Information Technologies and Jobless Recoveries

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Abstract

Jobless recoveries followed each of the last three recessions in the United States. This paper presents a model where firms use recessions as an opportunity to substitute workers with capital. This substitution of factors now involves Information Technologies because computers have recently become cheap and competitive compared to workers. As a consequence, specific jobs will disappear in the long-term—and they may disappear earlier if there is a recession. In recent recoveries, firms avoid hiring workers because they invest in computers instead. In effect, the recession accelerates the disappearance of specific jobs and their replacement with computers. Quantitatively, the model replicates the dynamics of employment in postwar recoveries. Empirically, this paper shows that the replacement of workers with computers may be responsible for the destruction of 5 million jobs over the 2007-2010 recession and recovery.

Keywords: jobless recoveries, computers, recession, acceleration, business cycles.

JEL codes: E22, E23, E24, E32.

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1 Introduction

Three years after the end of the 2007 recession, the recovery in employment stands at 2 percent above the level of the preceding trough, compared to 10 percent after the 1981 recession. Figure 1 shows the recoveries in employment around the NBER trough for the eleven postwar recessions. Employment often lagged the cyclical trough by about one quarter, but more recently the lag is nearly one year. Therefore, jobless recoveries, defined as the period between the trough of output and the trough of labor (Bachmann, 2011), have been lasting longer. Part of this difference is due to the slow recovery in output (Galí, Smets and Wouters, 2011) and another part is due to the slow recovery in employment even controlling for output. Put simply, after recent recession output has increased and employment has not.

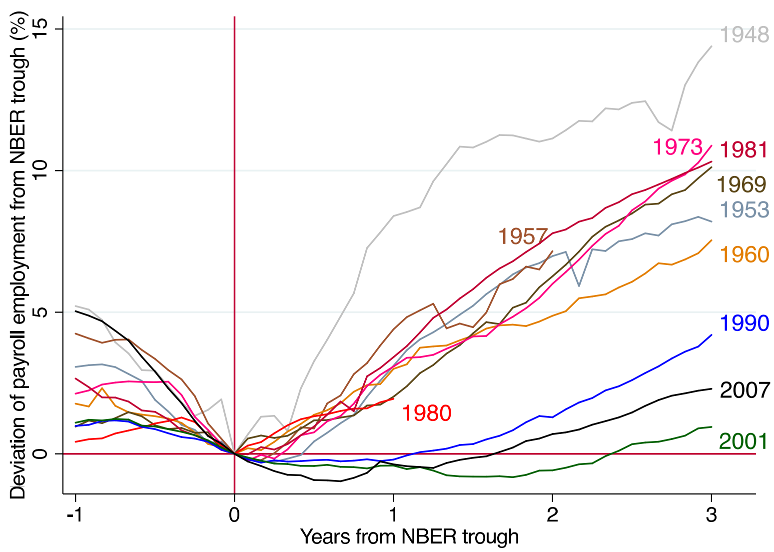

Figure 1: Longer jobless recoveries in employment during recent recoveries.
Source: Federal Reserve Economic Database. Later recessions use darker colors.

At the same time, computer investment has increased over the last decades. The seminal paper by Autor, Levy and Murnane (2003) investigates the effects of computers on the employment structure. It finds that, in the long-term, the increasing use of computers reduces the demand for *routine* workers, jobs that are high in automation and low in creativity and personal interactions, such as clerks. Simultaneously, it increases demand for *non-routine* workers, jobs low in automation and high in creativity and personal interactions, such as managers.[1]

This paper links these two branches of the literature and asks whether the increase in computer investment and the decrease in demand for routine workers can also account for jobless recoveries. There is already considerable research on computers and long-term productivity,[2] but little research on computers and the business cycle.[3]

This paper fills that gap and uses a simple model to study the effects of computer investment on the short-term behavior of employment. As in Autor, Levy and Murnane (2003), there are two types of capital, Information Technologies (IT) capital and non-IT capital. The cost of IT investment decreases continuously (and the cost of non-IT investment is constant). Also as in Autor, Levy and Murnane (2003), there are two types of workers, routine and non-routine. Routine workers are easier to replace with computers than non-routine workers. As in Hamermesh (1993), firms pay an adjustment cost to hire a *non-routine* worker (and no cost to hire a routine worker or to fire workers).

The growth rate of labor productivity has two sources in this model: the exogenous growth in Total Factor Productivity (TFP) and the endogenous substitution of capital for labor. Early in history, the cost of Information Technologies is high, firms invest little in computers, and they hire routine workers instead. The growth rate in labor productivity is due mainly to TFP. This is a *labor-intensive phase.* Later in history, the cost of Information Technologies decreases, computers become competitive compared to routine workers, and firms replace one with the other. The growth rate in labor productivity is due to both TFP and to the increasing use of computers. This is a *technological upgrading phase.* Even later in history, the decrease in the cost of Information Technologies slows down and firms also slow down their computer investments. The growth rate in labor productivity is again due mainly to TFP. This is a *capital-intensive phase.*

In the medium term, the growth rate of labor productivity increases between the labor-

[1] See also Autor, Katz and Kearney (2006), Goos and Manning (2007), Goldin and Katz (2008) and Autor (2010) for the related concept of polarization, whereby computer investment causes a decrease in the employment of middle-skill occupations and "hollows out" the employment structure.

[2] See Triplett (1999) for a review of the Solow paradox and the relation between IT investment and labor productivity.

[3] See Gordon (2010) for a suggestion of Information Technologies as a possible answer to the recent instability in the relationship between output and employment.

intensive phase and the technological upgrading phase, thus delivering an endogenous productivity "speedup." Given the productivity speedup in the data around the 1980s, the labor-intensive phase in the real economy lasts roughly until the 1980s, at which point the technological upgrading phase starts. Put simply, the cost of computers was already decreasing in the 1950s, but the effects on productivity become significant around the 1980s, when computers become competitive compared to routine workers.

In the short term, the model implies that routine employment is more responsive to TFP shocks compared to an economy without adjustment costs. Firms retain or hoard *non-routine* workers during recessions in order to avoid hiring them back in the recovery and paying the hiring cost. The burden of the adjustment of employment falls entirely on routine workers, who are particularly responsive to a recession.

The interaction between the trend in routine employment and the business cycle is the key mechanism in the model. Due to the responsiveness of routine workers, a recession during the technological upgrading phase accelerates the secular decline in routine employment.

This interaction between trend and cycle causes jobless recoveries. Due to the absence of firing of non-routine workers during the recession, firms have a stock of non-routine employment that is temporarily too high. To return to the ideal allocation (which would prevail without adjustment costs), firms refrain from hiring non-routine workers for some time during the recovery, i..e they dishoard non-routine labor. Firms also refrain from hiring routine workers because of the secular decline in routine employment. Overall, employment is stagnant even as output recovers, leading to a jobless recovery. Earlier in history, during the labor-intensive phase, the trend of routine employment is constant and routine employment recovers back to the pre-crisis level, leading to a "jobful" recovery. The transition from the labor-intensive to the technological upgrading phase displays longer jobless recoveries. Quantitatively, a calibration of the model with US GDP is able to match the dynamics of employment in postwar recoveries.

Additional evidence at the micro shows that this explanation may be empirically important. The Current Population Survey, matched to the Occupational Information Network, confirms that routine employment, which is more more easily replaced by computers, has a secular decrease that accelerates during recessions. For example, over the 2007-2010 recession and recovery, employment in routine occupations decreased by 4.8 million jobs, around 3 percent of employment. At the same time, non-routine employment is unaffected by cyclical fluctuations.

This paper relates to a growing literature on jobless recoveries, in particular to the two leading explanations of de-unionization (Berger, 2012) and routinization in labor supply (Jaimovich and Siu, 2012). A review of these models and their relationship to the model in this paper appears in Section 6.

The outline of the paper is as follows. Section 2 presents the model. Section 3 shows the results of the model in a special, analytically tractable case. Section 4 presents quantitative results and a calibration with US data. Section 5 provides additional evidence for the model. Section 6 discusses related literature and Section 7 concludes.

2 Model

This section presents a model to study the relationship between computers and jobless recoveries. Time is discrete, from 0 to infinity, and all agents have perfect foresight. A household consumes output, supplies labor, invests in capital, and rents the capital stock. A firm demands labor and capital and produces output.

2.1 The household

A representative household maximizes utility from consumption, net of disutility from labor supply

$$\max \sum_{t=0}^{\infty} \theta^t \left(\frac{C_t^{1-\eta} - 1}{1-\eta} - X_t \left(1+g\right)^t \frac{\varepsilon}{1+\varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right), \tag{2.1}$$

where X_t is a reduced-form labor wedge shock[4] and represents fluctuations driven by the household, and the remaining notation is standard.[5] The conventional case is $\eta = 1$

[4]See Hall (1997, page 226) for a similar example of using a preference shifter as a labor wedge. See also Balleer (2012) for the importance of the labor wedge for explaining labor market dynamics.

[5]Specifically, θ is the household's discount factor, C_t is consumption, η is the curvature of the utility from consumption, ε is the Frisch elasticity of labor supply, and L_t is labor supply.

and $g = 0$ but this paper also considers the possibility $\eta < 1$. The labor supply of the household is bounded above:

$$L_t \leq \bar{L},$$

where $\bar{L}$ represents the total amount of time available to work.

There are two types of capital, IT capital and non-IT capital. The distinction between IT and non-IT capital follows from Autor, Levy and Murnane (2003), who show that IT capital is associated with the routinization of the employment structure. The household accumulates capital with a perpetual inventory formula for each type of capital, IT capital $K_{I,t}$ and non-IT capital $K_{N,t}$:

$$K_{N,t+1} = (1 - \delta_N) K_{N,t} + I_{N,t}. \tag{2.2}$$

$$K_{I,t+1} = (1 - \delta_I) K_{I,t} + I_{I,t}, \tag{2.3}$$

The household has access to a technology that transforms output into investment: one unit of output becomes one unit of non-IT investment $I_{N,t}$ and one unit of output becomes $\exp(b_t)$ units of IT investment $I_{I,t}$, and. Alternatively, the cost of non-IT investment is 1 and the cost of IT investment is $\exp(-b_t)$. The logarithm b_t of the productivity of IT investment is the crucial variable to the long-term behavior of the model.

Considering consumption as the numeraire, the household has a budget constraint that balances consumption and investment with labor income and capital income:

$$C_t + I_{N,t} + \exp(-b_t) I_{I,t} = w_t L_t + r_{N,t} K_{N,t} + r_{I,t} K_{I,t} + \widehat{\text{profits}}_t, \tag{2.4}$$

where w_t is the wage, $r_{J,t}$ are the rental rates of capital ($J = I, N$), and $\widehat{\text{profits}}_t$ are the firm's total profits at the optimum.

The first crucial assumption is the long-term increase in the productivity b_t:

Assumption 1. *The logarithm b_t of the productivity of the IT-producing technology increases exogenously with time, up to an upper bound $\bar{b}$:*

$$b_t \nearrow \ in\ t, \quad \lim_t b_t = \bar{b}.$$

Alternatively, the cost of computers $\exp(-b_t)$ decreases with time. There is some disagreement on the exact rate of decrease of the cost of computers,[6] but there is wide agreement that it was high. Table 1 shows four examples, with estimates of the rate of decrease in the cost of computers ranging from 8 percent to 27 percent. Figure 2, from the Bureau of Labor Statistics, illustrates this rapid decrease: between 1960 and 2010, the cost of computers declined ten-thousand-fold.

Study	Time frame	Rate of decrease
Sichel (1997, page 122)	1987-1993	8 %
Bureau of Labor Statistics	1957-2010	18 %
Nordhaus (2007, page 142)	1850-2006	19 %
Berndt and Rappaport (2001, page 271)	1976-1999	27 %

Table 1: Estimates of the annual rate of decrease in the cost of computing power.

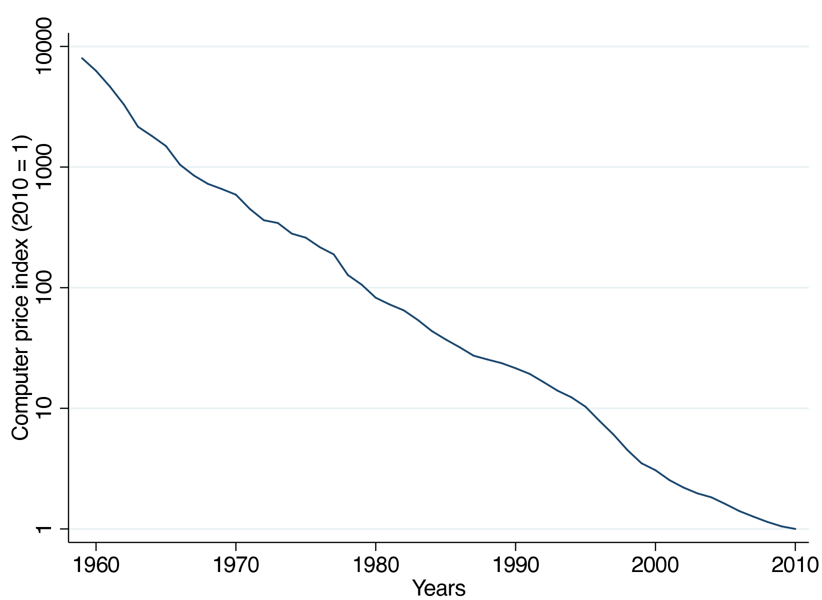

Figure 2: The cost of Information Technologies has been decreasing continuously.
Source: Bureau of Economic Analysis. This purchase price index adjusts for both quality and inflation.

