Technological Change: A Burden or a Chance

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1 Introduction

The photography industry underwent a disruptive change in technology during the 1990s when the traditional film was replaced by digital photography (see e.g. The Economist January 14th 2012). In particular Kodak was largely affected : by 1976 Kodak accounted for 90% of film and 85% of camera sales in America. Hence it was a near-monopoly in America. Kodaks revenues were nearly 16 billion in 1996 but the prediction is that it will decrease to 6.2 billion in 2011.

Kodak tried to get (squeeze) as much money out of the film business as possible and it prepared for the switch to digital film. The result was that Kodak did eventually build a profitable business out of digital cameras but it lasted only a few years before camera phones overtook it.

According to Mr Komori, the former CEO of Fujifilm of 2000-2003, Kodak aimed to be a digital company, but that is a small business and not enough to support a big company. For Kodak it was like seeing a tsunami coming and theres nothing you can do about it, according to Mr. Christensen in The Economist (January 14th 2012).

This paper focuses on industries that have to deal with technological change. The above example showed that this can be a burden. However there are enough industries where technological change brings fruitful times in terms of profits. One example is the video game industry, where innovation plays a big role. The publishers, Activision, saw their worldwide sales increase with $650m in the first five days, when the new video game “Call of Duty: Black Ops” replaced its predecessor, Call of Duty: Modern Warfare 2, in November 2010 (The Economist, December 10th 2011). (Another example is the IPhone launched by Apple was described by Time Magazine as ”the invention of the year 2007“. In 2011 net income was $7.31bn

(4.6bn) in the three months to 25 June, 125% higher than a year earlier and a record quarterly profit for the firm. Revenue was $28.6bn, also a quarterly record.)

We study the problem of a firm that produces with a current technology for which it faces a declining sales volume. It has two options: it can either exit this industry or invest in a new technology with which it can produce an innovative product. We distinguish between two scenarios in the sense that the resulting new market can be booming or ends up to be smaller than the old market used to be.

2 Change Model

Assumptions: $\gamma > \mu$.

3 Model

The firm currently produces an established product. The quantity, which has to be determined at each point of time, is q_1, the price is p_1 and the inverse demand function is given by

$$p_1 = \mu\theta - q_1,$$

in which

$$d\theta = \alpha_1 \theta dt + \sigma\theta dz.$$

where $\alpha_1 < 0$. We distinguish between two types of cost. On the one hand it faces a fixed cost K. On the other hand the firm has to incur unit production costs being equal to c. Due to the latter feature it can be optimal to temporarily suspend production, i.e. $q_1 = 0$ for some time.

The firm has the option to start producing an innovative product which requires paying a sunk cost I. Denoting the price and the quantity of the new product by p_2 and q_2, respectively, and the moment of the new product launch the firm's demand function changes into:

$$p_2 = \gamma\theta - q_2.$$

Because this innovative market grows faster than the old one, we assume a different speed of development. In particular the dynamics of θ now becomes

$$d\theta = \alpha_2 \theta dt + \sigma\theta dz.$$

with $\alpha_2 > \alpha_1$. In fact, we can define the stochastic process $\theta(t)$ as follows

$$d\theta(t) = \alpha(t)\theta(t)dt + \sigma\theta(t)dz.$$

with

$$\alpha(t) = \begin{cases} \alpha_1 & \text{for } t < \tau^* \\ \alpha_2 & \text{for } t \geq \tau^* \end{cases} \tag{1}$$

where this τ^* will be specified later.

The cost structure for the new product also changes after the new product launch. Whereas the fixed cost still equals K, there are no variable cost. We motivate this by observing that in the digital world the unit cost of a product is most of the time very small or negligible.

It can be the case that the new market is not profitable enough for an investment to be undertaken. Since the old market is decreasing over time it can be optimal for the firm to exercise the option to exit the market. We also allow for the possibility to exit market after the investment in the innovative product has taken place.

Therefore, the optimal stopping problem can be stated as follows:

$$\begin{aligned} \mathcal{V}(\theta_0) &= \sup_{\tau_1} \mathbb{E}\left[\int_0^{\tau_1} e^{-rt}\Pi_1(\theta_1(t))dt + e^{-r\tau_1}\right. \\ &\left. \max\left(0, \sup_{\tau_2 \mathbf{1}_{\{\tau_2 > \tau_1\}}} \mathbb{E}\left[\int_{\tau_1}^{\tau_2} e^{-r(t-\tau_1)}\Pi_2(\theta_2(t-\tau_1))dt - I \,\middle|\, \theta_2(0) = \theta_1(\tau_1)\right]\right) \,\middle|\, \theta_1(0) = \theta_0\right] \end{aligned} \tag{2}$$

Here, τ_1 denotes the first time at which the decision maker decides to invest in product 2 or exit the market. τ_2 denotes the time that the firm would decide to exit the market of product 2, in case it has invested in the first run.

To determine the value of investing in project 2, we first solve the subproblem that is stated at the right hand side of the maximization in equation (2). Considering a specific current value for $\theta_2(0)$ the net expected discounted profit of investing in project 2 is given by:

$$V_2(\theta_2(0)) = \sup_{\tau_2} \mathbb{E}^{\theta_2(0)}\left[\int_{\tau_1}^{\tau_2} e^{-r(t-\tau_1)}\Pi_2(\theta_2(t-\tau_1))dt - I\right] \tag{3}$$

$$= \sup_{\tau_2} \mathbb{E}^{\theta_2(0)}\left[\int_0^{\tau_2-\tau_1} e^{-rt}\Pi_2(\theta_2(t))dt - I\right] \tag{4}$$

$$= \sup_{\tilde{\tau}} \mathbb{E}^{\theta_2(0)}\left[\int_0^{\tilde{\tau}} e^{-rt}\Pi_2(\theta_2(t))dt - I\right] \tag{5}$$

where $\mathbb{E}^\theta$ denotes the expectation with respect to the probability law Q^θ of the process $\theta(t); t > 0$ starting at $\theta(0) = \theta \in \mathbb{R}^n$. The optimal stopping problem in (3) is a standard problem. The instantaneous profits in region 2 are given by

$$\Pi_2 = p_2 q_2 - K = (\gamma\theta - q_2)q_2 - K \tag{6}$$

The optimal quantity equals $q_2 = \frac{\gamma\theta}{2}$ which results in a profit of

$$\Pi_2(\theta) = \frac{\gamma^2\theta^2}{4} - K. \tag{7}$$

Taking into account that there is an option to exit the market, standard calculations lead to the following expression for the optimal value function V_2:

$$V_2(\theta_2(0)) = \frac{\gamma^2\theta_2(0)^2}{4(r-2\alpha_2-\sigma^2)} - \frac{K}{r} + A\theta_2(0)^{\beta_4}. \tag{8}$$

where β_4 is the negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\alpha_2\beta-r=0$.

