

MATHS I

2012-13 Test 5

1. Find the anti-derivatives of the following functions that pass at the given points.

(a) $f(x) = (x + 3)^2 + \frac{20e^{5x}}{5 + e^{5x}}$; at the point $(0, \ln 6)$

(b) $f(x) = \frac{\sin(5x)}{16 + \cos^2(5x)}$; at $(\pi/2, 5)$

(c) $(1 + 3x)^4 + e^{5x} + x^4 e^{3x^5}$; at $(0, 7/3)$

2. Find the anti-derivatives

(a) $f(x) = (x + 1)e^{2x}$.

(b) $\frac{e^{2x}}{e^x + e^{3x}}$. (Hint: $x = \ln t$)

3. Compute the integrals:

(a) $\int_0^{\pi/4} \sin 2x \cos^4 2x \, dx$.

(b) $\int_0^{+\infty} \frac{x^3}{e^{x^4}} \, dx$.

(c) $\int_{-\infty}^0 \frac{1}{(3x - 1)^4} \, dx$.

4. Compute the areas of the following sets:

(a) $A = \{(x, y) : 0, 25x^2 \leq y \leq x\}$

(b) $B = \{(x, y) : x \geq 1 \wedge -\frac{1}{x^3} \leq y \leq \frac{1}{x^2}\}$

5. Compute

(a) $\frac{d}{dx} \int_0^{\sin^2(3x)} t^3 \, dt$

(b) $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} e^{3t} \, dt}{4x^2}$