[6]See Nordhaus (2007, Table 10, page 153) for a compilation of studies and methods.

2.2 Technology

The technology to produce output Y_t uses four inputs, two types of capital, IT capital $K_{I,t}$ and non-IT capital $K_{N,t}$, and two types of labor, routine labor $L_{R,t}$ and non-routine labor $L_{N,t}$. The distinction between routine and non-routine labor follows from Autor, Levy and Murnane (2003), who show that employment has different trends depending on the characteristics of the occupation (routinizable or not). The production function is:

$$Y_t = A_t \, K_{N,t}^{\alpha} \, L_{N,t}^{\beta} \, \left(K_{I,t}^{\rho} + L_{R,t}^{\rho}\right)^{\frac{\gamma}{\rho}}, \tag{2.5}$$

where A_t is Total Factor productivity and represents fluctuations driven by technology. The production function has constant returns to scale, with $\alpha+\beta+\gamma = 1$. This production function has Cobb-Douglas aggregation of three factors: non-IT capital $K_{N,t}$, non-routine labor $L_{N,t}$, and a third factor, which is a CES (constant elasticity of substitution) aggregate between information capital $K_{I,t}$ and routine labor $L_{R,t}$.

This production function is increasingly common in the literature. Krusell et al. (2000) use it to explain the increase in income inequality with capital-skill complementarity, whereby an increase in capital investment contributes to increasing the skill premium by increasing the marginal product of skilled labor faster than that of unskilled labor. Autor and Dorn (2009, page 11) use it to explain the recent disappearance of middle-skill occupations (known as polarization and related to routinization): as firms invest more in IT capital, they increase employment of middle-skill routine occupations slower than low-skill or high-skill non-routine occupations.

The second crucial assumption is the gross substitutability of IT capital and routine employment in the production function:

Assumption 2. *The elasticity of substitution between IT capital and routine employment is greater than 1:*

$$\sigma = \frac{1}{1-\rho} > 1.$$

Autor, Levy and Murnane (2003) show that computer investment is correlated with a decrease in routine employment and an increase in non-routine employment. Assumption 2 captures that difference: $\sigma > 1$ implies that the elasticity of substitution between routine employment and computers is greater than the elasticity of substitution between non-routine employment and the CES aggregate of computers and routine employment. Intuitively, a computer can more easily replace automated occupations, such as bank tellers or cashiers, than non-routine occupations, such as managers and doctors.

2.3 The firm

A representative firm, owned by the household, operates under perfect competition and has profits

$$\text{profits}_t = Y_t - w_t\left(L_{N,t} + L_{R,t}\right) - r_{N,t}K_{N,t} - r_{I,t}K_{I,t} - c_N H_{N,t},$$

where c_N is the unit cost of hiring non-routine workers and $H_{N,t}$ is non-routine hiring of time t. The firm maximizes the present value of profits, discounted with a factor $D_{0,t}$ between times 0 and t, inherited from the household. The firm has two constraints on the accumulation of non-routine labor and on hiring in each period:

$$L_{N,t+1} \leq L_{N,t} + H_{N,t}, \tag{2.6}$$

$$H_{N,t} \geq 0. \tag{2.7}$$

The first equation implies that increases in the firm's stock of non-routine labor come from hiring $H_{N,t}$ and pay an adjustment cost. The second constraint ensures that hiring is never negative.

The third crucial assumption is a positive adjustment cost:

Assumption 3. *The cost c_N of hiring non-routine workers is positive:*

$$c_N > 0.$$

This assumption follows from the extensive literature documenting positive hiring costs: the survey by Hamermesh (1993, page 208) mentions that, in 1980, an employer spent 42 hours and $13,790 dollars (in 1990 terms) recruiting and training a new hire.

Assumption 3 has two implications. First, hiring costs are larger than firing costs, which is also consistent with Hamermesh: "The same study found separation costs to be much smaller, roughly $1,780." Second, hiring costs are larger for non-routine workers than for routine workers, which is also consistent with Hamermesh, who reports "an average hiring cost for all occupations of $910, but an average for managerial and professional workers of $4,660" for 1965." For simplicity, the model assumes that firing costs, as well as hiring costs for routine workers, are zero.

2.4 Equilibrium

The clearing of the labor market requires that labor supply equal labor demand:

$$L_t = L_{N,t} + L_{R,t}. \tag{2.8}$$

This condition, in combination with the utility function, implies that the household is indifferent between the two types of employment. Labor supply is perfectly substitutable between routine and non-routine workers and any difference in behavior between the two is due to labor demand. An extension of this model could allow for each occupation to have a different level of human capital and for costly reallocation between occupations. This cost of reallocation would induce an intertemporal tradeoff for the household, which would prefer to reallocate during recessions, and strengthen the mechanism described in this paper.[7] Note that the clearing of the labor market implies that the wage is endogenous in the model and equates demand and supply.

The clearing of the product market follows from the budget constraint, the definition of the firm's profits, and labor market clearing. The clearing of the capital market is implicit in the use of a single symbol for capital supply and capital demand.

An equilibrium of this economy is a set of quantities (consumption C_t, capital stocks $K_{I,t}$ and $K_{N,t}$, employment quantities L_t, $L_{N,t}$ and $L_{R,t}$, and output Y_t) and prices (rental rates $r_{I,t}$ and $r_{N,t}$, and wages w_t), conditional on exogenous variables (time t, TFP A_t, the

[7] This reallocation cost on labor supply is the approach of Jaimovich and Siu (2012), discussed in Section 6.

productivity b_t of the IT-producing technology, and the labor wedge X_t), such that the household maximizes utility subject to the budget constraint and the capital accumulation constraints, the firm maximizes profits subject to the non-routine labor accumulation constraint and non-routine hiring positivity constraint, and all markets clear.

Note that this model nests the Ramsey growth model, which corresponds to a two-factor production function, with $\gamma = 0$, no adjustment costs, with $c_N = 0$, and constant labor supply.

2.5 Existence and asymptotic balanced growth path

The full characterization of this model is in Appendix A.1. An equilibrium of this model is guaranteed to exist by the next lemma:

Lemma 4. *An equilibrium exists and it is unique.*

The existence follows from rewriting the market outcome as the solution to a central planner's problem in the form of a Bellman equation, and then using Blackwell's sufficient conditions for the contraction mapping theorem. (See Appendix A.1 for proofs of lemmas in this subsection.)

An appealing property of growth models is the balanced growth path, consistent with the "Kaldor facts" of a constant interest rate and a constant capital-output ratio (Kaldor, 1963, page 178). The following lemma characterizes the behavior of the asymptotic balanced growth path, where employment is stationary and all other quantities, aside from employment, grow at the same rate.

Lemma 5. *Consider the limiting economy, where TFP grows at rate $g_A > 0$, b_t tends to $\bar{b}$, consumption grows at rate g_C, and there are no labor wedge shocks, $X_t = X$. Employment is stationary if the growth in the disutility of labor supply verifies*

$$\log(1+g) = \frac{1-\eta}{\beta} g_A,$$

in which case consumption, output, and all quantities other than employment grow at rate g_A/β.

In particular, the conventional case of log-utility, with $\eta = 1$ and $g = 0$, ensures that employment is stationary in the long-term.

3 Results in a special case

The general model, with both growth and business cycle features, provides a framework to understand the link between the trend phenomenon of routinization and the cyclical phenomenon of jobless recoveries. As a first step in understanding the crucial mechanism of the model, this section uses a simplified version of the model that ignores the interaction between trend and cycle by removing all channels of intertemporal substitution. (This interaction is the focus of the next section.) In this simplified model, the crucial tradeoff is only the firm's choice of its input mix depending on input costs. This special case is tractable and delivers the analytical result of a productivity speedup and longer jobless recoveries.

3.1 Simplifications

The simplifications of the special case are as follows. There are no hiring costs, with $c_N = 0$, so the firm is free to adjust non-routine labor. The household is infinitely impatient, with $\theta \to 0$: at the limit, the economy behaves as if the household lived for one period and there were a new household in the next period. Capital accumulates immediately and depreciates fully after one period (Long and Plosser, 1983):

$$K_{N,t} = I_{N,t},$$
$$K_{I,t} = I_{I,t}.$$

In this special case, the firm has no frictions and makes zero profits. Since capital equals investment, the household's budget constraint in equation (2.4) is

$$C_t + (1 - r_{N,t}) K_{N,t} + (\exp(-b_t) - r_{I,t}) K_{I,t} \leq w_t L_t.$$

In equilibrium, the household sells capital to the firm at marginal cost, with $r_{N,t} = 1$ and $r_{I,t} = \exp(-b_t)$, and the budget constraint becomes

$$C_t = w_t L_t.$$

3.2 Productivity speedup and longer jobless recoveries

The increase in the productivity b_t of the IT-producing technology induces a long-term speedup in labor productivity, which in turn induces longer jobless recoveries. This subsection considers the case of no TFP shocks, with $A_t = A$, so the time-varying exogenous variables are the labor wedge shock X_t and the productivity b_t of the IT-producing technology. The analytical results in this subsection rely on shocks to labor wedge X_t.

To prove the analytical result of the productivity speedup and longer jobless recoveries, the next lemma shows that the wage is a non-linear function of the cost of computers. This non-linear aggregation is the key mechanism in the long-term behavior of the model. (See Appendix A.2 for all proofs in this subsection.)

Lemma 6. *In equilibrium, the wage is the unique solution to*

$$w_t^{\beta} \left(w_t^{1-\sigma} + r_{I,t}^{1-\sigma} \right)^{\frac{\gamma}{1-\sigma}} = A\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}. \tag{3.1}$$

This equation follows from the zero-profit condition of the firm. It shows that the assumption $\sigma > 1$ induces a non-linear aggregation of wage and the cost of computers $r_{I,t} = \exp(-b_t)$. When computers are too expensive, for b_t small, the term $r_{I,t}^{1-\sigma}$ vanishes from the equation and wages are insensitive to the cost of computers. When computers are competitive relative to workers, the term $r_{I,t}^{1-\sigma}$ is comparable to $w_t^{1-\sigma}$ and wages respond to the cost of computers. (Note that wages are endogenous in the model.)

The next proposition uses this non-linear aggregation to show that labor productivity is a log-convex function of the logarithm b_t of the IT producing sector. If the cost of computers has an exponential decrease, then labor productivity speeds up, i.e. its growth rate increases with time.

Proposition 7. *If $\sigma \in (1, 2)$, labor productivity is log-convex in the productivity b_t of the IT-producing sector:*

$$\pi_t \equiv \log \frac{Y_t}{L_t}, \qquad \frac{\partial^2 \pi_t}{\partial b_t^2} > 0.$$

If $\sigma \in (1, 2)$, labor productivity is log-convex in the technology b_t: for a given increase in b_t, the growth rate of labor productivity is higher when b_t is larger. Intuitively, when the cost of computers $r_{I,t}$ is high compared to wages w_t, computers are still expensive even after a decrease in their cost, the firm undertakes a small replacement of routine workers with computers, and the quantitative response of labor productivity is small. The IT capital-routine labor ratio is low and the CES aggregate $(K_I^\rho + L_R^\rho)^{1/\rho}$ in the production function is dominated by routine labor L_R. The economy is in a *labor-intensive phase*, characterized by a negligible stock of IT capital, $K_{I,t} \approx 0$ and a constant routine share $L_{R,t}/L_t$ of employment.

When the cost of computers becomes comparable to wages w_t, computers continue to become cheaper, the firm undertakes a large replacement of routine workers with computers, and the quantitative response of labor productivity is large. The IT capital-routine labor ratio increases and the CES aggregate becomes dominated by IT capital. The economy is in a *technological upgrading phase*, characterized by an increasing stock of IT capital $K_{I,t}$ and by a decreasing routine share $L_{R,t}/L_t$ of employment.

The firm substitutes computers for routine labor, which in turn reallocates to non-routine labor. Both of these reallocation channels increase labor productivity, and they increase it faster the lower the level of the cost of computers. Note that there are no threshold effects on b_t and labor productivity is a continuous function: the crucial point is that the slope of labor productivity depends on the level of b_t.