In order to derive the constant parameter A we study the decision to exit the market. Denoting the exit threshold by $\hat{\theta}_2$, we can write down the following value matching and smooth pasting conditions:

$$V_2(\theta)|_{\theta=\hat{\theta}_2} = 0, \tag{9}$$

$$\left.\frac{\partial V_2(\theta)}{\partial\theta}\right|_{\theta=\hat{\theta}_2} = 0, \tag{10}$$

from which we can derive that

$$\theta_2(\tilde{\tau}) = \sqrt{\frac{K}{\gamma^2 r}4(r-2\alpha_2-\sigma^2)\left(\frac{\beta_4}{\beta_4-2}\right)}, \tag{11}$$

$$A = \frac{1}{\theta_2(\tilde{\tau})^{\beta_4}}\frac{K}{r}\left(\frac{2}{2-\beta_4}\right). \tag{12}$$

where $\theta_2(\tilde{\tau})$ is the critical demand level above which it is optimal to exit the market.

This results in the following value function V_2

$$V_2(\theta) = \frac{\gamma^2\theta^2}{4(r-2\alpha_2-\sigma^2)} - \frac{K}{r} + \left(\frac{\theta}{\theta_2(\tilde{\tau})}\right)^{\beta_4}\frac{K}{r}\left(\frac{2}{2-\beta_4}\right). \tag{13}$$

Now we can write equation (2) as

$$\mathcal{V}(\theta_0) = \sup_{\tau_1} E\left[\int_0^{\tau_1} e^{-rt}\Pi_1(\theta_1(t))dt + e^{-r\tau_1}\max(0, V_2(\theta_2(0)=\theta_1(\tau_1))-I)\middle|\,\theta_1(0)=\theta_0\right] \tag{14}$$

Now we consider the situation before the investment. The firm has essentially three options. The first is to invest in product 2. The second is to suspend production. And the third is to exit the market. Let us first determine the current instantaneous profits. Maximizing the profit function w.r.t to the optimal output quantity, we derive that:

$$q_1 = \frac{\mu\theta - c}{2} \tag{15}$$

From this expression we see that the firm will suspend production provided that it has not exited already whenever θ is below $\frac{c}{\mu}$. The instantaneous profit when q_1 is positive equals:

$$\Pi_1(\theta) = \begin{cases} \left(\frac{\mu\theta-c}{2}\right)^2 - K & \text{for } \theta > \frac{c}{\mu}, \\ -K & \text{for } \theta \leq \frac{c}{\mu}. \end{cases} \tag{16}$$

Denoting the exit threshold by $\hat{\theta}_1$ we can consider the following two cases: If $\hat{\theta}_1 \geq \frac{c}{\mu}$ then the firm will never suspend production because it already has exited. If $\hat{\theta}_1 < \frac{c}{\mu}$ there exists a θ-interval, where it is optimal for the firm to suspend production. In this region the firm has two options: either to resume production if θ has increases sufficiently or to exit the market, which will happen when θ decreases even more. The following Proposition states the the there always exists the optimal policy for the stopping problem and specifies the optimal value function of the firm.

Proposition 1 *The optimal policy for the stopping problem of equation (2) always exists. The optimal value function is uniquely given by*

$$V(\theta) = \begin{cases} V_1(\theta) & \text{for } \theta \in D^* = (\hat{\theta}_1, \theta^*) \\ \Omega(\theta) & \text{otherwise.} \end{cases} \tag{17}$$

where we distinguish two cases for the function $V_1(.)$. If $\hat{\theta}_1 \geq \frac{c}{\mu}$ then the function is equal to

$$V_1(\theta) = \frac{\mu^2\theta^2}{4(r - 2\alpha_1 - \sigma^2)} - \frac{c\mu\theta}{2(r - \alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + A_1\theta^{\beta_1} + A_2\theta^{\beta_2} \tag{18}$$

while for case $\hat{\theta}_1 < \frac{c}{\mu}$ the function is equal to

$$V_1(\theta) = \begin{cases} \frac{\mu^2\theta^2}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu\theta}{2(r-\alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + B_1\theta^{\beta_1} + B_2\theta^{\beta_2} & \text{for } \theta \geq \frac{c}{\mu} \\ -\frac{K}{r} + B_3\theta^{\beta_1} + B_4\theta^{\beta_2} & \text{for } \theta < \frac{c}{\mu} \end{cases} \tag{19}$$

if $\theta^* > \frac{c}{\mu}$ and equal to

$$V_1(\theta) = -\frac{K}{r} + C_1\theta^{\beta_1} + C_2\theta^{\beta_2} \tag{20}$$

if $\theta^* < \frac{c}{\mu}$. The value of the firm in the stopping region is equal to $\Omega(\theta) = \max(0, V_2(\theta_2(0) = \theta))$. The optimal continuation region is $D^* = (\hat{\theta}_1, \theta^*)$. It is optimal to exit the market when $\theta < \hat{\theta}_1$ and invest in the new product when $\theta > \theta^*$. Otherwise, it is optimal to continue operations.

In order to derive the two thresholds θ^* and $\hat{\theta}_1$ we apply the value matching and smooth pasting conditions which leads to the following equation systems that implicitly define the thresholds for the different cases.

