

**15 Jan 2009**

1.
  - a)  $\dim M = 2, \dim N = 2, \dim M \cap N = 1$
  - b)  $p = (x, y, z), T_p(M \cap N) = \text{span}\{(x/y(y+v), v, 1)\}, v = -(2x^2 - 1)y/(2x^2 - 2y^2), T_p(M \cap N)^\perp = \text{span}\{(-1/x, 1/y, 1), (2x, -2y, -1)\}$
2.
  - a)  $(0, 1/2, 0)$
  - b)  $\pi^{1/2}$
  - c) mais perto:  $(2 - 2a, 3 - 3a, 4, 4a)$ , mais longe:  $(2 + 2a, 3 + 3a, 4 + 4a)$ ;
- $a = 29^{-1/2}$
3.
  - a)  $78\pi$
  - b)  $0$
4. não é aditiva
5.
  - a)  $2$

**30 Jan 2009**

- 1.
- b)  $\varphi(\theta) = (\cos \theta, \cos \theta, \sin \theta), \theta \in ] - \pi/2, \pi/2[$
2.
  - a)  $\sqrt{3}(e^{2\pi} - 1)$
  - b)  $e^{2\pi-1}$
3.  $x = y = z = 1$
4.
  - a)  $0$
  - b)  $0$
5.  $\{\emptyset, \Omega, A_0, A_1, A_0^c, A_1^c, A_0 \cup A_1, (A_0 \cup A_1)^c\}, \mu(B) = \#B/\#\Omega$
6.
  - a)  $0$
  - b)  $2 + f(0)$

**6 Jan 2010**

1.
  - a)  $\dim M = 3$
  - b)  $T_p M = \{(x, y, z, w) : x + w = 0\}, T_p M^\perp = \{(x, 0, 0, w) : x - w = 0\}$
2.
  - a) mais perto  $(2/3, 0, 2/3)$ , mais longe  $(2, 0, -2)$
  - b)  $(0, 1/2, 0)$
  - c)  $1/2$
  - d)  $(5 - \cos 2)/8$
3.
  - a)  $\{\emptyset, \Omega, A_0, A_1, A_0 \cup A_1, A_0^c, A_1^c, (A_0 \cup A_1)^c\}$

**27 Jan 2010**

1.
  - a)  $\theta \neq 0, \dim M_\theta = 2$
  - b)  $T_p M = \text{span}\{(1, 0, 0, -1), (0, \theta, 1, 0)\}, T_p M^\perp = \text{span}\{(0, -1, \theta, 0), (1, 0, 0, 1)\}$
2.
  - b)  $1/(e^\pi - 1) + \pi^3/24 - 1$
- 3.

a)  $]0, 1[ \times ] - 1, 0[ \times ] 0, 2[$

c)  $\pi/2$

4.

a)  $0, m(E)$ , decrescente

b) sse  $m(f^{-1}(a)) = 0$

**5 Jan 2011**

2.

a)  $2\pi(1 - 1/e)$

b)  $45/56$

c)  $2\pi$

3.  $(1 - e^{-1})/16$

4.  $\pi\alpha^2/6$

5.

b)  $1/2$

**26 Jan 2011**

1.

a)  $(0, 3/8, 0)$

b)  $\sqrt{\pi}$

c)  $(-1, 0, 0)$

2.  $\sqrt[4]{2}$

3.  $0$

4.

b)  $e^{-\lambda}(1 + 2\lambda + 3\lambda^2/2)$

c)  $(1 - e^{-\lambda})/\lambda$

6. integrável

**9 Jan 2012**

2.

a)  $T_p M = \text{span}\{(1, 1, \sqrt{2}), (1, -1, 0)\}$ ,  $T_p M^\perp = \text{span}\{(1, 1, -\sqrt{2})\}$

b)  $\sqrt{2}/3$

3.

a)  $2R/3$

b)  $x^2 + y^2 = 1/2$ ,  $z = 1/2$

c)  $\frac{3}{8} \frac{R^2 - r^2}{R^{3/2} - r^{3/2}}$

4.

a)  $3$

b)  $1 + 1/4 + 1/9$

**25 Jan 2012**

2.

a)  $3R/4$

b)  $(0, 3/4, 0)$

c)  $\sqrt{\pi}$

3.

a)  $(2x, 2y, -1)/\sqrt{4z + 1}$

b)  $0$

4.

a)  $55$

b)  $10$

**11 Jan 2013**

1.
  - b)  $2\pi^2$
2.
  - b)  $\frac{\pi}{2} \frac{e-1}{e^2}$
3.
  - a)  $3R/4$
  - b)  $(1, 1, 1, 0)$
4.
  - a)  $\dim=2$
  - b)  $(0, 0, \pm 1)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$  e  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 1\}$ ,  $\pm(x, y, 0)$  em  $S$ ,  $(0, \pm 1, 0)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$
  - c)  $\pi/2$

**31 Jan 2013**

1.
  - a)  $\gamma(t) = (\sin(t), \sin(2t), 0)$ ,  $t \in [0, 2\pi]$ ,  $\Gamma = \gamma([0, 2\pi])$  não é uma variedade
  - b)  $p = \gamma(\pi/2) = (1, 0, 0)$ ,  $T_p\Gamma = \text{span}\{(0, 1, 0)\}$ ,  $T_p\Gamma^\perp = \text{span}\{(1, 0, 0), (0, 0, 1)\}$
2.
  - b)  $(\pi/2)^3$
3.
  - a)  $(3/2, 3/2)$
  - b)  $2\sqrt{R}/3$
4.
  - a)  $\dim=2$
  - b)  $(0, 0, \pm 1)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$  e  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 1/2\}$ ,  $(0, \pm 1, 0)$  em  $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$ ,  $\pm(x, y, 1-z)/[\sqrt{2}(1-z)]$  em  $S$
  - c)  $\pi/4$