The threshold at $\sigma = 2$ is similar to the result in Acemoglu (2009, page 510), who finds a different behavior for an economy with directed technical change depending on whether the elasticity of substitution between skilled and unskilled labor is above or below 2.

The next proposition shows that these differential effects follow from the assumption $\sigma > 1$. If $\sigma = 1$, labor productivity is linear in the productivity of the IT-producing technology:

Corollary 8. *If $\sigma \to 1$, labor productivity is log-linear in IT productivity:*

$$\left.\frac{\partial^2 \pi_t}{\partial b_t^2}\right|_{\sigma \to 1} = 0.$$

Therefore, productivity growth is independent of the level of the cost of computers in the case of $\sigma \to 1$, i.e. there is no productivity speedup.

Before considering economic fluctuations, note that labor productivity is independent of the labor wedge shock X_t (see equation A.1 in Appendix A.2):

$$\frac{\partial \pi_t}{\partial x_t} = 0.$$

Therefore, labor productivity only has a trend, due to the increase in the productivity of the IT-producing technology, and no fluctuations (recall that $A_t = A$). This feature is useful for proving the next result.

The next proposition is the key result of this subsection. Recall that the length of a jobless recovery is the period between the trough of output and the trough of labor, when output increases and employment decreases.

Proposition 9. *Suppose that there are no TFP shocks ($A_t = A$), that households have log-utility ($\eta = 1$), and that X_t is periodic, with a single trough in each cycle. To a first-order approximation, a productivity speedup causes jobless recoveries to last longer.*

Productivity growth is the difference between output growth and employment growth. This proposition shows that the faster the growth rate of labor productivity, the longer output can increase with labor simultaneously decreasing. Overall, for the same increase in b_t, jobless recoveries last longer the higher the level of b_t, i.e. they last longer in the technological upgrading phase than in the labor-intensive phase. This result of longer jobless recoveries conforms to the aggregate statistics after the recessions of 1990, 2001 and 2007. It depends on the increase in the productivity of the IT-producing sector and on the assumption of gross substitutability between routine labor and computers, $\sigma > 1$. In the Cobb-Douglas case with $\sigma = 1$, productivity growth is constant, so there is no productivity speedup and no difference in the length of jobless recoveries. (See Figure 4 in Section 4.3 for an illustration.)

3.3 Illustration

To illustrate the mechanism numerically, this subsection calibrates the crucial parameters, with the remaining parameters calibrated in Section 4.1. As Proposition 7 emphasizes, the two important parameters are the elasticity of substitution σ between IT capital and routine labor and the path for the cost of computers. With $\sigma \in (1, 2)$ and a continuous decrease in the cost of computers, there is a speedup in labor productivity. An estimation of this production function at the sectoral level in recent recoveries implies an elasticity of substitution σ around 1.6 (details available from the author). This value is also consistent with the estimate of Krusell et al. (2000), who find an elasticity of substitution of 1.67 between unskilled labor and equipment.

For the cost of computers, this paper alternates between two specifications. The first is a smooth path for the productivity b_t of the IT-producing sector:

$$b_t = \bar{b} - \log\left(1 + e^{-\phi(t-t_0)}\right).$$

In this specification, the logarithm of IT productivity is concave.[8] Early in time there is a linear increase in the productivity b_t, i.e. $\Delta b_t \approx \phi$. Late in time the increase in b_t slows down, i.e. $\Delta b_t \approx 0$ and b_t approaches its upper bound $\bar{b}$. The rate of decrease ϕ is 27 percent per year, at the upper bound of the range in Table1, and the timing of the slowdown is $t_0 = 2010$.

The second specification is based on the cost of computers from the BLS (see Figure 2), with a linear interpolation before 2010 and a concave extrapolation after 2010 (similar to the slowdown in the first specification). This paper uses the specification from the BLS in all numerical examples involving real data and the smooth specification otherwise.

[8]The acceleration of b_t is negative:

$$\frac{d^2 b}{dt^2} = -\phi^2 \frac{e^{\phi(t-t_0)}}{\left(1 + e^{\phi(t-t_0)}\right)^2} < 0.$$

First consider the long-term behavior of the model. Figure 3 shows the behavior of the economy in the case of log-utility and without business cycles. The income and substitution effects cancel out and the economy has constant employment. Early in time, for $t \ll t_0$, the slope of the productivity of the IT producing technology is positive, but its level is too low and it has a minimal effect on productivity and on routinization. Intuitively, computers are too expensive and the firm relies on routine labor instead. The economy is in a labor-intensive phase, characterized by near zero IT capital (unreported in this figure), which lasts roughly until the 1980s in this example.

As the cost of computers becomes comparable to wages, the firm starts replacing routine workers with computers. Labor reallocates away from routine labor L_R and into non-routine labor L_N, away from jobs that are easily replaced by computers and into jobs that are more difficult to replace with computers. The firm produces more output for the same level of employment and has a productivity speedup. The economy is in a technological upgrading phase, characterized by an increasing stock of IT capital, which lasts roughly until the 2010s in this example.

As the increase in the productivity of the IT-producing technology decelerates, the firm slows down the replacement of routine workers and the economy stabilizes at its steady-state. The slowdown in the growth of b_t causes a slowdown in labor productivity, which in turn implies shorter jobless recoveries, just as the productivity speedup implied longer jobless recoveries. The economy is in a *capital-intensive phase*, characterized by a constant stock of IT capital, which starts roughly in the 2010s in this example.

The combination of a productivity speedup, when b_t has a linear increase, and a productivity slowdown, when b_t approaches its upper limit, implies that output, consumption, and capital follow an S-shaped path, as in Figure 3. This S-shaped pattern is a consequence of labor productivity as a log-convex function of b_t and b_t as a concave function of time. The convex part represents a productivity *speedup* and the concave part represents a productivity *slowdown*. This S-shaped pattern is similar to the product adoption literature: Rogers (1995, page 152) mentions that “A general finding of past research is that the adoption of an innovation ... is essentially ’S’-shaped when plotted on a cumulative basis.” (See also McKay and Reis, 2008, page 745.)

Note that there are no threshold effects in this model and that the transition between the three phases is continuous. Productivity growth is positive in the labor-intensive phase, but it tends to zero early in time. As the cost of computers decreases, the firm always adjusts its input mix, but that adjustment is quantitatively small when the cost of computers is large.

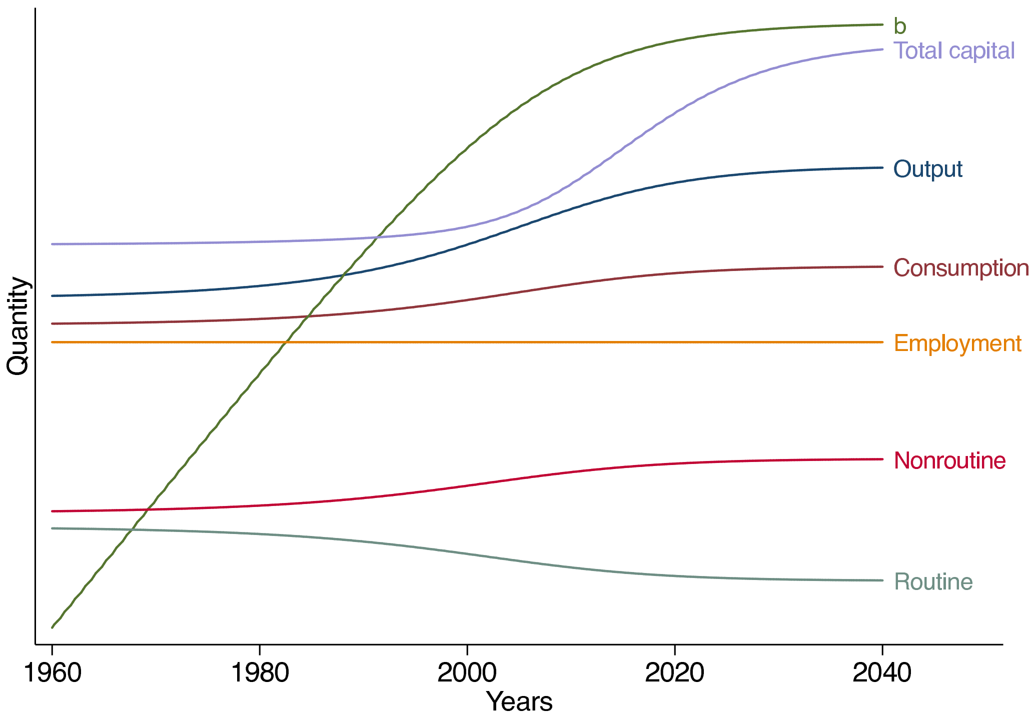

Figure 3: Special case of the model in the long-term: aggregate variables have an S-shape, except routine employment, which has a reverse S-shape.

After the long-term behavior of the model, consider business cycles and the dynamics of employment around recessions. The following simulation also uses log-utility and confirms the validity of this first-order approximation in Proposition 9. Total Factor Productivity is constant, with $A_t = 1$. The cost of IT is the series from the Bureau of Labor Statistics in Figure 2 with a linear trend at the quarterly frequency. The logarithm of the labor wedge follows an AR(1) process:

$$\log X_t = 0.98 \, \log X_{t-1} + 0.08 \times \mathcal{N}\,(0, 1)\,.$$

This specification roughly matches the persistence and variance of output implied by the model to those of US GDP.[9]

Given these exogenous variables, the model determines the paths of the other variables, including output and employment. Define a trough of a quantity when it has two preceding quarters of decrease and two succeeding quarters of increase. Recall that a jobless recovery occurs when the trough of labor is after the trough of output. Consider the following frequency of jobless recoveries:

$$\mathbb{P}\left(\text{jobless recovery with length } n\right) = \frac{\#\left\{\text{jobless recoveries with length } n\right\}}{\#\left\{\text{recoveries}\right\}}.$$

The denominator in this formula is between 1,000 and 6,000. Figure 4 plots the frequency of jobless recoveries for 100,000 paths of X_t: as time passes and the cost of IT decreases, the probability of a jobless recovery is larger and jobless recoveries are longer.

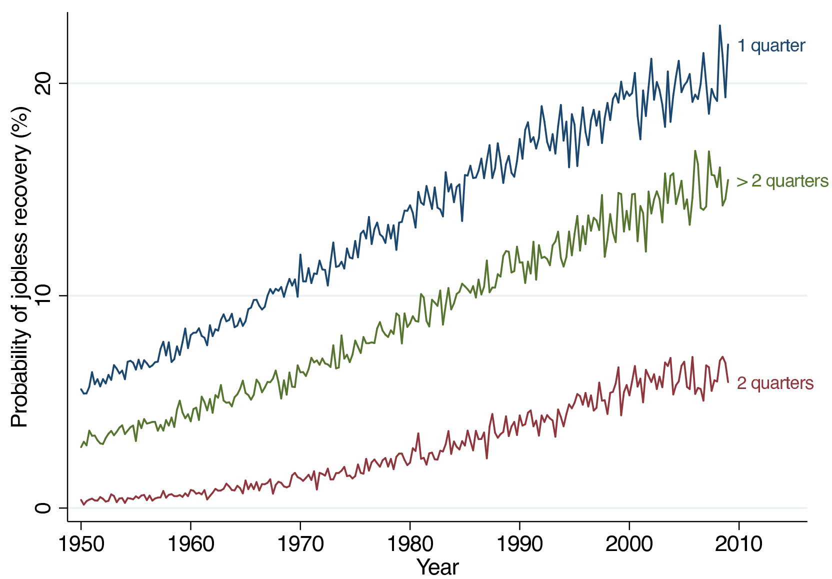

Figure 4: The probability of a jobless recovery increases with time. (See text for details.)

To confirm the relevance of the model for US business cycles in particular, the next numerical example uses supply shocks to TFP. A calibration with supply shocks and without demand shocks requires adjusting two parameters: the curvature of the utility

[9]These parameters imply an auto-correlation at one lag for log-output of 0.9578, compared to 0.9637 in the data, and a variance of log-output of 0.008, compared to 0.0029 in the data.

function, which is now $\eta = 0.5$, and the growth rate in the disutility of labor supply, which is now $g = 0.3$ percent (1.2 percent in annual terms).

The numerical solution of the model computes the series of TFP shocks that are exactly consistent with the behavior of output, so $Y_t(A_t, b_t) = GDP_t$. Figure 5 shows the dynamics of employment around recent recessions in the special case of the model, and Figure 6 is the empirical counterpart (identical to Figure 1, repeated for convenience). The calibration of the special case of the model matches the dynamics of employment in postwar recoveries.

This special case of the model emphasizes that the mechanism behind jobless recoveries is the replacement of routine workers with computers. In particular, the recession is simply an upward or downward scaling of all variables. The fully dynamic model, discussed in the next section, lets the firm choose the optimal time to fire workers.