Value matching and smooth pasting for case $\hat{\theta}_1 \geq \frac{c}{\mu}$:

$$V_1(\theta)|_{\theta=\hat{\theta}_1} = 0 \tag{21}$$

$$\left.\frac{\partial V_1(\theta)}{\partial \theta}\right|_{\theta=\hat{\theta}_1} = 0 \tag{22}$$

$$V_1|_{\theta=\theta*} = V_2 - I|_{\theta=\theta*} \tag{23}$$

$$\left.\frac{\partial V_1}{\partial \theta}\right|_{\theta=\theta*} = \left.\frac{\partial V_2}{\partial \theta}\right|_{\theta=\theta*} \tag{24}$$

$$\frac{\mu^2\hat{\theta}_1^2}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu\hat{\theta}_1}{2(r-\alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + A_1\hat{\theta}_1^{\beta_1} + A_2\hat{\theta}_1^{\beta_2} = 0 \tag{25}$$

$$\frac{2\mu^2\hat{\theta}_1}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu}{2(r-\alpha_1)} + \beta_1 A_1\hat{\theta}_1^{(\beta_1-1)} + \beta_2 A_2\hat{\theta}_1^{(\beta_2-1)} = 0 \tag{26}$$

$$\frac{\mu^2(\theta^*)^2}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu(\theta^*)}{2(r-\alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + A_1(\theta^*)^{\beta_1} + A_2(\theta^*)^{\beta_2} = \frac{\gamma^2(\theta^*)^2}{4(r-2\alpha_2-\sigma^2)} - \frac{K}{r} + \left(\frac{(\theta^*)}{\hat{\theta}_2}\right)^{\beta_4}\frac{K}{r}\left(\frac{2}{2-\beta_4}\right) - I \tag{27}$$

$$\frac{2\mu^2(\theta^*)}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu}{2(r-\alpha_1)} + \beta_1 A_1(\theta^*)^{(\beta_1-1)} + \beta_2 A_2(\theta^*)^{(\beta_2-1)} = \frac{2\gamma^2(\theta^*)}{4(r-2\alpha_2-\sigma^2)} + \beta_4\frac{1}{\hat{\theta}_2}\left(\frac{(\theta^*)}{\hat{\theta}_2}\right)^{(\beta_4-1)}\frac{K}{r}\left(\frac{2}{2-\beta_4}\right) \tag{28}$$

If $\theta^* > \frac{c}{\mu} > \hat{\theta}_1$ then

$$V_{1,1}|_{\theta=\frac{c}{\mu}} = V_{1,2}|_{\theta=\frac{c}{\mu}} \tag{29}$$

$$\left.\frac{\partial V_{1,1}}{\partial\theta}\right|_{\theta=\frac{c}{\mu}} = \left.\frac{\partial V_{1,2}}{\partial\theta}\right|_{\theta=\frac{c}{\mu}} \tag{30}$$

$$V_{1,1} = 0 \tag{31}$$

$$\left.\frac{\partial V_{1,1}}{\partial\theta}\right|_{\theta=\hat{\theta}_1} = 0 \tag{32}$$

$$V_{1,2} = V_2 - I \tag{33}$$

$$\left.\frac{\partial V_{1,2}}{\partial\theta}\right|_{\theta=\theta^*} = \left.\frac{\partial V_2}{\partial\theta}\right|_{\theta=\theta^*} \tag{34}$$

$$\frac{\mu^2\left(\frac{c}{\mu}\right)^2}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu\left(\frac{c}{\mu}\right)}{2(r-\alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + B_1\left(\frac{c}{\mu}\right)^{\beta_1} + B_2\left(\frac{c}{\mu}\right)^{\beta_2} = -\frac{K}{r} + B_3\left(\frac{c}{\mu}\right)^{\beta_1} + B_4\left(\frac{c}{\mu}\right)^{\beta_2} \tag{35}$$

$$\frac{2\mu^2\left(\frac{c}{\mu}\right)}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu}{2(r-\alpha_1)} + \beta_1 B_1\left(\frac{c}{\mu}\right)^{(\beta_1-1)} + \beta_2 B_2\left(\frac{c}{\mu}\right)^{(\beta_2-1)} = \beta_1 B_3\left(\frac{c}{\mu}\right)^{(\beta_1-1)} + \beta_2 B_4\left(\frac{c}{\mu}\right)^{(\beta_2-1)} \tag{36}$$

$$-\frac{K}{r} + B_3\hat{\theta}_1^{\beta_1} + B_4\hat{\theta}_1^{\beta_2} = 0 \tag{37}$$

$$\beta_1 B_3\hat{\theta}_1^{(\beta_1-1)} + \beta_2 B_4\hat{\theta}_1^{(\beta_2-1)} = 0 \tag{38}$$

$$\frac{\mu^2(\theta^*)^2}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu(\theta^*)}{2(r-\alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + B_1(\theta^*)^{\beta_1} + B_2(\theta^*)^{\beta_2} = \frac{\gamma^2(\theta^*)^2}{4(r-2\alpha_2-\sigma^2)} - \frac{K}{r} + \left(\frac{(\theta^*)}{\hat{\theta}_2}\right)^{\beta_4}\frac{K}{r}\left(\frac{2}{2-\beta_4}\right) - I \tag{39}$$

$$\frac{2\mu^2(\theta^*)}{4(r-2\alpha_1-\sigma^2)} - \frac{c\mu}{2(r-\alpha_1)} + \beta_1 B_1(\theta^*)^{(\beta_1-1)} + \beta_2 B_2(\theta^*)^{(\beta_2-1)} = \frac{2\gamma^2(\theta^*)}{4(r-2\alpha_2-\sigma^2)} + \beta_4\frac{1}{\hat{\theta}_2}\left(\frac{(\theta^*)}{\hat{\theta}_2}\right)^{(\beta_4-1)}\frac{K}{r}\left(\frac{2}{2-\beta_4}\right) \tag{40}$$

