

# MATHEMATICS I

## 2012-13 Test (3)

1. Determine the domain  $D$  of the following functions

(a)  $f(x) = \frac{\sqrt{x^2 - 9}}{\ln x + 10}$

(b)  $f(x) = \frac{\sqrt{|x| - 9}}{e^{x+10} - 1}$

(c)  $f(x) = \frac{\ln(1 - \sin^2 x)}{|\cos x| - 1}$ .

2. Study the existence of limit of the following functions at the respective points

(a)  $f(x) = \sin^3\left(\frac{2x+5}{7x+10}\right) \cdot \arctan(3x^2 + 4x)$  at 0

(b)  $f(x) = \frac{x^4 \sqrt{\sin^2(x+1)}}{\sqrt{x^4 + x^2}}$  at 0

(c)  $f(x) = \frac{x \sin(x-1)}{(x-1)^2}$

3. Study the domain of definition and of continuity of the following functions

(a)  $f(x) = \frac{x}{\sqrt{1 - \cos x}}$  is  $x \neq 0$  and 0 at 0

(b)  $f(x) = \ln 1 + |x|$

(c)  $f(x) = \frac{|x^2 + 2x - 3|}{x + 2}$

4. Show that:

(a) any polynomial with an odd degree has at least one zero

(b)  $x^8 + 3x^4 - 1$  has at least two real different roots

5.

(a) Let  $f(x) = \ln|x-2|$  if  $x \leq 0$  and  $f(x) = x^2 + mx + p$  if  $x > 0$ .

Determine  $m$  and  $p$  such that  $f$  is continuous at every point.

(b) Let  $f$  and  $g$  be continuous functions such that

$$f(0) = 0, \quad g(0) > 0, \quad \lim_{x \rightarrow +\infty} f(x) > \lim_{x \rightarrow +\infty} g(x).$$

Show that there is  $x > 0$  such that  $f(x) = g(x)$ .