4 Quantitative results

This section considers the fully dynamic version of the model. Compared to the special case of the model above, the main difference is that the firm chooses the optimal timing of hiring and firing. The hiring cost for non-routine workers causes firms to hoard or retain non-routine workers and to fire routine workers even more, compared to a hypothetical situation without hiring costs. The interaction between this cyclical responsiveness and the secular decline causes routine employment to decrease especially during recessions. In effect, the recession causes long-term adjustment to occur in the short-term and accelerates the secular decline in routine employment during recessions that occur in the technological upgrading phase.

4.1 Calibration

The calibration of this model uses the same values as Section 3.3, with an elasticity of substitution between IT capital and routine labor $\sigma = 1.6$. For the path of the

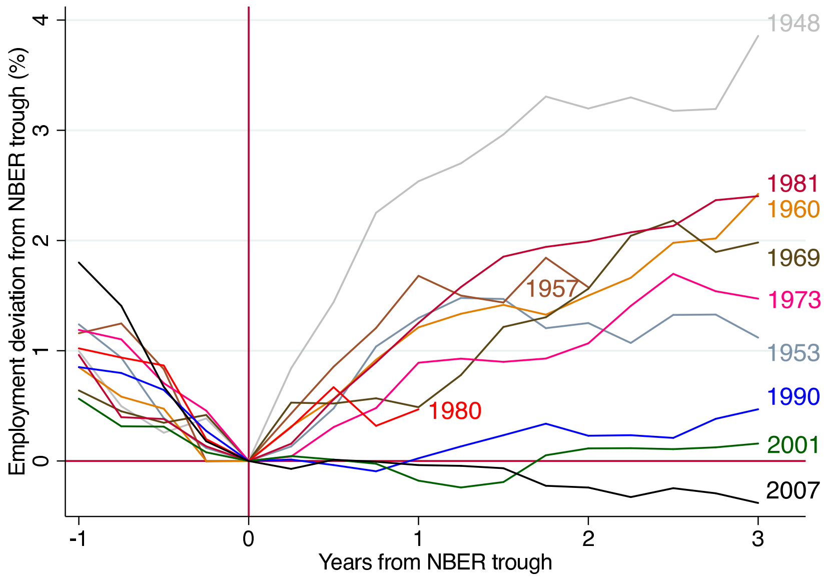

Figure 5: Dynamics of employment around recessions in the special case of the model.
Later recessions use darker colors.

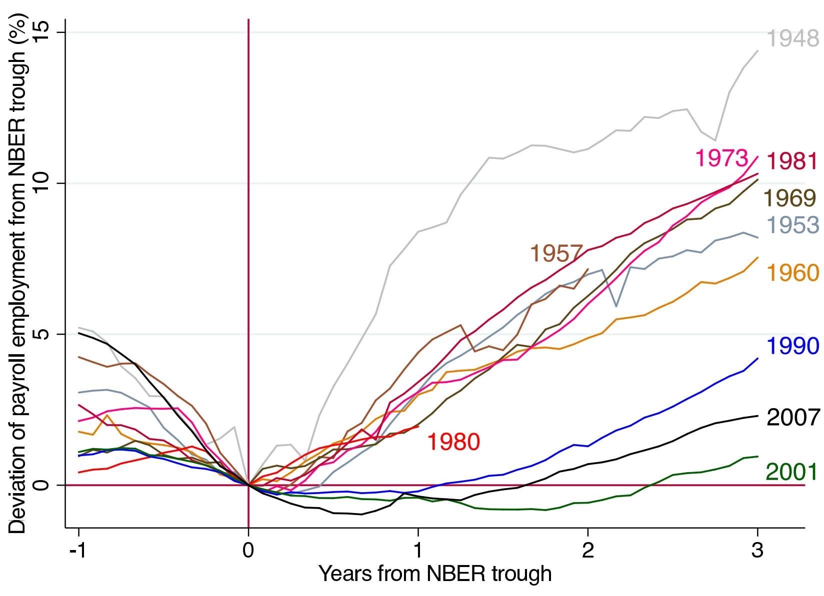

Figure 6: Dynamics of employment around recessions in the data (same as Figure 1 in the introduction).
Source: Federal Reserve Economic Database. Employment: payroll survey. Later recessions use darker colors.

productivity of the IT-producing technology, the simulations in this section use a smooth concave function:

$$b_t = \bar{b} - \log\left(1 + e^{-\phi(t-t_0)}\right),$$

with $\phi = 14\%$, in the middle range of the estimates in the literature, and $t_0 = 2010$. The numerical example with the actual series of US GDP uses the cost of IT from the BLS (see Figure 2). The hiring cost equals the disutility of labor supply, with $c_N = 1$. This value implies that the cost of hiring a new worker is around one quarter of wages and is consistent with previous literature.[10] Alternatively, a benchmark model uses no hiring costs, with $c_N = 0$.

The share of non-IT capital is $\alpha = 0.3$, the standard share of capital in aggregate income. The non-routine share $\beta = 0.39$ of aggregate output is from the Current Population Survey in 2007, identifying workers as non-routine if they are below the median.[11] The quarterly discount factor is $\theta = 0.99$. The utility from consumption is logarithmic with $\eta = 1$ and there is no growth in the disutility of labor supply with $g = 0$. For the elasticity of labor supply, there is a long-lasting debate: microeconomists, focusing on sub-populations, find an elasticity smaller than unity, while macroeconomists, focusing on the whole population, find an elasticity greater than unity (Chetty, 2009, page 5). Keane (2011, page 1042) surveys this literature and finds an elasticity of 0.85 (averaged across different studies). Given this debate, the calibration uses $\varepsilon = 1$. The depreciation of normal capital is $\delta_N = 2.5$ percent (10 percent in annual terms) and the depreciation of IT capital is $\delta_I = 7.5$ percent (30 percent in annual terms).

Parameter	α	β	γ	σ	c_N	θ	η	g	ε	δ_I	δ_N
Value	0.3	0.39	0.31	1.6	1 or 0	0.99	1	0%	1	7.5%	2.5%

Table 2: Parameter values.

[10] The adjustment costs in Berger (2012, page 23) are 7 months of wages. The actual value is of little importance, as long as it induces non-routine hoarding during the recession.

[11] Multiplying the non-routine share of labor income of 56 percent in 2007 by the labor share of income of 70 percent yields $\beta \approx 0.39$.

4.2 Simulations

This subsection decomposes the behavior of routine employment during a recession into three parts, a trend component and two cyclical components. The *trend* component corresponds to the effect of the declining cost of computers, which causes the secular decline in routine employment that gains importance in the 1980s. The *frictionless cyclical* component corresponds to the hypothetical response of routine employment to the recession in the absence of adjustment costs ($c_N = 0$). The *frictional cyclical* component corresponds to the additional response of routine employment to the recession in the presence of frictions, e.g. to the response of routine employment with adjustment costs compared to the case without adjustment costs. The first two components, trend and cycle without frictions, are present in the special case of the model. The third component, cycle with frictions, is due to intertemporal substitution in the firm's behavior and is absent from the special case of the model.

The model is analytically intractable and requires a numerical solution.[12] The specification for TFP shocks follows the standard AR(1) process in Kydland and Prescott (1982):

$$\log A_t = 0.95 \log A_{t-1} + 0.009 \times \mathcal{N}(0,1).$$

These simulations use a stationary path for TFP, with no trend growth. (Note that this calibration of TFP shocks concerns only these simulations, while the fit of US data in the next subsection computes the implied TFP shocks directly from the data.) The simulations use 600 paths for TFP A_t and solves two models, first without adjustment costs ($c_N = 0$) and then with adjustment costs ($c_N = 1$). The next OLS regressions, restricted to the time periods where TFP is decreasing with $\Delta \log A_t < 0$, estimate the elasticity of employment to a TFP shock:

$$\Delta \log L_{J,c_N,t} = constant + coefficient \times \Delta \log A_t + error, \qquad J = N, R, \quad c_N = 0, 1.$$

The results of this regression are in Table 3. (Due to the large size of the sample, this table omits standard errors.) The response of non-routine employment to a TFP shock

[12] I thank Chris Conlon for suggesting AMPL as an efficient solver for this type of problem.

is smaller with hiring costs, while that of routine employment is larger. Hiring costs cause a responsiveness of routine employment that is nearly 20% higher than without hiring costs. (Similar results hold when restricting to the time periods with non-routine hoarding in the solution with frictions.) Therefore, these simulations confirm that the recession accelerates the secular decline of routine employment, both compared to the expansion and to an economy without hiring costs. (Furthermore, the simulations also show that the probability and the length of a jobless recovery increases with time, similar to the special case of the model.)

$\Delta \log L_{j,c_N,t} / \Delta \log A_t$	$c_N = 0$	$c_N = 1$
L_N	71.9%	0.06%
L_R	71.5%	83.4%

Table 3: Estimates of the elasticity of response of employment to a TFP shock.

The combination of the three components implies that the secular decline of routine employment accelerates during recent recessions. The typical behavior of routine employment is V-shaped during early recessions and L-shaped during late recessions. Furthermore, this asymmetric behavior of routine employment causes jobless recoveries. During a recovery from an early recession, non-routine employment is dishoarded and remains constant, while routine employment recovers. Both output and total employment recover. During a recovery from a late recession, non-routine employment is also dishoarded, while routine employment does not recover. Output recovers but total employment remains stagnant. Figure 7 shows the representative dynamics of employment during a business cycle.

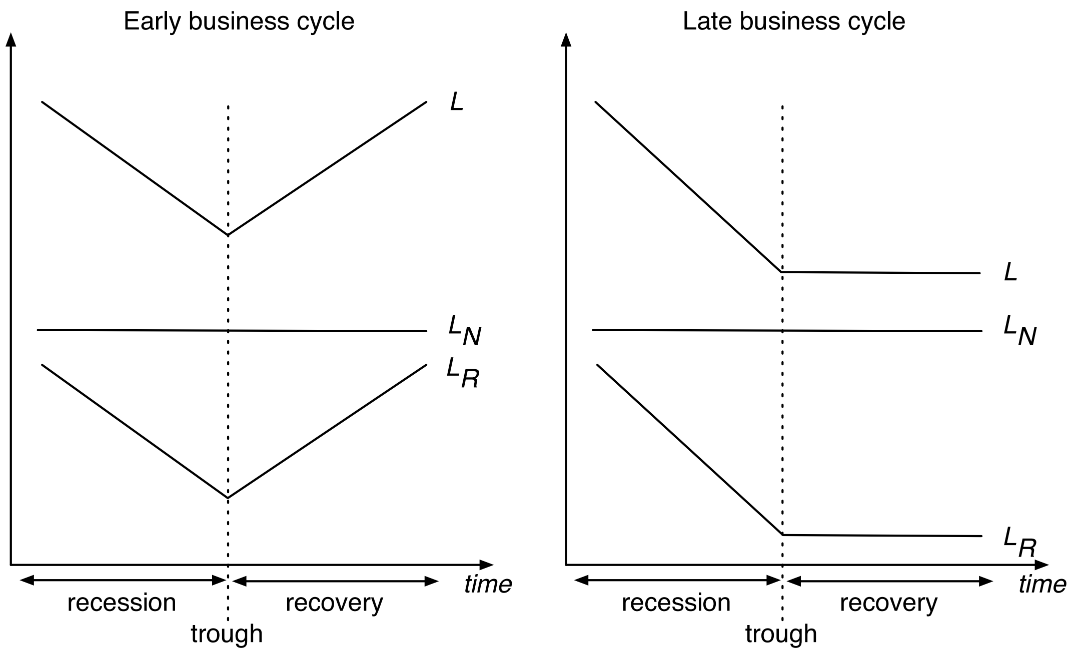

Figure 7: Representative dynamics for employment during early and late business cycles.

4.3 US data

This subsection calibrates the model with the series of US GDP. The numerical solution to the model ignores labor wedge shocks, imposing $X_t = 1$, and computes the shocks to TFP A_t that match US GDP exactly.[13]

Figure 8 shows that the model matches the dynamics of employment around the last five recessions. Figure 9 shows employment around the last four recessions and shows that the mechanism of the model is non-routine labor hoarding. The firm hoards non-routine workers during recessions, rather than firing them in a recession and hiring them again in a recovery. This hoarding causes the firm to fire routine workers more than without non-routine hoarding.[14] In all recessions of Figure 9, the firm hoards non-routine labor and

[13]Specifically, consider the characterization of the equilibrium in Appendix A.1. The analytical solution to the model uses output as an endogenous variable and TFP as an exogenous variable. The numerical solution solves the system of equations, replacing output with actual GDP and computing the series of TFP shocks that are exactly consistent with output. This procedure avoids computing a nested fixed point and allows an efficient calibration that solves in a few seconds.