If $\frac{c}{\mu} > \theta^* > \hat{\theta}_1$ then

$$
\begin{aligned}
V_{1,1} &= 0 && (41)\\
\left.\frac{\partial V_{1,1}}{\partial \theta}\right|_{\theta=\hat{\theta}_1} &= 0 && (42)\\
V_{1,1} &= V_2 - I && (43)\\
\left.\frac{\partial V_{1,1}}{\partial \theta}\right|_{\theta=\theta^*} &= \left.\frac{\partial V_2}{\partial \theta}\right|_{\theta=\theta^*} && (44)
\end{aligned}
$$

$$
\begin{aligned}
-\frac{K}{r} + C_1\hat{\theta}_1^{\beta_1} + C_2\hat{\theta}_1^{\beta_2} &= 0 && (45)\\
\beta_1 C_1 \hat{\theta}_1^{(\beta_1-1)} + \beta_2 C_2 \hat{\theta}_1^{(\beta_2-1)} &= 0 && (46)\\
-\frac{K}{r} + C_1 (\theta^*)^{\beta_1} + C_2 (\theta^*)^{\beta_2} &= \frac{\gamma^2 (\theta^*)^2}{4(r-2\alpha_2-\sigma^2)} - \frac{K}{r} + \left(\frac{(\theta^*)}{\hat{\theta}_2}\right)^{\beta_4} \frac{K}{r}\left(\frac{2}{2-\beta_4}\right) - I && (47)\\
\beta_1 C_1 (\theta^*)^{(\beta_1-1)} + \beta_2 C_2 (\theta^*)^{(\beta_2-1)} &= \frac{2\gamma^2 (\theta^*)}{4(r-2\alpha_2-\sigma^2)} + \beta_4 \frac{1}{\hat{\theta}_2}\left(\frac{(\theta^*)}{\hat{\theta}_2}\right)^{(\beta_4-1)} \frac{K}{r}\left(\frac{2}{2-\beta_4}\right) && (48)
\end{aligned}
$$

Proposition 3: *The optimal return function $V_1(.)$ is convex. The value function in region 2 $V_2(.)$ is convex as well.*

4 Numerical Results

In a first step we try to find numerical examples for all 3 cases:

where Case 1 means that there is no suspension region. Case 2 means that the firm exits from the suspension region and invests from the production region. And Case 3 means that the firm exists as well as invests from the suspension region.

- Table 4 contains examples of Case 1 and of Case 2.
- Table 5 shows an example for Case 3.
- Comparing Tables 1 and 2 the uncertainty is increases from $\sigma = 0.1$ to $\sigma = 0.175$.

5 Comparative Statics

Proposition 4: *The optimal return function $V_2(.)$ is nondecreasing in α_1.*

Some results:

$$
\begin{array}{ll}
\frac{\partial \hat{\theta}_2}{\partial K} > 0 & \frac{\partial V_2}{\partial} \\
\frac{\partial \hat{\theta}_2}{\partial \gamma} < 0 & \\
\frac{\partial \hat{\theta}_2}{\partial \sigma} & \\
\frac{\partial \hat{\theta}_2}{\partial \alpha_2} < 0 &
\end{array}
\qquad (49)
$$

Table 1: Parameter Values: $\sigma = 0.1$, $r = 0.1$, $c = 0.1$, $\mu = 1$, $\gamma = 1.1$, $K = 1$, $I = 100$ so $\frac{c}{\mu} = 0.1$

	$\alpha_1 = -0.01$	$\alpha_1 = -0.02$	$\alpha_1 = -0.03$	$\alpha_1 = -0.03$	$\alpha_1 = -0.02$
	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = -0.01$	$\alpha_2 = -0.01$
θ^*	7.8002	7.06726	6.61769	10.0606	11.5093
$\hat{\theta}_1$	1.72736	1.80236	1.85641	1.85648	1.80258
$\hat{\theta}_2$	1.28565	1.28565	1.28565	1.49744	
A_1	0.0000800014	$7.57819*10^{-6}$	$5.41195*10^{-7}$	$8.38366*10^{-9}$	$1.1636*10^{-7}$
A_2	23.0758	21.0037	19.0366	19.0372	21.0069
Case	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$		

Table 2: Parameter Values: $\sigma = 0.175$, $r = 0.1$, $c = 0.1$, $\mu = 1$, $\gamma = 1.1$, $K = 1$, $I = 100$ so $\frac{c}{\mu} = 0.1$

	$\alpha_1 = -0.01$	$\alpha_1 = -0.02$	$\alpha_1 = -0.03$	$\alpha_1 = -0.03$	$\alpha_1 = -0.02$
	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = -0.01$	$\alpha_2 = -0.01$
θ^*	7.33158	6.569	6.09172	9.73558	11.2149
$\hat{\theta}_1$	1.3327	1.43258	1.51613	1.54878	1.46914
$\hat{\theta}_2$	0.942478	0.942478	0.942478	1.19307	1.19307
A_1	0.0750437	0.0396422	0.0190254	0.00170862	0.00300053
A_2	9.22285	10.347	11.0699	11.2851	10.6419
Case	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$		
Case	No Suspension	No Suspension	No Suspension	No Suspension	No Suspension

Table 3: Parameter Values: $\sigma = 0.1$, $r = 0.1$, $c = 0.1$, $\mu = 1$, $\gamma = 1.1$, $K = 1$, **I=1000** so $\frac{c}{\mu} = 0.1$

	$\alpha_1 = -0.01$	$\alpha_1 = -0.02$	$\alpha_1 = 1.25$	$\alpha_1 = 1.5$	$\alpha_1 = 1.75$
	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = 0.019$	$\alpha_2 = 0.008$	$\alpha_2 = 0.004$
θ^*	26.1195				
$\hat{\theta}_1$	1.72793				
A_1	$6.35371*10^{-7}$				
A_2	23.0896				
Case	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$				

Table 4: Parameter Values: $\sigma = 0.1$, $r = 0.1$, $c = 1$, $\mu = 0.2$, $\gamma = 1.1$, $K = 1$, **I=100** so $\frac{c}{\mu} = 5$

	$\alpha_1 = -0.01$	$\alpha_1 = -0.02$	$\alpha_1 = -0.03$	$\alpha_1 = -0.03$	$\alpha_1 = -0.02$
	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = 0.01$	$\alpha_2 = -0.01$	$\alpha_2 = -0.01$
θ^*	5.91384	5.71062		6.76049	6.862
$\hat{\theta}_1$	3.84592	4.00893		5.20311	5.056
$\hat{\theta}_2$				1.49744	
Case	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$		$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$
Case	Suspension	Suspension		No Suspension	No Suspension

Table 5: Parameter Values: $\sigma = 0.1$, $r = 0.1$, $c = 1$, $\mu = 0.2$, $\gamma = 2$, $K = 1$, **I=100** so $\frac{c}{\mu} = 5$

