

MATHS I

2012-13 Test 4

1. Compute the derivatives of the following functions and find the values at the given points.

(a) $f(x) = e^{x^2+4x+2\sin(4x)}$; at $x = 0$

(b) $f(x) = \frac{\sin^4(5x)}{\cos(4x)}$; at $x = \pi/2$

(c) $f(x) = \sqrt{\arctan(1+4x)}$; at $x = 0$

2. Answer the following questions

(a) Is it possible to say that, using Rolle's theorem, the function $f(x) = 4 - |5 - x|$ has a critical point in $[1, 9]$? Justify.

(b) Show, using Lagrange's theorem, that $\arctan e^x < x + \frac{\pi}{4}$, if $x > 0$.

3. Answer the following questions:

(a) Write Mac-Laurin's formula of $f(x) = (1+x)^\alpha$ with order 4.

(b) Discuss the fact that the function $f(x) = e^{x^3}$ is analytic and write its power series.

4. Consider the function defined by

$$f(x) = \begin{cases} \arctan(x^2 - 6x + 8), & \text{if } x \leq 2 \\ 2 + \frac{1}{3-x}, & \text{if } x > 2 \end{cases}$$

and find the following elements:

(a) domain;

(b) limits in $+\infty$, $-\infty$, 2 and 3,

(c) domain of differentiability;

(d) intervals of monotonicity and relative maxima and minima, if possible.

5. Study the following function (domain, limits, asymptotic lines, monotonicity, extreme points, concavities) and sketch its graph.

$$f(x) = \frac{1}{x} e^{\frac{1}{x}}$$