[14]As an example of this behavior at the micro level, Bewley (1999, page 230) explains that managers

adjusts with routine labor. After the 1982 recession, the firms hires back routine workers, but after the 2007 recession, the firm does not hire them back and invests in computers instead (see Figure 15 in Appendix A.3 for the path of IT investment in the data and in the model). Overall, the recession accelerates the secular decrease in routine employment, a prediction tested in Section 5. In recent recoveries, non-routine employment stagnates as the firm dishoards its stock of non-routine labor, and routine employment stagnates as it returns to its declining trend. Thus, the model predicts that total employment stagnates even as output recovers.

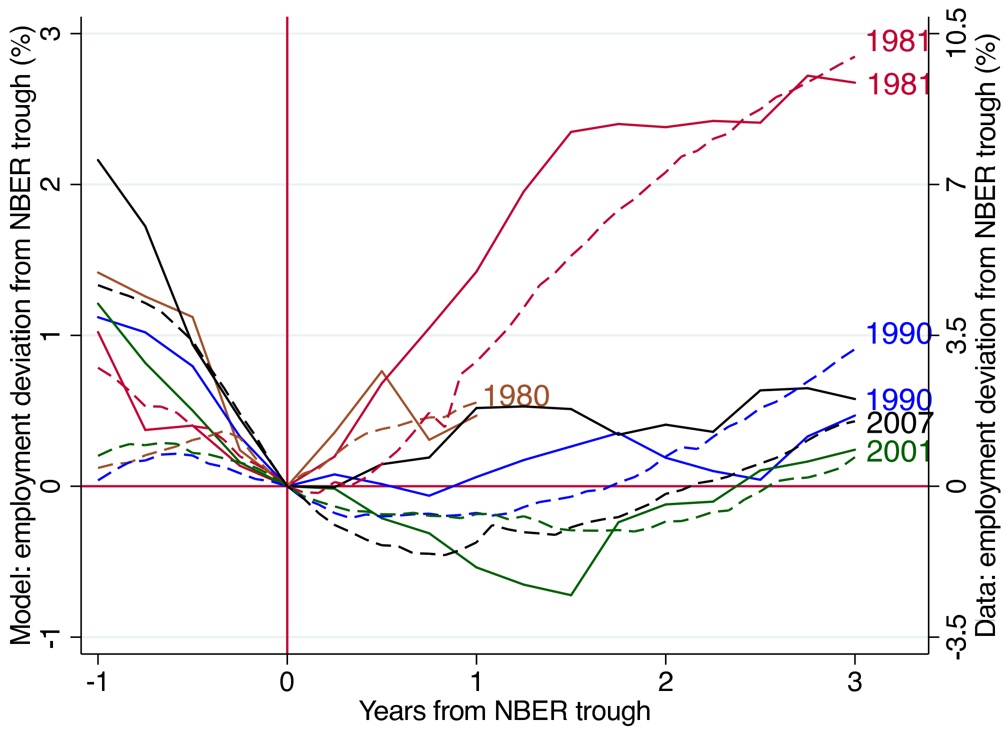

Figure 8: The model matches the dynamics of employment around recessions.
Model: left axis and solid line. Data: right axis and dashed line. Later recessions use darker colors. For clarity, this figure has only the last five business cycles.

Among the current explanations for the jobless recoveries, Berger (2012) provides a calibration, reproduced in Figure 10. Similar to the calibration in this paper, he uses the series of productivity shocks that are exactly consistent with the path of output over

have limited time and prefer to reorganize their firms during recessions, when the opportunity cost is low, rather than during expansions, when alternative projects draw their attention.

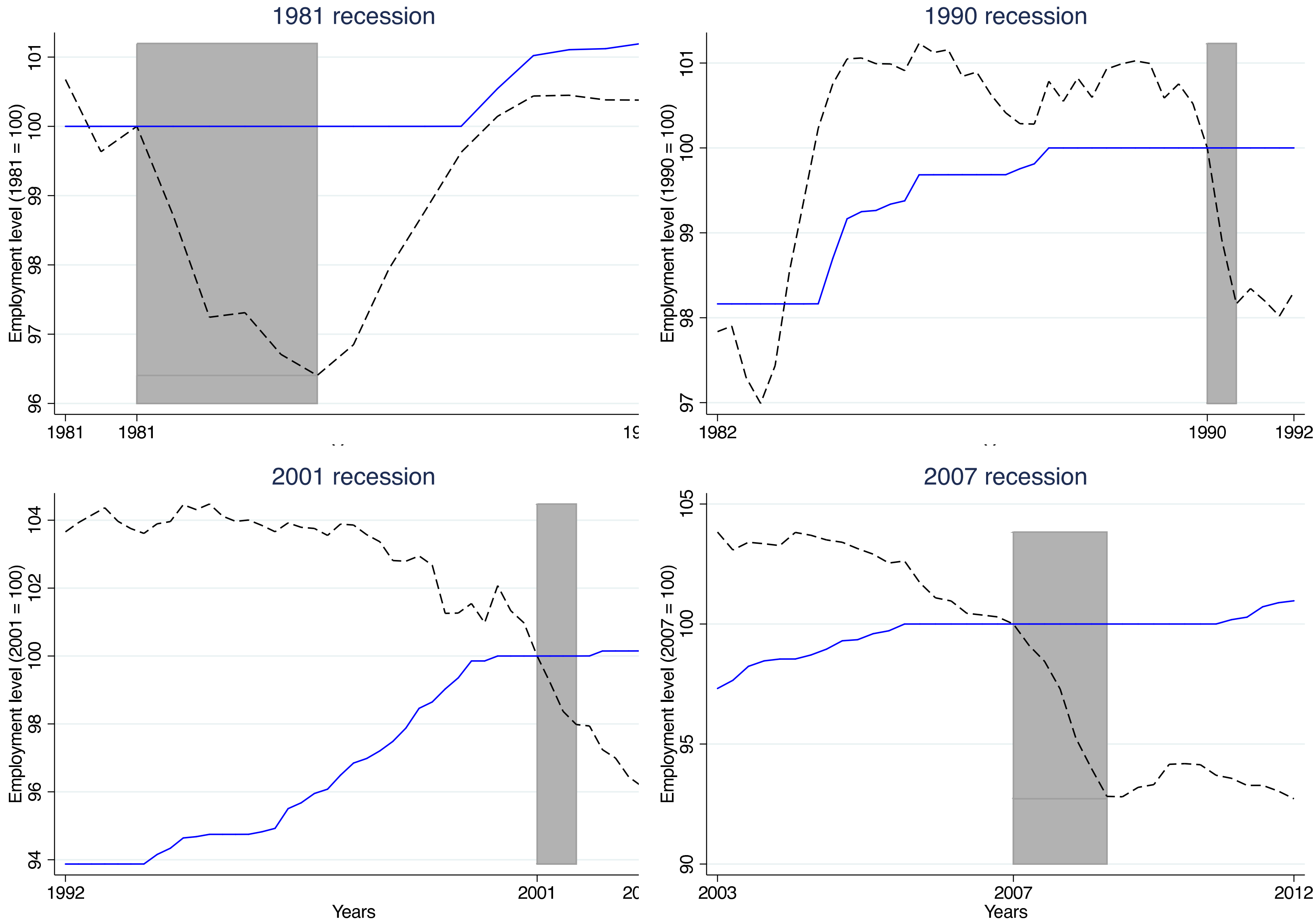

Figure 9: Time series of disaggregate employment for four recent recessions in the calibration of the model. Shaded areas are NBER recessions. Routine employment in dashed black, non-routine employment in solid blue.

the postwar period. His model is able to match the path of employment for the most recent recession. In contrast, the calibration in this paper is able to match the path of employment over other postwar recessions as well.

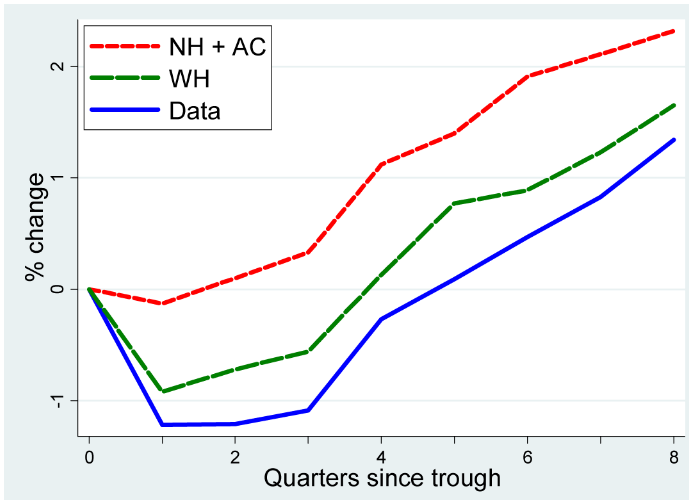

Figure 10: Dynamics of employment in the model of heterogeneous workers in the recession of 2007 (Berger, 2012, Figure 6, page 30).
"WH" is Berger's model with worker heterogeneity. "NH+AC" is an alternative model without heterogeneity and with adjustment costs.

5 Additional evidence

The model predicts that firms hoard non-routine workers during recessions and shift the burden of adjustment to routine workers. This section considers the empirical counterpart of that prediction with the Current Population Survey matched to the Occupational Information Network.

For a measure of routinization, the seminal paper by Autor, Levy and Murnane (2003) uses two characteristics of occupations: the degree of automation and the importance of manual or cognitive tasks. The paper shows that computers replace jobs that are both automated and cognitive, such as clerks and bookkeepers. In contrast, computers complement jobs that are non-automated and cognitive, such as engineers and researchers. Computers have no impact on manual jobs, such as janitors or gardeners. Under this hypothesis, jobs that are automated and cognitive constitute the category of routine employment L_R, which can be more easily replaced by computers, while the other jobs constitute the category of non-routine employment L_N. Therefore, the relevant measures for routinization are the degree of automation, the importance of assisting other people, and the level of creativity. An index of routinization combines these three measures with:

$$\text{routinization}_j = \text{automation}_j - \text{assisting others}_j - \text{level of creativity}_j,$$

where j indexes occupations. Occupations are then aggregated into employment quartiles, depending on their employment level in 2007 and their routinization measures.

Figure 11 plots the time-series of each quartile from 2003 to 2010,[15] and shows that routine and non-routine occupations behave differently. The least routinizable occupations, in the first quartile, represent expanding jobs, intensive in personal interactions and creativity and with little scope for automation. They have the strongest long-term increase in the decade and no recession in 2007-2009. Occupations that are neither routine nor non-routine, in the second quartile, represent cyclical jobs: they increase during expansions and decrease during recessions. The most routinizable occupations, in the third and fourth quartiles, represent declining jobs, intensive in automation and with little scope for personal interactions or creativity. They increase little in the expansion and decrease in the recession. Between 2007 and 2010, the employment in upper quartiles of routinization decreased by 4.8 million jobs, more than 3 percent of employment in 2007. This differential behavior conforms to the predictions of the model (see Figure 9 and the discussion at the end of Section 4.3).

Previous recessions require another classification system (see Appendix A.4 for details). This classification system is coarser and has structural breaks in 1992 and 2003, but still points to an acceleration of the firing of routine workers during recessions: in the 1990s and 2000s, the more routinizable occupations grew slower in the expansion and decreased more in the recession (see Figure 12). In contrast, during the 1980s, all quartiles of employment had a similar rate of increase during the expansion.

[15] The occupational classification of the CPS changed every decade. Figure 11 uses the 6-digit Standard Occupation Classification of 2000 (SOC2000), which served for the CPS between 2003 and 2010.

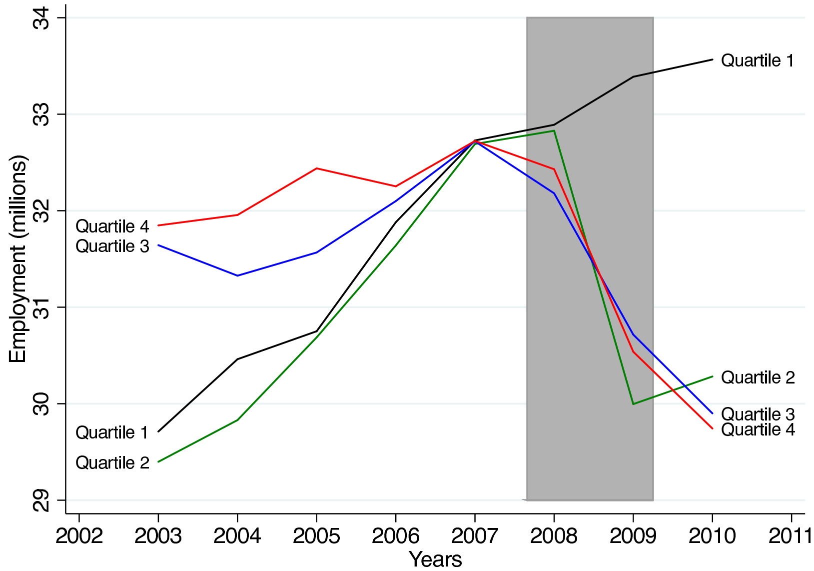

Figure 11: Employment by routinization quartiles in 2007. (The shaded area is the 2007-2009 recession.)