	$\alpha_1 = -0.01$	$\alpha_1 = -0.01$	$\alpha_1 =$	$\alpha_1 =$	$\alpha_1 =$
	$\alpha_2 = 0.02$	$\alpha_2 = 0.02$	$\alpha_2 =$	$\alpha_2 =$	$\alpha_2 =$
	$\gamma = 2$	$\gamma = 1.1$	$\gamma =$	$\gamma =$	$\gamma =$
θ^*	2.74882	4.99786			
$\hat{\theta}_1$	1.78775	3.25045			
$\hat{\theta}_2$	0.615062	1.11829			
Case	$\frac{c}{\mu} > \theta^* > \hat{\theta}_1$	$\frac{c}{\mu} > \theta^* > \hat{\theta}_1$			
Case	Suspension	Suspension			

Table 6: Parameter Values: $\alpha_1 = -0.02$, $\alpha_2 = 0.01$, $r = 0.1$, $c = 1$, $\mu = 0.2$, $\gamma = 1.2$, $K = 2$, **I=100** so $\frac{c}{\mu} = 5$

	$\sigma = 0.075$	$\sigma = 0.1$	$\sigma = 0.125$	$\sigma = 0.15$	$\sigma = 0.175$
θ^*	5.30487	5.37763	5.43402	5.45344	5.40902
$\hat{\theta}_1$	4.51649	4.08685	3.60464	3.0972	2.58437
$\hat{\theta}_2$	1.80293	1.66667	1.52512	1.37752	1.222
Case	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$
Case	Suspension	Suspension	Suspension	Suspension	Suspension
P[first invest]	50.79%	51.71%	52.83%		
$E[T^*]$	1.11915	1.83247	2.61843		

Table 7: Parameter Values: $\alpha_1 = -0.02$, $\alpha_2 = -0.01$, $r = 0.1$, $c = 1$, $\mu = 0.2$, $\gamma = 1.2$, $K = 2$, **I=100** so $\frac{c}{\mu} = 5$

	$\sigma = 0.075$	$\sigma = 0.1$	$\sigma = 0.125$	$\sigma = 0.15$	$\sigma = 0.175$
θ^*	6.31107	6.29435	6.91264	7.0917	7.26531
$\hat{\theta}_1$	5.6842	5.24238	4.58644	4.02971	3.47407
$\hat{\theta}_2$	2.06704	1.94122	1.81263	1.68131	1.54665
Case	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \hat{\theta}_1 > \frac{c}{\mu}$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$	$\theta^* > \frac{c}{\mu} > \hat{\theta}_1$
Case	No Suspension	No Suspension	Suspension	Suspension	Suspension

5.1 List of Things we planned to do

1. Derive analytical results for effect of parameters on thresholds (see the derivations of Kwong and Decamps et al.).
2. Thresholds as a function of K.
3. Most important parameters: σ, α_1, μ
4. Probability of innovating or exit [*Just possible with simulations*]
5. Probability as function of σ, α_1, μ [*Just possible with simulations*]
6. Expected time of innovation or exit, respectively [*Just possible with simulations*]
7. Once in the suspension area - what is the probability of undertaking investment? [*Just possible with simulations*]
8. How long do you stay in the suspension area (before production, invest or exit) [*Just possible with simulations*]
9. Expected time of exit after innovation has taken place. [*Derived, have to write down still*]
10. Existence of suspension area: when is it there depending on ?
11. Is is possible to innovate directly from the suspension area? [*Yes*]
12. Can we derive conditions that are sufficient to end up in one of the three cases?[*Working on*]
13. Add different volatility parameters σ_1 and σ_2. [*Later*]

A Proofs

A.1 Proof of Proposition 1

Lemma 1:*Suppose that there exists a solution* $\left(\hat{\theta}_1, \theta^*, A_1, A_2\right)$ *(and* $\left(\hat{\theta}_1, \theta^*, B_1, B_2, B_3, B_4\right)$*,respectively) to equations ()-() that satisfies the constraints:*

$$V_1(\theta) \geq h(\theta) \mathit{for}\ \theta \in (\hat{\theta}_1, \theta^*) \tag{50}$$

And for the case $\hat{\theta}_1 \geq \frac{c}{\mu}$

$$\frac{2\mu^2}{4(r - 2\alpha_1 - \sigma^2)} + \beta_1(\beta_1 - 1)A_1\theta^{\beta_1 - 2} + \beta_2(\beta_2 - 1)A_2\theta^{\beta_2 - 2} \geq h''(\theta) \mathit{for}\ \theta \in \{\hat{\theta}_1, \theta^*\} \tag{51}$$

and for the case $\hat{\theta}_1 < \frac{c}{\mu}$

$$\left\{ \begin{array}{ll} \frac{2\mu^2}{4(r-2\alpha_1-\sigma^2)} + \beta_1(\beta_1 - 1)B_1\theta^{\beta_1-2} + \beta_2(\beta_2 - 1)B_2\theta^{\beta_2-2} & \mathit{if}\ \theta \geq \frac{c}{\mu} \\ \beta_1(\beta_1 - 1)B_3\theta^{\beta_1-2} + \beta_2(\beta_2 - 1)B_4\theta^{\beta_2-2} & \mathit{if}\ \theta < \frac{c}{\mu} \end{array} \right\} \geq h''(\theta) \mathit{for}\ \theta \in \{\hat{\theta}_1, \theta^*\} \tag{52}$$

Proof of Lemma 1: Now assume that $V_1(.)$ in equation (**??**) is a candidate for the optimal value function. In the following we verify that $V_1(.)$ it indeed satisfies all the sufficient conditions for being the optimal value function specified in Theorem 10.4.1 of Oksendal (2003). $V_1(.)$ is continuous differentiable in R for both cases since we impose the smooth pasting conditions at the threshold $\hat{\theta}_1$ and θ^* (see equations () and ()). For case $\hat{\theta}_1 < \frac{c}{\mu}$ this holds in view of the value matching and smooth pasting conditions imposed at the point $\theta = \frac{c}{\mu}$. Furthermore, $V_1(.)$ is twice continuously differentiable except at $\theta = \hat{\theta}_1$ and θ^*. Condition (ii) of Theorem 10.4.1 of Oksendal (2003) holds by definition. Conditions (iii), (viii) and (ix) of the theorem in Oksendal hold trivially because θ follows a geometric Brownian motion. Since $V_1(.)$ is a polynomial in θ the second order derivatives of $V_1(.)$ are finite near $\theta = \hat{\theta}_1$ and θ^* which relates to condition (v). Moreover, we introduce the partial differential operator L applied to the process $\{\theta(t); t \geq 0\}$:

