuller tests strongly suggest that the log-levels of these variables are non-stationary, I(1) time series. Following the Johansen (1988) procedure, results from trace and maximum eigenvalue tests with a small sample correction suggested by Reimers (1992) do not allow us to safely dismiss the null hypothesis of no cointegration. We then proceed to estimate a VAR in first differences of log-levels. Starting with four lags, a constant and a deterministic trend, model reduction F-tests have lead us to consider a more parsimonious formulation – an order 3 VAR with a constant but without trend[6].

Orthogonal disturbances are identified through a Choleski decomposition with public investment ordered first, as in Pereira (2000). Because of the lags in government decision-making, it is assumed that public investment does not respond contemporaneously to any structural disturbances to the remaining variables.

Table 1 – Long-term estimated elasticities of private output with respect to investment

	Public Investment	Private Investment
VAR 1 (all variables endogenous)	0.1220	0.6407
VAR 2 (exogenous private investment)	0.0769	not applicable

Table 1 summarises results for long-term investment elasticities, derived from the converged accumulated IRFs. Using the output to investment average ratios in the ten final years of the

[6] Further model reductions are not acceptable as residuals would display autocorrelation.

sample, it becomes possible to compute the several marginal productivities and the implied rates of return presented in Table 2.

Table 2 – Macroeconomic returns from public investment

	Marginal productivity	Rate of return
Partial-cost dynamic feedbacks	4.117	7.33%
Full-cost dynamic feedbacks	2.058	3.68%
Ceteris paribus	2.595	4.88%

The partial-cost dynamic feedbacks rate of return is very close to Pereira's (2000) estimate of 7.8 percent[7]. However, as public investment crowds in private investment, the full-cost dynamic feedbacks rate of return is much smaller – somewhat below 4 percent. For a similar reason (the exclusion of indirect output effects of public investment through induced private investment), the *ceteris paribus* rate of return is also smaller, lying close to 5 percent.

4. Conclusions

This paper has analysed how to compute the rate of return on public investment in a VAR framework. VAR models allow us to take account of the dynamic response of private inputs to a shock to public investment, and hence to study whether the latter is a source of crowding in or crowding out. We have discussed and refined Pereira´s (2000) method of incorporating dynamic feedbacks into the determination of the rate of return.

[7] Three factors explain why results do not exactly coincide: (i) samples are different; (ii) due to data revisions, even figures for common years may somewhat differ; (iii) the VAR order is not the same.

Pereira's method excludes from the computations the costs of induced private investment. This approach, though correct for fiscal purposes, overestimates the macroeconomic returns to public capital if there is crowding in, and underestimates them if crowding out occurs. We hence suggest a rate of return which accounts for both public and private investment costs. An empirical application to US data shows that differences between alternative rates of return are non-trivial: considering the whole investment effort has actually halved Pereira's estimated returns.

References

Aschauer, D., 1989a, Is public expenditure productive? Journal of Monetary Economics 23(2), 177- 200.

Aschauer, D., 1989b, Does public capital crowd out private capital? Journal of Monetary Economics 24(2), 171-188.

Batina, R., 2001, The Effects of Public Capital on the Economy. Public Finance and Management 1(2), 113-134.

Doornik, J. and D. Hendry, 2001, GiveWin, An Interface to Empirical Modelling. London, Timberlake Consultants.

Johansen, S., 1988, Statistical analysis of cointegration vectors. Journal of Economic Dynamics and Control 12, 231-254.

Pereira, A. M., 2000, Is All Public Capital Created Equal? The Review of Economics and Statistics. 82(3), 513-518.

Reimers, H.-E., 1992, Comparisons of tests for multivariate cointegration. *Statistical Papers* 33, 393-397.