1.

(a) $Pf(x) = \dfrac{1}{3}(x+3)^3 + 4\ln(5+e^{5x}) + C, \quad C \in \mathbf{R};$

$F(x) = \dfrac{1}{3}(x+3)^3 + 4\ln(5+e^{5x}) - 9 - 3\ln 6.$

(b) $Pf(x) = -\dfrac{1}{20}\arctan\left(\dfrac{1}{4}\cos(5x)\right) + C, \quad C \in \mathbf{R};$

$F(x) = -\dfrac{1}{20}\arctan\left(\dfrac{1}{4}\cos(5x)\right) + 5$

(c) $Pf(x) = \dfrac{1}{15}(1+3x)^5 + \dfrac{1}{5}e^{5x} + \dfrac{1}{15}e^{3x^5} + C, \quad C \in \mathbf{R};$

$F(x) = \dfrac{1}{15}(1+3x)^5 + \dfrac{1}{5}e^{5x} + \dfrac{1}{15}e^{3x^5} + 2$

2.

(a) $Pf(x) = \dfrac{1}{2}xe^{2x} + \dfrac{1}{4}e^{2x} + C, \quad C \in \mathbf{R}$

(b) $Pf(x) = \arctan e^x + C, \quad C \in \mathbf{R}.$

3.

(a) $\dfrac{1}{10}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{9}$

4.

(a) Área $(A) = \dfrac{8}{3}.$

(b) Área $(B) = \dfrac{3}{2}.$

5.

(a) $6\sin^7(3x)\cos(3x)$

(b) $\dfrac{1}{4}$