MATHEMATICS I

2012-13 Test (1)

1. Consider the matrix $C = \begin{bmatrix} 5 & -2 & -4 \\ -2 & 2 & b \\ 3 & -6 & -6 \end{bmatrix}$, with $b \in \mathbb{R}$ and $B = \frac{1}{6}\begin{bmatrix} 2 & 2 & 0 \\ 0 & -3 & -2 \\ 1 & 4 & 1 \end{bmatrix}$. Determine $b \in \mathbb{R}$ so that

a) B is the inverse of C.

b) $\det\left[C\,(B+I)\right] = 8$.

2. If $A = \begin{bmatrix} 4 & 1 & 0 \\ 0 & a & 1 \\ -2 & 5 & 2 \end{bmatrix}$ find $a \in \mathbb{R}$ such that A is invertible.

3. Consider $A = \begin{bmatrix} 0 & 4 & -4 \\ -6 & -2 & -1 \\ 0 & 10 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 2/21 & -1/6 & -1/14 \\ -3/28 & \beta & 1/7 \\ -5/14 & 0 & 1/7 \end{bmatrix}$, $(\beta \in \mathbb{R})$, e $C = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 28 & 0 \\ 0 & 0 & 14 \end{bmatrix}$. Knowing that A and B and inverses,

a) determine β.

b) find the matrix X that satisfies $AXC^{-1} = A + I$ (in case you have not solved a) take $\beta = 0$).

4. Determine $a \in \mathbb{R}$ such that $\begin{bmatrix} 2 & 3 \\ -1 & 0 \\ 5 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & 2 \\ 6 & a \\ 2 & 0 \\ 1 & -2 \end{bmatrix}^T = \begin{bmatrix} 6 & 24 & 4 & -4 \\ 0 & -6 & -2 & -1 \\ 8 & 46 & 10 & -3 \end{bmatrix}$.

5. Discuss for each parameter the solutions of the following systems:

a) $\left\{\begin{array}{c} x + 4y + 3z = 10 \\ 2x - 7y - 2z = 10 \\ x + 5y + \alpha z = \beta \end{array}\right.$

b) $\left\{\begin{array}{c} x + 2y + 3z + 4t = 2 \\ \alpha y + 3t = 1 \\ 5y + z - t = 2 \end{array}\right.$