Maths I

Exercises

Mathematical Analysis

6 Differential calculus R

6.1. Compute the derivatives of the following functions:

1) $f(x) = x^2$

2) $f(x) = 2x + 2$

3) $f(x) = \frac{1}{2}x^2$

4) $f(x) = 2x^2 + 4x + 4$

5) $f(x) = c$

6) $f(x) = 2x^2 + 4$

7) $f(x) = 2x^5 + 8x^2 + x - 78$

8) $f(x) = \frac{1}{x^2} + 3x^{\frac{1}{3}}$

9) $f(x) = \frac{3}{x^4} - \sqrt[4]{x} + x$

10) $f(x) = 6x^{1/3} - x^{0.4} + \frac{9}{x^2}$

11) $f(x) = \frac{1}{\sqrt[3]{x}} + \sqrt{x}$

12) $f(x) = (x^4 + 4x + 2)(2x + 3)$

13) $f(x) = (2x - 1)(3x^2 + 2)$

14) $f(x) = (x^3 - 12x)(3x^2 + 2x)$

15) $f(x) = (2x^5 - x)(3x + 1)$

16) $f(x) = \frac{2x+1}{x+5}$

17) $f(x) = \frac{3x^4+2x+2}{3x^2+1}$

18) $f(x) = \frac{x^{\frac{3}{2}}+1}{x+2}$

19) $f(x) = \frac{x^3+2}{x^3}$

20) $f(x) = \frac{x^2+x}{2x-1}$

21) $f(x) = \frac{16x^4+2x^2}{x}$

22) $f(x) = (x + 5)^2$

23) $g(x) = (x^3 - 2x + 5)^2$

24) $f(x) = \sqrt{1-x^2}$

25) $f(x) = \frac{(2x+4)^3}{4x^3+1}$

26) $f(x) = (2x+1)\sqrt{2x+2}$

27) $f(x) = \frac{2x+1}{\sqrt{2x+2}}$

28) $f(x) = \sqrt{2x^2+1}(3x^4+2x)^2$

29) $f(x) = \frac{2x+3}{(x^4+4x+2)^2}$

30) $f(x) = \sqrt{x^3+1}(x^2-1)$

31) $f(x) = ((2x+3)^4+4(2x+3)+2)^2$

32) $f(x) = \sqrt{1+x^2}$

33) $f(x) = (3x^2+e)e^{2x}$

34) $f(x) = e^{2x^2+3x}$

35) $f(x) = e^{e^{2x^2}+1}$

36) $f(x) = 2^{x-3}\sqrt{x^3-2} + \ln x$

37) $f(x) = \ln x - 2e^x + \sqrt{x}$

38) $f(x) = \ln(\ln(x^3(x+1)))$

39) $f(x) = \ln(2x^2+3x)$

40) $f(x) = \ln^4 x + \ln x^4 + 4\ln x$

41) $f(x) = \ln(\sin x)$

42) $f(x) = \ln \frac{1+x}{1-x}$

43) $f(x) = \ln\sqrt{x^2+1}$

44) $f(x) = x^2 \ln x$

45) $f(x) = 3e^x - 4\cos(x) - \frac{1}{4}\ln x$

46) $f(x) = \sin(x) + \cos(x)$

47) $f(x) = \arcsin\frac{x}{2}$

48) $f(x) = \arccos(2x^2)$

49) $f(x) = \sin(2x) - \sin^2 x + 2\sin x$

50) $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$

51) $f(x) = e^x(\sin x + \cos x)$

52) $f(x) = e^{ax}\sin(ax)$

6.2. Compute:

a) $\frac{d}{dx}e^{x^2}$

b) $\frac{d}{dx}e^{2^x}$

c) $\frac{d}{dx} \arctan(x^4)$

d) $\frac{d}{dx} \arctan(2x+4)$

e) $\frac{d}{dx} \ln(x^4)$

f) $\frac{d}{dx} \ln(2x+4)$

g) $\frac{d}{dx} \frac{1}{1+x^4}$

h) $\frac{d}{dx} \frac{1}{2x+4}$

i) $\frac{d}{dx} \arctan e^x$

6.3. Consider the following functions and points:

$f(x) = \frac{x^3}{3} + x^2 + 5, \quad (3, 23)$

$f(x) = x^3 - 3x + 1, \quad (1, -1)$

$f(x) = (x^2 + 1)(2 - x), \quad (2, 0)$

(a) Determine for which values of x the tangent line to f is horizontal.

(b) Write the equation of the tangent line to f at the given points.

6.4. Verify that Rolle's theorem can be applied to the following functions:

a) $f(x) = x^2 - 3x + 2$ on $[1, 2]$

b) $f(x) = |x - 1|$ on $[0, 2]$.

6.5. Let f and g differentiable functions on $[a, b]$ such that $f(a) = g(a)$ and $f(b) = g(b)$. Show that there is $c \in]a, b[$ such that $f'(c) = g'(c)$.

6.6. Using the mean value theorem, prove the following inequalities:

a) $e^x \geq 1 + x$

b) $\ln(1 + x) < \sqrt{x}$ if $x > 0$.

6.7. Compute

a) $\lim_{x \to 0} \dfrac{\cos(2x)}{x^{3/2}}$

b) $\lim_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

c) $\lim_{x \to +\infty} \dfrac{\ln x}{\sqrt[3]{x}}$

d) $\lim_{x \to 0} \dfrac{e^x}{e^x + \sqrt{x}}$

e) $\lim_{x \to +\infty} \dfrac{e^x}{\sin 1/x}$

f) $\lim_{x \to 3} \dfrac{1}{x-3} - \dfrac{5}{x^2 - x - 6}$.

6.8. Write the Taylor series of the following functions:

a) $f(x) = \sin x$, around 0

b) $f(x) = \cos x$, around 0

c) $f(x) = e^x$, around 0

d) $f(x) = \ln x$, around 1.

6.9. Study the following functions (domain, asymptotic lines, intervals of monotonicity, extrems, concavities) and sketch the graphs:

a) $f(x) = x^4 - 10x^2 + 9$

b) $f(x) = \frac{x-1}{x+1}$

c) $f(x) = \sqrt{x^2 - 1}$

d) $f(x) = e^{\frac{1}{\ln x}}$

e) $f(x) = \frac{x}{\ln x}$

f) $f(x) = e^{\frac{-1}{x}}$

g) $f(x) = e^{-x^2}$

h) $f(x) = x^2 \ln x$