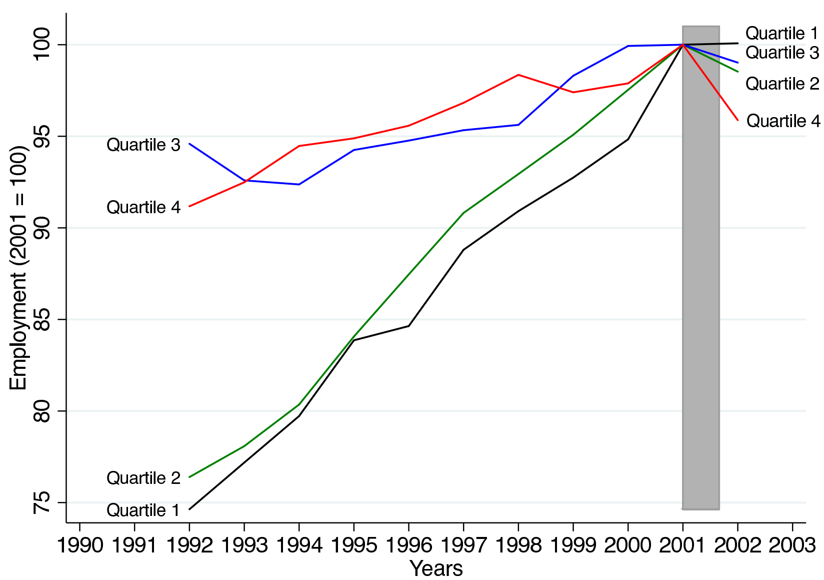

Figure 12: Employment by routinization quartile in 2001. (The shaded area is the 2001 recession.)

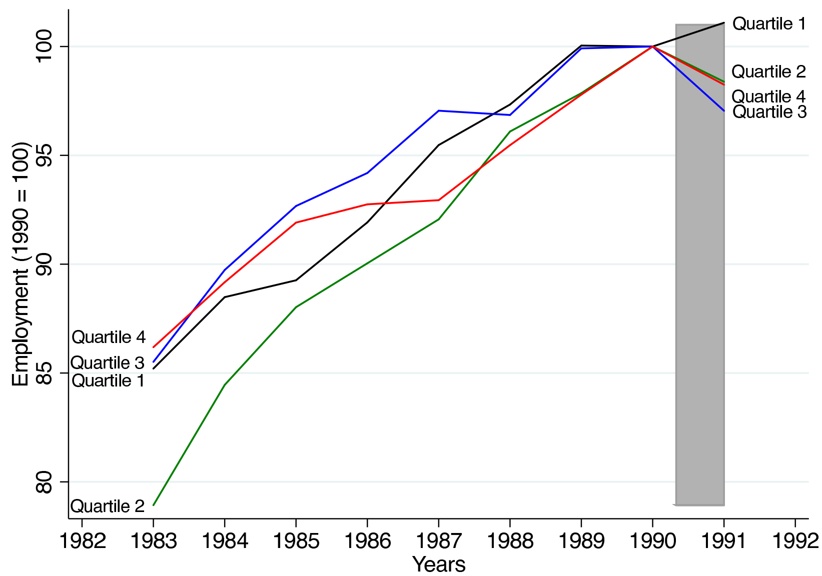

Figure 13: Employment by routinization quartile in 1990. (The shaded area is the 1990 recession.)

6 Related literature

This paper is related to a growing literature on jobless recoveries, dominated by two leading explanations. The first explanation, by Jaimovich and Siu (2012), uses a period of retraining in the reallocation from routine to non-routine occupations. Workers prefer to reallocate when the opportunity cost is low, i.e. during recessions if wages are procyclical. Jaimovich and Siu, as well as this paper, explain jobless recoveries with a distinction between routine and non-routine workers. The models are different in that Jaimovich and Siu use a labor supply mechanism, with workers unwilling to search, while this paper uses a labor demand mechanism, with firms unwilling to hire. Moreover, two advantages of the model in this paper are that wages can be acyclical, since the cyclical mechanism relies on the hiring cost rather than the wage, and the divergence between routine and non-routine employment is endogenous with a continuous mechanism instead of a structural break.

The second leading explanation, by Berger (2012), uses a labor demand mechanism, in which the recent decrease in unionization allows firms to fire unproductive workers more easily during recessions. Berger, like this paper, matches the emergence of longer jobless recoveries after the 1980s by distinguishing between two types of workers. In addition, this paper provides contributions both theoretical and empirical. On the theoretical side, it endogenizes a continuous productivity speedup without structural breaks or threshold effects. On the empirical side, it suggests that jobless recoveries are a recurrent issue in economic history, linked to the decrease in the cost of an essential input and to Industrial Revolutions in general. Future work can distinguish these two hypotheses: for example, the rapid decrease in the cost of electricity during the first half of the 20th century coincided with an *increase* in unionization. The model in this paper predicts jobless recoveries during that period, whereas Berger's model predicts the opposite.

Another popular explanation for jobless recoveries concerns offshoring. The routinization literature has difficulty differentiating the two explanations of jobs replaced by computers and sent overseas. The first half of the 20th century can also distinguish these two hypotheses, since offshoring was infeasible at that time.

There are other explanations for jobless recoveries, highlighting different mechanisms. Calvo, Coricelli and Ottonello (2012) show that recoveries from financial crises tend to be more jobless, presumably because collateral constraints cause firms to invest in capital-intensive rather than labor-intensive projects. Their paper has the merit of analyzing the specific cause of the crisis whereas this paper abstracts from it. Albeit different, the mechanisms of financial frictions and of Information Technologies strengthen each other: firms should invest in IT capital particularly after a recession in order to replace routine workers and to take advantage of easier financing.

Koenders and Rogerson (2005) offer an alternative view and explain jobless recoveries with counter-cyclical restructuring: the longer an expansion, the more inefficiencies accumulate, the more managers increase productivity during the recession, and the more jobless the recovery. Their paper has the merit of classifying jobless recoveries according to the length of the previous expansion whereas this paper classifies jobless recoveries with their occurrence in history. The mechanism of counter-cyclical restructuring and of Information Technologies also strengthen each other: managers should restructure firms particularly in recent recessions that follow long expansions, such as the 1990 and 2001 recessions.

7 Conclusion

This paper proposes a new explanation for recent jobless recoveries, arguing that firms now use recessions as an opportunity to replace workers with computers. This behavior has been happening recently, as opposed to earlier in history, because computers are now cheap and competitive relative to routine workers such as clerks. This behavior occurs especially around the trough of the business cycle: in the recession, firms hoard non-routine workers such as managers and fire routine workers; in the recovery, firms dishoard non-routine workers, refrain from hiring back routine workers, and invest in computers instead.

This mechanism should apply to any large and continuous decrease in the cost of an essential input. During the first half of the 20th century, electricity had a rapid cost decrease and coincided with a productivity speedup in the 1920s and with the jobless recovery from the Great Depression.[16] A companion paper uses plant-level data from the Great Depression to estimate the causal effect of the decline in the cost of electricity on the level of employment.

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[16]Kehoe and Prescott (2008, page 6) write that "a satisfactory theory of the U.S. Great Depression ... needs to explain why hours fells so sharply from 1929 to 1933 and stayed so depressed afterward even though productivity recovered."

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A Appendix

A.1 Equilibrium and proofs of the model

Equilibrium of the model. Denote $\nu_{I,t}$ and $\nu_{N,t}$ the Lagrange multipliers of the capital accumulation constraints (equations 2.3 and 2.2), μ_t the multiplier of the budget constraint (equation 2.4), $\psi_{N,t}$ the multiplier on the non-routine accumulation constraint (inequality 2.6), $\vartheta_{N,t}$ the multiplier on the non-routine hiring positivity constraint (inequality 2.7), and $\iota_{I,t}$ and $\iota_{N,t}$ the multipliers on the positivity constraint for investment:

$$I_{I,t} \geq 0, \quad I_{N,t} \geq 0.$$

The upper bound $\bar{L}$ on labor supply is taken to be so large that it never binds and this subsection ignores the Lagrange multiplier (the upper bound serves for the result of existence and uniqueness in Lemma 4). The first-order conditions of the household's program are:

$$\begin{aligned}
C_t^{-\eta} &= \mu_t \\
X_t \left(1+g\right)^t L_t^{\frac{1}{\varepsilon}} &= \mu_t w_t \\
\mu_t r_{N,t} &= \theta^{-1} \nu_{N,t-1} - \nu_{N,t} \left(1-\delta_N\right) \\
\mu_t r_{I,t} &= \theta^{-1} \nu_{I,t-1} - \nu_{I,t} \left(1-\delta_I\right) \\
\nu_{N,t} &= \mu_t 1 - \iota_{N,t} \\
\nu_{I,t} &= \mu_t \exp\left(-b_t\right) - \iota_{I,t}
\end{aligned}$$

The household's subjective discount factor, inherited by the firm, is

$$D_{0,t} = \theta^t \frac{\mu_t}{\mu_0} = \theta^t \frac{C_t^{-\eta}}{C_0^{-\eta}}.$$

Alternatively, one could include bond markets in the model, with an interest rate r_{t+1} between times t and $t+1$, in which case the household would have the following Euler equation:

$$\theta\left(1+r_{t+1}\right)=\frac{\mu_t}{\mu_{t+1}}=\frac{C_t^{-\eta}}{C_{t+1}^{-\eta}},$$

and the firm would discount profits at the following interest rate

$$\prod_{s=1}^{t}\frac{1}{1+r_s}=\prod_{s=1}^{t}\left(\theta\frac{\mu_s}{\mu_{s-1}}\right)=\theta^t\frac{\mu_t}{\mu_0}=D_{0,t}.$$

The household's program has two complementarity slackness conditions:

$$I_{N,t}\iota_{N,t}=I_{I,t}\iota_{I,t}=0.$$

The first-order conditions of the firm imply:

$$\begin{aligned}
MPL_{N,t}=\beta A_t K_{N,t}^{\alpha}L_{N,t}^{\beta-1}\left(K_{I,t}^{\rho}+L_{R,t}^{\rho}\right)^{\gamma/\rho}&=w_t+\frac{D_{0,t-1}}{D_{0,t}}\psi_{N,t-1}-\psi_{N,t}\\
MPL_{R,t}=\gamma A_t K_{N,t}^{\alpha}L_{R,t}^{\rho-1}L_{N,t}^{\beta}\left(K_{I,t}^{\rho}+L_{R,t}^{\rho}\right)^{\frac{\gamma}{\rho}-1}&=w_t\\
MPK_{N,t}=\alpha A_t K_{N,t}^{\alpha-1}L_{N,t}^{\beta}\left(K_{I,t}^{\rho}+L_{R,t}^{\rho}\right)^{\gamma/\rho}&=r_{N,t}\\
MPK_{I,t}=\gamma A_t K_{I,t}^{\rho-1}K_{N,t}^{\alpha}L_{N,t}^{\beta}\left(K_{I,t}^{\rho}+L_{R,t}^{\rho}\right)^{\frac{\gamma}{\rho}-1}&=r_{I,t}\\
\vartheta_{N,t}&=p_t c_N-\psi_{N,t}
\end{aligned}$$

where MPF is the marginal product of factor F. The firm makes zero marginal profits, since it equates marginal cost to marginal profit, but it may make positive or negative total profits, reverted to or financed by the household.

The firm's program has one complementarity slackness condition:

$$\vartheta_{N,t}H_{N,t}=0.$$

In addition, there are the physical constraints of the model (equations 2.3-2.8) and the following transversality conditions:

$$\lim_{t\to\infty}D_{0,t}K_{N,t}=\lim_{t\to\infty}D_{0,t}K_{I,t}=0.$$

Proof of Lemma 4. Given that this model has no market failures, the market equilibrium coincides with the optimum of a benevolent social planner who maximizes the household's utility:

$$\max \quad \sum_{t=0}^{\infty} \theta^t \left(\frac{C_t^{1-\eta} - 1}{1-\eta} - (1+g)^t X_t \frac{\varepsilon}{1+\varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right),$$

subject to the physical constraints in equations (2.3-2.2), (2.5-2.8), and to the following resource constraint (implied by the definition of profits, the budget constraint, and the labor market equilibrium):

$$Y_t = C_t + I_{N,t} + \exp(-b_t) I_{I,t} + c_N H_{N,t}.$$

The Bellman formulation for the planner's problem uses four state variables and seven control variables:

$$V(K_{N,t}, K_{I,t}, L_{N,t}, t) = \max_{C_t, F_{N,t} H_{N,t}, I_{I,t}, I_{N,t}, L_{I,t}, L_t} \left\{ \frac{C_t^{1-\eta} - 1}{1-\eta} - X_t (1+g)^t \frac{\varepsilon}{1+\varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right.$$
$$\left. + \theta V(K_{N,t+1}, K_{I,t+1}, L_{N,t+1}, t+1) \right\}$$

subject to the same physical constraints.