$$L = \frac{\partial}{\partial t} + \alpha(\theta)\theta\frac{\partial}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2}{\partial\theta^2} \tag{53}$$

and show that conditions (vi) and (vii) hold. Condition (vii), i.e. $LV_1(\theta) + \Pi_1(\theta) = 0$ for $\theta \in (\hat{\theta}_1, \theta^*)$ can easily be verified with straightforward calculations. Condition (vi) has yet to be verified: $LV(\theta) \leq \Pi_1(\theta)$ for $\theta \in R\backslash\{\hat{\theta}_1, \theta^*\}$. In the stopping region it holds

$$LV(\theta) = Lh(\theta) = \left\{ \begin{array}{ll} -\frac{\gamma^2\theta^2}{4} + K & \text{if } \theta > \theta^* \\ 0 & \text{if } \theta < \hat{\theta}_1 \end{array} \right. \tag{54}$$

Considering equations (52), (33) and (34) (or (52), (23) and (24) equivalently) it follows that $\lim_{\theta \nearrow \theta^*} L[V(\theta) - h(\theta)] \geq 0$. Furthermore,

$$\lim_{\theta \searrow \theta^*} LV(\theta) = -\Pi(\theta^*) \geq \lim_{\theta \searrow \theta^*} Lh(\theta) = K - \frac{\gamma^2\theta^2}{4}$$

holds trivially for case $\theta < \frac{c}{\mu}$ and for case $\theta \geq \frac{c}{\mu}$ because

$$\frac{\gamma^2\theta^2}{4} > \left(\frac{\mu\theta - c}{2}\right)^2, \tag{55}$$

$$\theta > -\frac{c}{\gamma - \mu}, \tag{56}$$

because $\gamma > \mu$. By equations (), () and () it follows that $\lim_{\theta \searrow \theta_1^*} L[V(\theta) - h(\theta)] \geq 0$. Furthermore,

$$\lim_{\theta \nearrow \hat{\theta}_1} LV(\theta) = -\Pi(\hat{\theta}_1) \geq 0 = \lim_{\theta \nearrow \hat{\theta}_1} Lh(\theta)$$

has to hold for case $\hat{\theta}_1 > \frac{c}{\mu}$ and for case $\hat{\theta}_1 \leq \frac{c}{\mu}$. For $\hat{\theta}_1 > \frac{c}{\mu}$:

$$-\Pi(\hat{\theta}_1) > 0 \tag{57}$$

$$K - \left(\frac{\mu\hat{\theta}_1 - c}{2}\right)^2 > 0 \tag{58}$$

For this case we can not show that required relation. Do we have to consider this case? $\Leftrightarrow$ Would the firm exit if $\theta > \frac{c}{\mu}$, i.e. production stage?

For $\hat{\theta}_1 \leq \frac{c}{\mu}$:

$$-\Pi(\hat{\theta}_1) \geq 0 \tag{59}$$

$$K \geq 0 \tag{60}$$

A.1.1 Equations for thresholds

In the following we show that for each of the three equation systems for the thresholds there exists a unique solution:

For case $\hat{\theta}_1 \geq \frac{c}{\mu}$ we consider a solution $(\theta^*, \hat{\theta}_1, A_1, A_2)$ to Equations (21) - (24). We eliminate A_1 and A_2 from equations (21) - (24), and obtain

$$F_1 = \left(1 - \frac{2}{\beta_2}\right)\left[a\hat{\theta}_1^{(2-\beta_1)} - (a-d)\theta^{*(2-\beta_1)}\right] - b\left(1 - \frac{1}{\beta_2}\right)\left[\hat{\theta}_1^{(1-\beta_1)} - \theta^{*(1-\beta_1)}\right] + c\left[\hat{\theta}_1^{(-\beta_1)} - \theta^{*(-\beta_1)}\right] \tag{61}$$

$$- e\theta^{*(-\beta_1)} + A\theta^{*(\beta_4-\beta_1)}\left(1 - \frac{\beta_4}{\beta_2}\right) - I\theta^{*(-\beta_1)} = 0 \tag{62}$$

$$F_2 = \left(1 - \frac{2}{\beta_1}\right)\left[a\hat{\theta}_1^{(2-\beta_2)} - (a-d)\theta^{*(2-\beta_2)}\right] - b\left(1 - \frac{1}{\beta_1}\right)\left[\hat{\theta}_1^{(1-\beta_2)} - \theta^{*(1-\beta_2)}\right] + c\left[\hat{\theta}_1^{(-\beta_2)} - \theta^{*(-\beta_2)}\right] \tag{63}$$

$$- e\theta^{*(-\beta_2)} + A\theta^{*(\beta_4-\beta_2)}\left(1 - \frac{\beta_4}{\beta_1}\right) - I\theta^{*(-\beta_2)} = 0 \tag{64}$$

$$\frac{\partial F_1}{\partial \hat{\theta}_1} = (2-\beta_1)\left(1-\frac{2}{\beta_2}\right)a\hat{\theta}_1^{(1-\beta_1)} - b(1-\beta_1)\left(1-\frac{1}{\beta_2}\right)\hat{\theta}_1^{(-\beta_1)} - \beta_1 c\hat{\theta}_1^{(-1-\beta_1)} \tag{65}$$

$$\frac{\partial F_2}{\partial \hat{\theta}_1} = (2-\beta_2)\left(1-\frac{2}{\beta_1}\right)a\hat{\theta}_1^{(1-\beta_2)} - b(1-\beta_2)\left(1-\frac{1}{\beta_1}\right)\hat{\theta}_1^{(-\beta_2)} - \beta_2 c\hat{\theta}_1^{(-1-\beta_2)} \tag{66}$$

$$\frac{\partial F_1}{\partial \theta^*} = -(2-\beta_1)\left(1-\frac{2}{\beta_2}\right)(a-d)\theta^{*(1-\beta_1)} + b(1-\beta_1)\left(1-\frac{1}{\beta_2}\right)\theta^{*(-\beta_1)} + \beta_1 c\theta^{*(-1-\beta_1)} \tag{67}$$

