Maths I

Exercises

Mathematical Analysis

7 Anti-derivatives and integrals

7.1. Compute the anti-derivatives of the following functions

1) $f(x) = x^2$
2) $f(x) = 2x + 2$
3) $f(x) = \frac{1}{2}x^2$
4) $f(x) = 2x^2 + 4x + 4$
5) $f(x) = c$
6) $f(x) = 2x^2 + 4$
7) $f(x) = 2x^5 + 8x^2 + x - 78$
8) $f(x) = \dfrac{1}{x^2} + 3x^{\frac{1}{3}}$
9) $f(x) = \dfrac{3}{x^4} - \sqrt[4]{x} + x$
10) $f(x) = 6x^{1/3} - x^{0.4} + \dfrac{9}{x^2}$
11) $f(x) = \dfrac{1}{\sqrt[3]{x}} + \sqrt{x}$
12) $f(x) = (x^4 + 4x + 2)(2x + 3)$
13) $f(x) = (2x - 1)(3x^2 + 2)$
14) $f(x) = (x^3 - 12x)(3x^2 + 2x)$
15) $f(x) = (a + bx^3)^2$
16) $f(x) = \sqrt{2ax}$
17) $f(x) = \dfrac{1}{\sqrt{x}}$
18) $f(x) = \cos 5x \sin 5x$
19) $f(x) = \sin^5 4x \cos 4x$
20) $f(x) = 4e^{5x}$
21) $f(x) = xe^{4x^2}$
22) $f(x) = (x + 5)^2 e^{(x+5)^3}$

23) $f(x) = \dfrac{1}{1+x}$; $f(x) = \dfrac{1}{1+x^2}$; $f(x) = \dfrac{x}{1+x^2}$; $f(x) = \dfrac{x}{(1+x^2)^2}$

24) $f(x) = \dfrac{e^x}{1+e^x}$; $f(x) = \dfrac{e^x}{1+e^{2x}}$; $f(x) = \dfrac{e^x}{(1+e^x)^2}$

25) $f(x) = \dfrac{\cos x}{1+\sin x}$; $f(x) = \dfrac{\cos x}{1+\sin^2 x}$; $f(x) = \dfrac{\cos x}{(1+\sin x)^2}$; $f(x) = \cos x(1+\sin x)^2$

26) $f(x) = \dfrac{\ln x}{x}$; $f(x) = \dfrac{\ln^5 x}{x}$; $f(x) = \dfrac{1}{x(1+\ln x)}$; $f(x) = \dfrac{1}{x(1+\ln^2 x)}$

7.2. Compute the anti-derivatives of the rational functions

1) $f(x) = \dfrac{1}{(x+1)(x+2)}$

2) $f(x) = \dfrac{x}{x+1}$

3) $f(x) = \dfrac{1}{x(x+1)}$

4) $f(x) = \dfrac{x^2-5x+1}{x^2-5x+8}$

5) $f(x) = \dfrac{x^2}{x^2+1}$

6) $f(x) = \dfrac{x^2+2x}{x^2-1}$

7) $f(x) = \dfrac{x}{x^4+4}$

8) $f(x) = \dfrac{2x^3}{x^4-1}$

7.3. Determine by parts the anti-derivatives of the following functions

1) $f(x) = xe^x$; $f(x) = x^2e^x$; $f(x) = x^2e^{3x}$

2) $f(x) = \ln x$; $f(x) = \arctan x$;

3) $f(x) = x\sin x$

4) $f(x) = x\cos 3x$

5) $f(x) = \dfrac{x}{e^x}$

6) $f(x) = \dfrac{x^2+2x}{x^2-1}$

7) $f(x) = x^2\ln x$

8) $f(x) = x\arctan x$

9) $f(x) = \sin 2x\cos 3x$

10) $f(x) = x\sin x\cos x$

7.4. Use the substitution method to find the anti-derivatives

1) $f(x) = \dfrac{x+\sqrt{x}}{x-\sqrt{x}}$

2) $f(x) = \dfrac{x^3}{\sqrt{2-x^2}}$

3) $f(x) = \dfrac{\sqrt{x}-1}{\sqrt[3]{x}+1}$

4) $f(x) = \dfrac{e^{3x}}{1-e^{2x}}$

5) $f(x) = \dfrac{\cos x}{\sin^2 x - 2}$

6) $f(x) = 2 + \sqrt{1-x^2}$

7) $f(x) = \dfrac{e^{2x}}{\sqrt{1+e^x}}$

7.5. Find the anti-derivatives

1) $f(x) = (-2x+5)e^{-x}$

2) $f(x) = \dfrac{x}{\sqrt{x+1}}$

3) $f(x) = e^{\sqrt{x}}$

4) $f(x) = xe^{-x^2}$

5) $f(x) = x(x^2+1)^{20}$

6) $f(x) = x\cos x$

7) $f(x) = \dfrac{1}{e^{2x}-3e^x}$

8) $f(x) = \dfrac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}$

9) $f(x) = \dfrac{x^6+1}{x+1}$

10) $f(x) = \dfrac{\sin x}{1+\sin x}$

7.6. Compute the integrals

1) $\int_1^2 x^2 - 2x + 3dx$

2) $\int_0^8 \sqrt{2x} + \sqrt[3]{x}dx$

3) $\int_1^4 \dfrac{1+\sqrt{x}}{x^2}dx$

4) $\int_0^{\pi/4} \cos^2 x dx$

5) $\int_e^{e^2} \dfrac{1}{x\ln x}dx$

6) $\int_0^{-3} \dfrac{1}{\sqrt{25+3x}}dx$

7.7. Compute the area of the region bounded by the parabola $y = \frac{x^2}{2}$ and the lines $x = 1$, $x = 3$ and $y = 0$.

7.8. Find the area between the curves

(a) $y = x^2 + 2x + 1$, $y = x^2 - 2$, $x = 0$, $y = 0$ e $x = 2$.

(b) $y = \dfrac{1}{x}$, $y = e^{x/4}$, $x = 0$, $x = 1$ e $x = 2$.

(c) $y = x^3 + 1$ e $y = 2x^2 + x - 1$.

(d) $y = 2 - x^2$ e $y^3 = x^2$.

7.9. Compute the improper integrals

(a) $\int_0^{+\infty} \dfrac{1}{1+x^2}dx$

(b) $\int_1^{+\infty} \dfrac{1}{x^2}dx$

(c) $\int_1^{1} \dfrac{1}{x^2}dx$

(d) $\int_0^{1} \dfrac{1}{\sqrt{x}}dx$

(e) $\int_0^{+\infty} \dfrac{\arctan x}{x^2+1}dx$

(f) $\int_0^{1} \ln x dx$

7.10. Compute the derivatives of the following functions

(a) $\int_0^x t^4\, dt$

(b) $\int_{-x}^x t^4\, dt$

(c) $\int_0^{x^2} e^{t^2}\, dt$

(d) $\int_{\sin x}^{\cos x} e^t dt$

7.11. Compute the limits

(a) $\lim_{x\to 0} \dfrac{\int_0^x \cos t\, dt}{x}$

(b) $\lim_{x\to 0} \dfrac{\int_0^x \sin^2 t\, dt}{x^3}$