MATHEMATICS I

2012-13 Test (2)

1. Let D be the domain of $f(x) = f\,(x) = \dfrac{2}{\sqrt{2-|x-1|}}$ and C the set of the terms of the sequence $u_n = 12\ln(e + n\sin(\dfrac{1}{n}))$.

(a) Determine the domain of f and represent on the real line the set $A = C \cup D$.

(b) Find the boundary and the set of accumulation points of A.

(c) Decide if A is open or closed.

2. Study the convergence of the sequences, computing its limit if it exists

(a) $u_n = \left(\frac{2n+5}{7n+10}\right)^n$

(b) $u_n = \frac{2n}{\sqrt{7n^2+1}} + \frac{2n}{\sqrt{7n^2+2}} + \frac{2n}{\sqrt{7n^2+3}} + \cdots + \frac{2n}{\sqrt{7n^2+n+1}}$

(c) $u_n = (-1)^n n(e^{1/n} - 1)$

3. Determine for which values of x the follosing series converge and compute its sum

(a) $\displaystyle\sum_{n=0}^{+\infty}(2x-5)^n$

(b) $\displaystyle\sum_{n=0}^{+\infty}(4|x-2|-7)^n$

(c) $\displaystyle\sum_{n=0}^{+\infty}(9-|2x|)^n$

4. What is the convergence interval of the following power series?

(a) $\displaystyle\sum_{n=0}^{+\infty}\frac{n!}{(n+1)^n}x^n$

(b) $\displaystyle\sum_{n=0}^{+\infty}\frac{(1-|4x|)^n}{(n!)^n}x^n$

(c) $\displaystyle\sum_{n=0}^{+\infty}(\ln n)(|4x|-3)^n$

5.

(a) Determine the fraction represented by 1.2(02).

(b) Find the set of the solutions of $-6x < x^2 + 9 \leq 10x$

(c) Prove that any series with positive terms whose partial sums are bounded is convergent.