$$+ \quad e\beta_1\theta^{*(-1-\beta_1)} + (\beta_4-\beta_1)\left(1-\frac{\beta_4}{\beta_2}\right)A\theta^{*(\beta_4-\beta_1-1)} + \beta_1 I\theta^{*(-1-\beta_1)} \tag{68}$$

$$\frac{\partial F_2}{\partial \theta^*} = -(2-\beta_2)\left(1-\frac{2}{\beta_1}\right)(a-d)\theta^{*(1-\beta_2)} + b(1-\beta_2)\left(1-\frac{1}{\beta_1}\right)\theta^{*(-\beta_2)} + \beta_2 c\theta^{*(-1-\beta_2)} \tag{69}$$

$$+ \quad \beta_2 e\theta^{*(-1-\beta_2)} + (\beta_4-\beta_2)A\theta^{*(\beta_4-\beta_2-1)}\left(1-\frac{\beta_4}{\beta_1}\right) + \beta_2 I\theta^{*(-1-\beta_2)} \tag{70}$$

where $a = \frac{\mu^2}{4(r-2\alpha_1-\sigma^2)} > 0$, $b = \frac{c\mu}{2(r-\alpha_1)} > 0$, $c = \frac{c^2}{4r} - \frac{K}{r}$, $d = \frac{\gamma^2}{4(r-2\alpha_2-\sigma^2)} > 0$ and $e = \frac{K}{r} > 0$. If we can show that the determinate of the following matrix is $\neq 0$ then we can conclude that there is a unique solution to this equation system.

$\begin{bmatrix} \frac{\partial F_1}{\partial \hat{\theta}_1} & \frac{\partial F_1}{\partial \theta^*} \\ \frac{\partial F_2}{\partial \hat{\theta}_1} & \frac{\partial F_2}{\partial \theta^*} \end{bmatrix}$ Therefore, we want to show that $\frac{\partial F_1}{\partial \hat{\theta}_1}\frac{\partial F_2}{\partial \theta^*} - \frac{\partial F_2}{\partial \hat{\theta}_1}\frac{\partial F_1}{\partial \theta^*} \neq 0$.

xxx*This proof is already done but we have to write it down still.*xxx

For case $\hat{\theta}_1 < \frac{c}{\mu} < \theta^*$ we consider a solution $(\theta^*, \hat{\theta}_1, B_1, B_2, B_3, B_4)$ to Equations (29) - (34). We eliminate B_1, B_2, B_3 and B_4 from equations (29) - (**??**), and obtain xxx*Still has to be derived*xxx

For case $\frac{c}{\mu} > \theta^* > \hat{\theta}_1$ we consider a solution $(\theta^*, \hat{\theta}_1, C_1, C_2)$ to Equations () - (). We eliminate C_1, C_2 from equations () - (), and obtain

$$F_1 = -e\hat{\theta}_1^{-\beta_1} + d\theta^{*(2-\beta_1)}\left(1-\frac{2}{\beta_2}\right) + A\theta^{*(\beta_4-\beta_1)}\left(1-\frac{\beta_4}{\beta_2}\right) - I\theta^{*(-\beta_1)} = 0 \tag{71}$$

$$F_2 = -e\hat{\theta}_1^{-\beta_2} + d\theta^{*(2-\beta_2)}\left(1-\frac{2}{\beta_1}\right) + A\theta^{*(\beta_4-\beta_2)}\left(1-\frac{\beta_4}{\beta_1}\right) - I\theta^{*(-\beta_2)} = 0 \tag{72}$$

The partial derivatives are

$$\frac{\partial F_1}{\partial \hat{\theta}_1} = \beta_1 e\hat{\theta}_1^{(-1-\beta_1)} \tag{73}$$

$$\frac{\partial F_2}{\partial \hat{\theta}_1} = \beta_2 e\hat{\theta}_1^{(-1-\beta_2)} \tag{74}$$

$$\frac{\partial F_1}{\partial \theta^*} = \theta^{*(-1-\beta_1)}\left[d(2-\beta_1)\left(1-\frac{2}{\beta_2}\right)\theta^{ast(2)} + A(\beta_4-\beta_1)\left(1-\frac{\beta_4}{\beta_2}\right)\theta^{*(\beta_4)} + \beta_2 I\right] \tag{75}$$

$$\frac{\partial F_2}{\partial \theta^*} = \theta^{*(-1-\beta_2)}\left[d(2-\beta_2)\left(1-\frac{2}{\beta_1}\right)\theta^{*(2)} + A(\beta_4-\beta_2)\left(1-\frac{\beta_4}{\beta_1}\right)\theta^{*(\beta_4)} + \beta_2 I\right] \tag{76}$$

We want to show that $\frac{\partial F_1}{\partial \hat{\theta}_1}\frac{\partial F_2}{\partial \theta^*} - \frac{\partial F_2}{\partial \hat{\theta}_1}\frac{\partial F_1}{\partial \theta^*} \neq 0$.

$$\frac{\partial F_1}{\partial \hat{\theta}_1}\frac{\partial F_2}{\partial \theta^*} - \frac{\partial F_2}{\partial \hat{\theta}_1}\frac{\partial F_1}{\partial \theta^*} = \tag{77}$$

$$\beta_1 e\hat{\theta}_1^{(-1-\beta_1)}\frac{\partial F_2}{\partial \theta^*} - \beta_2 e\hat{\theta}_1^{(-1-\beta_2)}\frac{\partial F_1}{\partial \theta^*} \tag{78}$$

where $C := \beta_1 \theta^{*(1+\beta_2)} \frac{\partial F_2}{\partial \theta^*} = \beta_2 \theta^{*(1+\beta_1)} \frac{\partial F_1}{\partial \theta^*}$ so that $eC\left[\hat{\theta}_1^{(-1-\beta_1)} \theta^{*(-1-\beta_2)} - \hat{\theta}_1^{(-1-\beta_2)} \theta^{*(-1-\beta_1)}\right] \neq 0$ holds if $\theta^* \neq \hat{\theta}_1$.