There are three Blackwell conditions for the Bellman operator to be a contraction mapping. First, the set of controls is bounded: labor variables are bounded by maximum employment $\bar{L}$, and quantity variables are bounded by maximum production $\bar{Y}_t$:

$$\bar{Y}_t = A_t K_{N,t}^{\alpha} L_{N,t}^{\beta} \left(K_{I,t}^{\rho} + \left(\bar{L} - L_{N,t} \right)^{\rho} \right)^{\gamma/\rho}.$$

Thus, both the disutility from labor supply and the utility from consumption are bounded. Therefore, the Bellman operator in maps the space of bounded functions into itself.

Second, monotonicity holds. Denote z the set of control variables, ω the set of state variables, $\omega' = \tau(\omega, z)$ the transition rule, and T the Bellman operator:

$$(TV)(\omega) = \max_{z} \left(U(\omega, z) + \theta V(\tau(\omega, z)) \right),$$

where $U(\omega, z)$ is the utility from consumption net of the disutility from labor supply. Denote V and W two value functions with $V(\omega) \leq W(\omega)$. Then:

$$\begin{aligned}
V(\tau(\omega, z)) &\leq W(\tau(\omega, z)), \\
U(\omega, z) + \theta V(\tau(\omega, z)) &\leq U(\omega, z) + \theta W(\tau(\omega, z)), \\
U(\omega, z) + \theta V(\tau(\omega, z)) &\leq \max_{z'} \left\{ U(\omega, z') + \theta W(\tau(\omega, z')) \right\} \quad \forall z, \\
\max_{z} U(\omega, z) + \theta V(\tau(\omega, z)) &\leq \max_{z'} \left\{ U(\omega, z') + \theta W(\tau(\omega, z')) \right\}, \\
(TV)(\omega) &\leq (TW)(\omega).
\end{aligned}$$

Third, discounting holds:

$$\begin{aligned}
\left[T(V + a) \right](\omega) &= \max_{z} \left\{ U(\omega, z) + \theta \left[V + a \right](\tau(\omega, z)) \right\}, \\
&= \max_{z} \left\{ U(\omega, z) + \theta V(\tau(\omega, z)) + \theta a \right\}, \\
&= \max_{z} \left\{ U(\omega, z) + \theta V(\tau(\omega, z)) \right\} + \theta a, \\
&\leq (TV)(\omega) + \theta a.
\end{aligned}$$

Thus, the three Blackwell conditions hold and the Bellman operator is a contraction mapping. The contraction mapping theorem guarantees existence and uniqueness of the equilibrium of the model (see Stokey and Lucas, 1989, page 54). It also guarantees stability of the equilibrium.

Proof of Lemma 5. In the limiting balanced growth path, where consumption grows at rate g_C, the equilibrium rental rates of capital are constant:

$$\begin{aligned}
r_N &= \theta^{-1} (1 + g_C)^{\eta} - 1 + \delta_N, \\
r_I &\to \theta^{-1} \exp\left(-\bar{b}\right) (1 + g_C)^{-\eta} - 1 + \delta_I.
\end{aligned}$$

The firm's limiting subjective one-period discount factor also converges:

$$\frac{D_{0,t+1}}{D_{0,t}} = \theta \left(\frac{C_{t+1}}{C_t}\right)^{-\eta} \rightarrow \theta \left(1 + g_C\right)^{-\eta}.$$

The factor price frontier, implied by the firm's first-order conditions, is:

$$\alpha^\alpha \beta^\beta \gamma^\gamma A_t = r_{N,t}^\alpha \left(w_t + \frac{D_{0,t-1}}{D_{0,t}} \psi_{N,t-1} - \psi_{N,t}\right)^\beta \left(r_{I,t}^{1-\sigma} + w_t^{1-\sigma}\right)^{\frac{\gamma}{1-\sigma}}.$$

The left-hand side of the factor price frontier diverges. On the right-hand side, the two rental rates of capital converge, the one-period discount factor also converges, and the multipliers $\psi_{N,t-1}$ and $\psi_{N,t}$ are bounded between 0 and c_N. All terms on the right-hand side converge or are bounded, except for wages w_t. Therefore, wages also diverge and grow indefinitely at a rate implied by the limiting factor price frontier:

$$\alpha^\alpha \beta^\beta \gamma^\gamma A_t = r_N^\alpha w_t^\beta \underbrace{\left(1 + \frac{D_{0,t-1}}{D_{0,t}} \frac{\psi_{N,t-1}}{w_t} - \frac{\psi_{N,t}}{w_t}\right)^\beta}_{\rightarrow 1} \underbrace{\left(r_{I,t}^{1-\sigma} + w_t^{1-\sigma}\right)^{\frac{\gamma}{1-\sigma}}}_{\rightarrow r_I^\gamma} \quad \Rightarrow \quad g_w = \frac{g_A}{\beta}.$$

Given the behavior of factor prices (constant rental rates of capital and unbounded wages), it is now possible to show that the limiting capital-output ratios are constant:

$$\begin{aligned}
\frac{K_{N,t}}{Y_t} &= \frac{\alpha}{r_{N,t}}, \\
\frac{K_{I,t}}{Y_t} &= \frac{\gamma}{r_{I,t}} \left(1 + \left(\frac{w_t}{r_{I,t}}\right)^{1-\sigma}\right)^{-1} \rightarrow \frac{\gamma}{r_I}.
\end{aligned}$$

The labor supply equation from the household, in the case of no labor wedge shocks with $X_t = X$, is

$$X \left(1 + g_D\right)^t L_t^{\frac{1}{\varepsilon}} = C_t^{-\eta} w_t,$$

where $g_D = g$ denotes the growth rate of the disutility of labor supply to avoid confusion. This equation implies the following growth rate for employment:

$$g_L = \varepsilon \left(g_w - \eta g_C - \log\left(1 + g_D\right)\right).$$

As wages grow indefinitely, the relative cost of IT capital decreases to zero and employment reallocates entirely from routine employment to non-routine employment:

$$L_{N,t} \to L, \qquad L_{R,t} \to 0.$$

The limiting production function is a three-factor Cobb-Douglas:

$$Y_t \to A_t K_{N,t}^{\alpha} L_{N,t}^{\beta} K_{I,t}^{\gamma},$$

and it implies the following equation between limiting growth rates:

$$g_Y = g_A + \alpha g_{K_N} + \beta g_L + \gamma g_{K_I}.$$

Using the constant capital-output ratios and the limiting growth rate of employment, the growth rate of output is:

$$g_Y = \frac{1+\varepsilon}{\beta\left(1+\eta\varepsilon\right)} g_A - \frac{\varepsilon}{1+\varepsilon\eta} \log\left(1+g_D\right)$$

At the limit, the investment-capital ratios are constant:

$$\frac{I_{J,t}}{K_{J,t}} = \frac{K_{J,t+1}}{K_{J,t}} - \left(1-\delta_J\right) \to g_{K_J} + \delta_J = g_Y + \delta_J, \qquad J = I, N.$$

Therefore, the investment-output ratios are also constant and the two types of investment grow at rate $g_Y = g_A/\beta$. The resource constraint implies that consumption tends to a constant share of output:

$$\begin{aligned}
\frac{C_t}{Y_t} &= 1 - \frac{I_{N,t}}{Y_t} - \exp\left(-b_t\right) \frac{I_{I,t}}{Y_t} \\
&= 1 - \frac{I_{N,t}}{K_{N,t}} \frac{K_{N,t}}{Y_t} - \exp\left(-b_t\right) \frac{I_{I,t}}{K_{I,t}} \frac{K_{I,t}}{Y_t} \\
&\to 1 - \left(\delta_N + \frac{g_A}{\beta}\right) \frac{\alpha}{r_N} - \exp\left(-\bar{b}\right) \left(\delta_I + \frac{g_A}{\beta}\right) \frac{\gamma}{r_I}.
\end{aligned}$$

Therefore, consumption grows at the same rate as output:

$$g_C = g_Y.$$

Finally, to find the growth rate of the disutility of labor supply that ensures constant employment at the limit, write:

$$0 = g_L = \varepsilon \left(g_w - \eta g_C - \log\left(1 + g_D\right)\right).$$

After some algebra, this equation implies:

$$\log\left(1 + g_D\right) = \frac{1-\eta}{\beta} g_A,$$

and, after some more algebra, the growth rates of consumption, output, the two types of capital stock and investment are the same as wages:

$$g_C = g_Y = g_{K_I} = g_{K_N} = g_{I_I} = g_{I_N} = g_w = \frac{g_A}{\beta}.$$

A.2 Proofs in the special case of the model

Proof of Lemma 6. This proof omits the time index t. For the equilibrium condition, note that the resource constraint on the product market and the household's budget constraint imply zero profits for the firm. Denoting Π_j the indexed product operator (different from the logarithm π of labor productivity), consider a multifactor Cobb-Douglas production function, $Y = A \prod_j F_j^{\alpha_j}$, with constant returns to scale $\sum \alpha_j = 1$. Denote the marginal cost of each factor F_j with mc_j. Optimization of this production function implies constant factor shares:

$$F_j = \frac{\alpha_j Y}{mc_j}.$$

Raising to the power α_j and multiplying over j yields:

$$\frac{Y}{A} = \prod_j F_j^{\alpha_j} = \prod_j \left(\frac{\alpha_j}{mc_j}\right)^{\alpha_j} \times Y^{\sum \alpha_j},$$

which, in turn, implies:

$$\frac{1}{A} \prod_j \left(\frac{mc_j}{\alpha_j}\right)^{\alpha_j} = 1.$$

In the production function, the marginal cost of the first two factors, K_N and L_N, is 1 and w. The marginal cost of the third Cobb-Douglas factor, the CES aggregate, requires more detail.

Consider a firm that is selling the third Cobb-Douglas factor at marginal cost mc_I to maximize profits:

$$\max_{K_I, L_R} \quad mc_I \left(K_I^\rho + L_R^\rho\right)^{\frac{1}{\rho}} - r_I\, K_I - w\, L_R.$$

The ratio of first-order conditions on capital K_I and labor L_R imply:

$$\left(\frac{K_I}{L_R}\right)^{\rho-1} = \frac{r_I}{w} \quad \Rightarrow \quad \frac{K_I}{L_R} = \left(\frac{w}{r_I}\right)^{\sigma}$$

The first-order condition for labor implies:

$$mc_I \left(K_I^\rho + L_R^\rho\right)^{\frac{1}{\rho}-1} L_R^{\rho-1} = w.$$

Rearrange this expression, use $\sigma\rho = \sigma - 1$, and use the above capital-labor ratio to obtain:

$$\begin{aligned}
mc_I &= \left(1 + \left(\frac{K_I}{L_R}\right)^{\rho}\right)^{1-\frac{1}{\rho}} w \\
&= \left(1 + \left(\frac{w}{r_I}\right)^{\rho\sigma}\right)^{\frac{1}{1-\sigma}} w \\
&= \left(1 + w^{\sigma-1} r_I^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \left(w^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
mc_I &= \left(r_I^{1-\sigma} + w^{1-\sigma}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

With the marginal cost of the CES aggregate factor, the zero profit condition of the three factor Cobb-Douglas production function is

$$\frac{1}{A}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{w}{\beta}\right)^{\beta}\left(\frac{\left(r_I^{1-\sigma}+w^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{\gamma}\right)^{\gamma}=1.$$

This equation is the equilibrium condition for the wage, where the marginal cost of production equals the marginal revenue. The left-hand side is strictly increasing in w, equals 0 for $w=0$ and tends to infinity for $w\to\infty$. Therefore, there is a unique wage that verifies the equation.