A.2 Proposition 2

Proposition 2: *The investment is never optimal if and only if the following three conditions hold:*

Proof V_e is the value of waiting to exit without having the option to innovate. Case $\hat{\theta}_1$:

$$V_e(\theta) = \left\{ \frac{\mu^2 \theta^2}{4(r - 2\alpha_1 - \sigma^2)} - \frac{c\mu\theta}{2(r - \alpha_1)} + \frac{c^2}{4r} - \frac{K}{r} + B\theta^{\beta_2} \right. \tag{79}$$

Value match and smooth paste at $\hat{\theta}_e$:

$$V_1(\hat{\theta}_e) = 0 \tag{80}$$

$$\left. \frac{\partial V_1(\theta)}{\partial \theta} \right|_{\theta = \hat{\theta}_e} = 0 \tag{81}$$

which leads to the following equation that threshold $\hat{\theta}_e$ has to satisfy

$$\frac{\mu^2 \hat{\theta}_e^2}{4(r - 2\alpha_1 - \sigma^2)} \left(\frac{\beta_2 - 2}{\beta_2} \right) - \frac{c\mu\hat{\theta}_e}{2(r - \alpha_1)} \left(\frac{\beta_2 - 1}{\beta_2} \right) + \frac{c^2}{4r} - \frac{K}{r} \tag{82}$$

Employing the value matching and smooth pasting conditions gives the following exit threshold:

$$\hat{\theta}_e = \frac{-b - \sqrt{b^2 - 4ad}}{2a} \tag{83}$$

where $a = \frac{\mu^2}{4(r - 2\alpha_1 - \sigma^2)} \left(\frac{\beta_2 - 2}{\beta_2} \right)$, $b = \frac{c\mu}{2(r - \alpha_1)} \left(\frac{\beta_2 - 1}{\beta_2} \right)$, $d = \frac{c^2}{4r} - \frac{K}{r}$. We can rule out the smaller of the two roots because it occurs in the θ-area where q optimally would be negative, implying that this is an irrelevant region because $\hat{\theta}_{e,2} < \frac{c}{\mu}$. A sufficient condition for that is $\frac{-b}{2a} < \frac{c}{\mu}$.

$$\frac{-b}{2a} < \frac{c}{\mu} \tag{84}$$

$$\frac{c}{\mu} \frac{(r - 2\alpha_1 - \sigma^2)}{r - \alpha_1} \frac{(\beta_2 - 1)}{(\beta_2 - 2)} < \frac{c}{\mu} \tag{85}$$

$$\frac{(r - 2\alpha_1 - \sigma^2)}{r - \alpha_1} < \frac{(\beta_2 - 2)}{(\beta_2 - 1)} \tag{86}$$

$$(-\alpha_1 - \sigma^2)\beta_2 + \sigma^2 > -r \tag{87}$$

$$\sigma^2 + r > \frac{\alpha_1 + \sigma^2}{\sigma^2} \left(-(\alpha_1 - \frac{\sigma^2}{2}) - \sqrt{(\frac{\alpha_1 - \sigma^2}{2})^2 + 2\sigma^2 r} \right) \tag{88}$$

sufficient condition for this to be satisfied is

$$\sigma^2 + r > \frac{\alpha_1 + \sigma^2}{\sigma^2} \left(-(\alpha_1 - \frac{\sigma^2}{2}) \right) \tag{89}$$

$$\tag{90}$$

If $\alpha_1 + \sigma^2 < 0$ this holds. Otherwise,

$$\sigma^2 + r > \frac{\alpha_1 + \sigma^2}{\sigma^2}\left(-(\alpha_1 - \sigma^2)\right) = -(\alpha_1^2 - \sigma^4) > \frac{\alpha_1 + \sigma^2}{\sigma^2}\left(-(\alpha_1 - \frac{\sigma^2}{2})\right) \tag{91}$$

which leads to

$$\sigma^2 + r > -\frac{(\alpha_1^2 - \sigma^4)}{\sigma^2} \tag{92}$$

$$r\sigma^2 > -\alpha_1^2 \tag{93}$$

Therefore, B is equal to

$$B = \frac{1}{\hat{\theta}_e^{\beta_2}}\left[-\frac{\mu^2\hat{\theta}_e^2}{4(r - 2\alpha_1 - \sigma^2)} + \frac{c\mu\hat{\theta}_e}{2(r - \alpha_1)} - \frac{c^2}{4r} + \frac{K}{r}\right] \tag{94}$$

If the following three conditions hold, then the firm never innovates: First, it has to hold that $V_e - V_2 + I > 0$ for $\theta \to \infty$. Second, $V_e - V_2 + I$ should be decreasing in θ. The first condition holds when $\frac{\mu^2}{4(r-2\alpha_1-\sigma^2)} - \frac{\gamma^2}{4(r-2\alpha_2-\sigma^2)} > 0$. The second condition requires that

$$2\left[\frac{\mu^2}{4(r - 2\alpha_1 - \sigma^2)} - \frac{\gamma^2}{4(r - 2\alpha_2 - \sigma^2)}\right]\theta - \frac{c\mu}{2(r - \alpha_1)} = 0 \tag{95}$$

$$\theta_{min} = \tag{96}$$

and $\left[\frac{\mu^2}{4(r-2\alpha_1-\sigma^2)} - \frac{\gamma^2}{4(r-2\alpha_2-\sigma^2)}\right]\theta^2 - \frac{c\mu\theta}{2(r-\alpha_1)} + I + \frac{c^2}{4r}$ has to be positive at θ_{min}. Third,

$$B\theta^{\beta_2} - \frac{2K}{r(2 - \beta_4)}\left(\frac{\theta}{\hat{\theta}_2}\right)^{\beta_4} > 0. \tag{97}$$

$$V_2(\theta) = \frac{\gamma^2\theta^2}{4(r - 2\alpha_2 - \sigma^2)} - \frac{K}{r} + \left(\frac{\theta}{\hat{\theta}_2}\right)^{\beta_4}\frac{K}{r}\left(\frac{2}{2 - \beta_4}\right). \tag{98}$$

We want to show that $V_e > V_2 - I$.

Sufficient condition for $\theta^* = \infty$ are: $\gamma < \mu$, $\alpha_2 < \alpha_1$ and $c = 0$.

A.3 Proof of Proposition 3

xxx*Showed already but still have to type in latex.*xxx

A.4 Proof of Proposition 4

xxx*Showed already but still have to type in latex.*xxx