Proof of Proposition 7. labor productivity π_t is independent of the logarithm of the labor wedge shock $x_t=\log X_t$:

$$\pi_t=\log\frac{w_t}{\beta+\gamma\left(1+r_{I,t}^{1-\sigma}w_t^{\sigma-1}\right)^{-1}}. \tag{A.1}$$

The second derivative of the logarithm of labor productivity π_t with respect to b_t is:

$$\begin{aligned}\frac{\partial^2\pi_t}{\partial b_t^2}=&\left(\sigma-1\right)\alpha^{\frac{2\alpha(\sigma-1)}{\gamma}}\beta^{\frac{2\beta(\sigma-1)}{\gamma}}\gamma^{2\sigma-1}\left(\beta+\gamma\right)^2 r_{I,t}^{1-\sigma}w_t^{\sigma-1}\\ &\times\left(1+r_{I,t}^{1-\sigma}w_t^{\sigma-1}\right)\left(\alpha^{\frac{\alpha(\sigma-1)}{\gamma}}\beta^{\frac{\beta(\sigma-1)}{\gamma}}\gamma^{\sigma}+\beta w_t^{\frac{(\sigma-1)(\beta+\gamma)}{\gamma}}\right)^2\\ &\times\left(\beta\left(2-\sigma\right)r_{I,t}^{1-\sigma}w_t^{\sigma-1}+\sigma\left(\beta+\gamma\right)\right)\\ &/\left(\left(\beta+\gamma\right)+\beta r_{I,t}^{1-\sigma}w_t^{\sigma-1}\right)^2,\end{aligned}$$

where the rental cost of IT capital is $r_{I,t}=\exp\left(-b_t\right)$. Since $\sigma\in(1,2)$, all the terms in this expression are strictly positive.

Proof of Corollary 8. Take the limit $\sigma\to1$ in the second derivative of labor productivity in the proof of Proposition 7.

Proof of Proposition 9. Denote log-employment with $l\left(t\right)$, log-output with $y\left(t\right)$, and the logarithm of labor productivity with $\pi\left(t\right)$:

$$y\left(t\right)=\pi\left(t\right)+l\left(t\right).$$

A linear approximation of employment growth around its trough t_l yields:

$$\dot{l}(t) = \underbrace{\dot{l}(t_l)}_{=0} + (t - t_l)\,\ddot{l}(t_l) + o(t - t_l).$$

Writing output growth as productivity growth plus employment growth, and using the linear approximation above:

$$\dot{y}(t) = \dot{\pi}(t) + \dot{l}(t) = \dot{\pi}(t) + \ddot{l}(t_l)(t - t_l) + o(t - t_l).$$

The trough of output, denoted t_y, verifies:

$$\dot{y}(t_y) = 0 = \dot{\pi}(t_y) + \ddot{l}(t_l)(t_y - t_l) + o(t_y - t_l) \quad \Leftrightarrow \quad t_l - t_y = \frac{\dot{\pi}(t_y)}{\ddot{l}(t_l)} + o(t_l - t_y).$$

In the model with $\eta = 1$, the dynamics of labor are:

$$\log L_t = \frac{\varepsilon}{1+\varepsilon} \log X_t.$$

Since X_t is periodic with a single trough, $\ddot{l}(t_l)$ is also periodic and has the same value at all troughs, denoted $\ddot{l}_{trough}$. Since labor has a trough, $\ddot{l}_{trough} > 0$. To a first-order approximation, the length of the jobless recovery is proportional to productivity growth $\dot{\pi}$:

$$t_l - t_y \approx \frac{\dot{\pi}(t_y)}{\ddot{l}_{trough}}.$$

Note that this proposition assumes that the dynamics of the labor market are periodic. Alternatively, consider a model where productivity growth has a speedup and no fluctuations, and output has only fluctuations and no trend. In that case, jobless recoveries would also last longer to a first-order approximation:

$$t_l - t_y \approx \frac{\dot{\pi}(t_l)}{\ddot{y}_{trough}}.$$

The meaning of the first-order approximation is that $t_y - t_l$ be small compared to the period of the business cycle in X_t. In economic terms, it assumes that the length of the jobless recovery is a fraction of the length of the business cycle (for example, the duration peak-to-peak). Mathematically, the Taylor series expansion of employment growth with the Lagrange form of the remainder is:

$$\dot{l}(t_y) = (t_y - t_l)\,\ddot{l}(t_l) + \frac{1}{2}(t_y - t_l)^2\,\dddot{l}(t_0)\,, \quad t_0 \in (t_y, t_l)\,.$$

If $t_y - t_l$ is small, in the sense that $\dddot{l}(t_0)$ is changes little in the interval (t_y, t_l) compared to its variations over the business cycle, then $\ddot{l}(t_0)$ is close to $\ddot{l}(t_l)$ and the first-order approximation is valid. Figure 14 contains a graphical version of this interpretation.

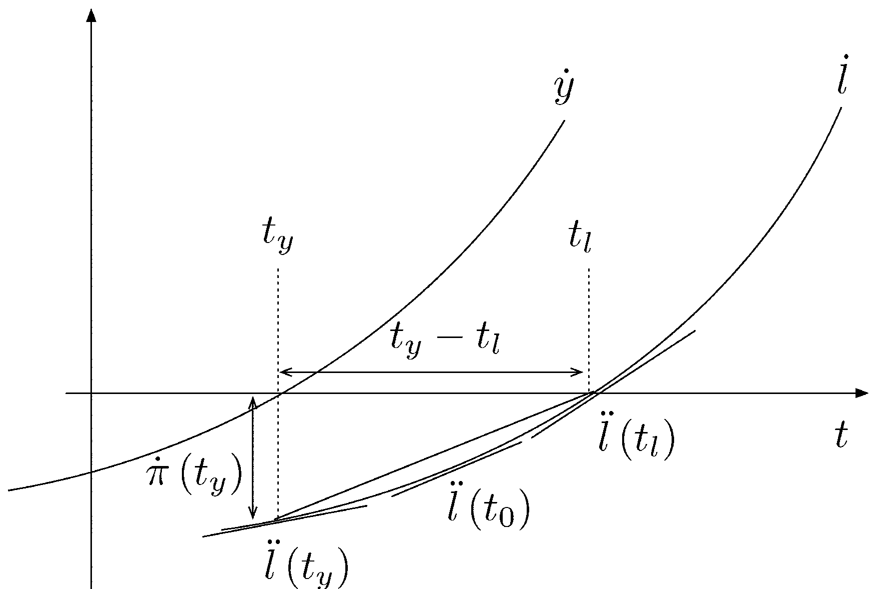

Figure 14: Diagram with the relation between the length of jobless recoveries and productivity growth.

A.3 Cyclicality of investment

This paper explains jobless recoveries with computers. Interestingly, it does not predict an acceleration of IT investment around recessions. Since there are no adjustment costs to IT investment, IT is free to adjust, so it falls in recessions and increases in recoveries. Figure 15 shows the behavior of IT investment in the data and in the model. The model matches the behavior of IT investment: after a recession, IT investment simply catches up with its long-term trend, rather than accelerating or increasing to a permanently higher level.

Another prediction is that the investment share of output is procyclical and decreases in recessions, for two reasons. First, the absence of adjustment costs to capital implies that it is free to adjust to the recession. Second, the household has an incentive to smooth consumption but no incentive to smooth investment, so a recession affects primarily investment. Figure 16 shows that the calibration of the model in the general case also matches the cyclicality of the investment share of output.

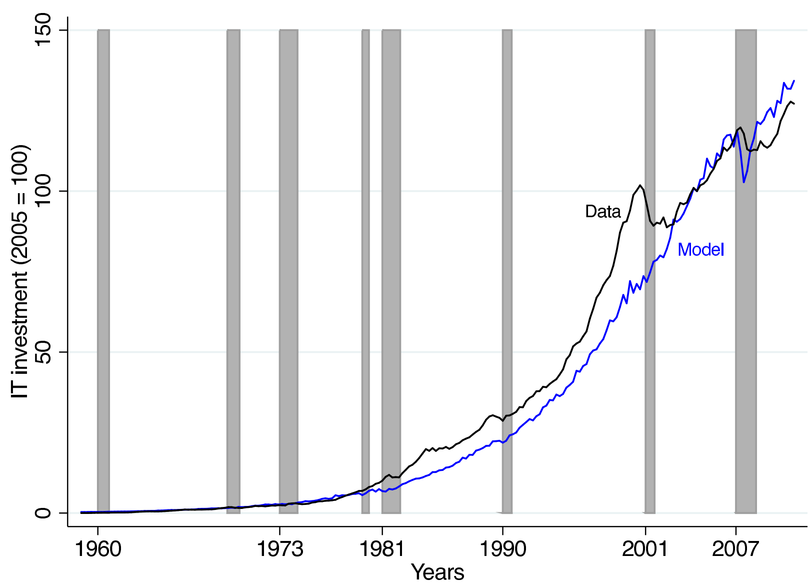

Figure 15: After recessions, IT investment returns back to trend, both in the data and in the model.

Source: Bureau of Economic Analysis, Private Fixed Investment in Equipment and Software by Type (Table 5.5.5U) and model with a smooth cost of computers to display a smooth series for IT investment (with $\phi = 14\%$).

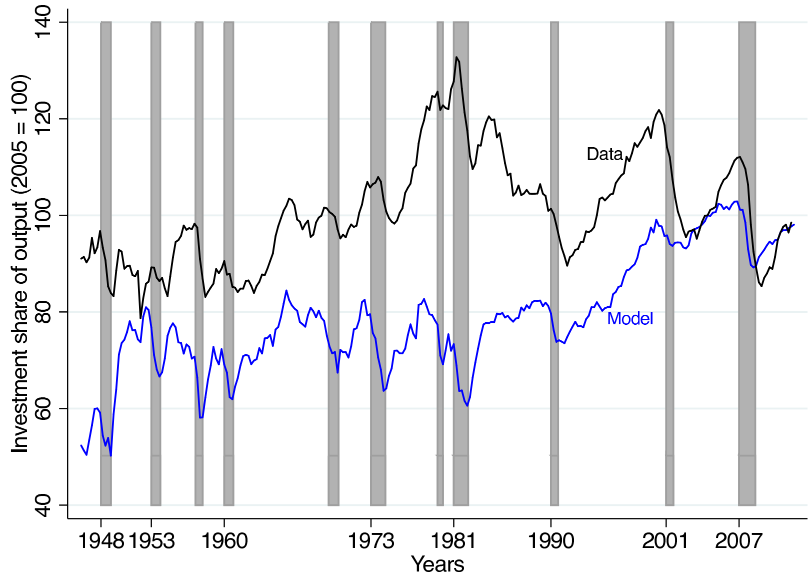

Figure 16: The investment share of output is procyclical, both in the data and in the model.

Source: Federal Reserve Economic Database, Private Nonresidential Fixed Investment divided by Gross Domestic Product (both in nominal terms) and model with the cost of computers from the BEA.

A.4 Occupational matching

The evidence on routinization draws from two data sources: the March Current Population Survey (CPS), provided by the Public Use Micro-Samples (PUMS), and the Occupational Information Network (ONET).

From the CPS, a person is defined as employed if she was at work during the week before the interview week, or if she was not at work but had a job. The aggregation across individuals uses the March supplement weight. There is a Standard Occupational Classifications (SOC) for each decade, and the CPS upgraded to the latest classification in 1971, 1983, 1992, 2003, and 2011. These classifications are not directly comparable across decades. PUMS provide two codes for occupations: OCC, which contains the original coding by the Census Bureau, and OCC1990, an attempt by PUMS at an occupational classification consistent across decades. The OCC1990 variable is only broadly consistent across decades: in 1983, there are 3 million "retail sales clerks" who become "sales persons, n.e.c." and reverse to "retail sales clerks" in 2003. The assessment of occupational changes in the short-term segments the analysis by decade.

In ONET, the occupational codes use the SOC 2010 classification. The three measures relevant for routinization are the level of automation, the importance of assisting and caring for others, and the level of creativity. The latter two concepts, caring for others and creativity, have two measures, "level" and "importance" which are almost perfectly correlated, so a missing observation of one measure is imputed with the other.

The analysis of the 2003-2010 decade uses two crosswalks: a first crosswalk between the PUMS coding of OCC and the standard code in SOC 2010 (crosswalk directly provided by PUMS), and a second crosswalk between SOC 2000 and SOC 2010 in ONET. Occupations with no routinization measure use the average of the 5-digit level above, and, if it is still missing, the average of the 4-digit level above. This procedure assigns a measure of routinization to 90 percent of employed workers in 2007. The aggregation of employment by occupation from the CPS produces a panel of occupational employment during the 2003-2010 decade. These occupations are then allocated into quartiles of routinization using their occupational measures from ONET and their employment levels in 2007.

The analysis of previous decades uses a crosswalk between OCC and OCC1990. An aggregation of the panel of occupations during 2003-2010 at the OCC1990 level produces a measure of routinization at the coarser OCC1990 level. A comparison of routinization measures between the OCC level and the OCC1990 level allows the exclusion of outlier occupations, where the difference between the routinization index of OCC and of OC1990 is greater than 2 (having standardized the three original measures of automation, personal interactions, and creativity to be mean zero and standard deviation 1). These outliers are mostly due to re-classifications into "not elsewhere classified." This approach produces a measure of routinization at the OCC1990 level that is consistent with the more precise measure at the OCC level. An aggregation of the CPS at the OCC1990 level produces a panel of occupations for each decade (1973-1982, 1983-1992, and 1993-2002), then allocated into quartiles of routinization.