Cyclicality and Firm-size in Private Firm Defaults

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Abstract

The Basel II and III Accords treat loans to small firms as less sensitive to macroeconomic cyclicality than loans to larger firms. We investigate, in an intensity regression framework, whether this is appropriate using a sample of private firms. In our models, we find that accounting ratios are important mainly for ranking firms, and we identify the macro variables which are important for default prediction over time. We find little support—both in a Cox regression setting and using an additive intensity model—for the hypothesis that smaller firms are less sensitive to macroeconomic cyclicality.

Keywords: Default prediction · Private firms · Basel II and III

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1 Introduction

Small and medium-sized enterprises (SMEs) typically depend more heavily on funding from banks than do larger firms. It is therefore conceivable that SMEs are hit harder during a financial crisis in which banks' capital constraints are binding. Perhaps as a recognition of this dependence, the Basel II Accord awards preferential treatment of bank loans to SMEs, effectively and significantly lowering capital charges for lending to the SME-segment. Technically, the reduction follows by prescribing lower asset correlation with a common risk-driver when calculating capital charges. To the extent that asset correlation arises because of common dependence on macroeconomic shocks, the reduction corresponds to assuming a smaller sensitivity of SMEs to macroeconomic fluctuations or cyclicality. These deductions in capital charges were recently reaffirmed and extended in the Basel III Accord and in the fourth Capital Requirements Directive (CRD IV). In this paper we investigate, in an intensity regression framework, whether there is empirical support for the assumption of less sensitivity to macroeconomic cyclicality for smaller firms.

Our data is a sample of private firms obtained from a large Danish financial institution. We use both a Cox regression framework and an additive intensity specification, to be explained below. Using our estimated Cox model, we find that solely discriminating with respect to firm size, and keeping all other firm characteristics equal, default intensities for smaller firms do in fact exhibit less sensitivity to macro variables, in the sense that the coefficients on the macro variables are of smaller magnitude for smaller firms compared to larger firms. However, when we account for the Cox model's non-linearity, and use averaging techniques adapted from other non-linear regression models, the results indicate that smaller firms may "on average" be as cyclical, or perhaps even more cyclical, than larger firms. Because of this ambiguity in the Cox specification, we also conduct the investigation using an additive specification of our model for default intensities. Due to the linearity of its effects, an additive model allows us to directly compare the coefficients of macro variables for small and large firms. In this setting, we find no evidence that there is different sensitivity to macro variables for smaller firms compared to larger firms.

To ensure to the validity and robustness of our results, our regression specification also includes accounting ratios that control for characteristics other than size. This is to account for variations in firm-specific default risk not related to size in our tests of macro-sensitivity: By including firm-specific controls, we rule out the possibility that our division into large and small firms is merely capturing other characteristics that differentiate the two types of firms. Furthermore, our regression results indicate that accounting ratios and macro variables play distinct roles in default prediction for private firms: Accounting ratios are necessary for credit scoring, in that they help us rank firms according to their default likelihood, but cannot by themselves capture the cyclicality of default rates over times—on the other hand, macro variables are indispensable for accurate prediction of portfolio credit risk over time, but do not aid in the ranking of firms with respect to default likelihood. These results indicate that our method of focusing on the coefficients of the macro variables for small vs. large firms is adequate, as it is the macro variables that add the cyclicality component to our prediction models.

The flow of the rest of the paper is as follows. Following this introduction is a brief review of the most related literature. Section 2 details our data, variable selection, and estimation methodology. Section 3 presents our regression results, where we show that accounting ratios and macro variables play distinct roles in default prediction for private firms. Section 4 presents our tests for the macro-sensitivity of small vs. large firms, both in a Cox regression and in an additive intensity setup. Section 5 shows some results related to robustness and model check. Section 6 concludes.

1.1 Related literature

Statistical models using accounting ratios to estimate default probabilities date back to at least Beaver (1966) and Altman (1968), followed by Ohlsen (1980) and Zmijewski (1984)—we use many of the same accounting ratios as in these studies. Shumway (2001) was among the first to demonstrate the advantages of intensity models with time-varying covariates compared to traditional discriminant analysis, and was also among the first to include equity return as a market-based predictor of default probabilities—we use a similar estimation setup, although we do not have market-based variables for our private firms. Chava and Jarrow (2004) improved the setup of Shumway (2001) using covariates measured at the monthly level and showed the importance of industry effects—our data frequency is also at the monthly level and we correct for industry effects in all our regressions.

Structural models of credit risk, like the models of Black and Scholes (1973), Merton (1974), and Leland (1994), usually assume that a firm defaults when its assets drop to a sufficiently low level relative to its liabilities. The connection between structural models and intensity models was formally established by Duffie and Lando (2001), who showed that when the firm's asset value process is not perfectly observable, a firm's default time has a default intensity that depends on the firm's observable characteristics as well as other covariates. Studies demonstrating the importance of covariates implied from structural models, like distance-to-default or asset volatility, include Duffie, Saita, and Wang (2007), Bharath and Shumway (2008), Lando and Nielsen (2010), and Chava, Stefanescu, and Turnbull (2011) among many others.

Default studies using data on public firms and employing macroeconomic covariates include McDonald and de Gucht (1999), Peseran, Schuermann, Treutler, and Weiner (2006), Duffie et al. (2007), Lando and Nielsen (2010), Figlewski, Frydman, and Liang (2012), among many others. Recent default studies of private firms that also employ macroeconomic covariates include Carling, Jacobson, Lindé, and Roszbach (2007), who use Swedish data, and Bonfim (2009), who use Portuguese data. We employ many of the same macro variables as in these studies.

The provisions in the Basel II Accord permitting *"[banks] to separately distinguish exposures to SME borrowers (defined as corporate exposures where the reported sales for the consolidated group of which the firm is a part is less than 50 million euro) from those to large firms"* and specifying the reduction for SMEs through the asset correlation formula can be found in Articles 273 and 274 of The Basel Committee on Banking Supervision's (2006) report. Lopez (2004) examines the relationship between Basel II's assumed asset correlation and firm size, and finds an increasing relationship between the two using a sample of US, Japanese, and European firms. Using data on French and German firms, Dietsch and Petey (2004) find that SMEs are riskier than larger firms, and that the asset correlations for SMEs are weak and decrease with firm size. Chionsini, Marcucci, and Quagliariello (2010) find evidence in support of the the size-sensitive treatment in the Basel II Accord for italian SMEs, through not during severe financial crises like that of 2008-09.

The corresponding provisions in the Basel III Accord and the recently adopted CRD IV can be found in Articles 153.4 and 501.1 of The European Parliament and the Council of the European Union's (2013) report. Discussions of the treatment of SMEs in the Basel III Accord and CRD IV prior to the adoption can, for instance, be found in reports by The Association of Chartered Certified Accountants (ACCA) (2011a,b) and The European Banking Authority (EBA) (2012)—see also the references therein.

In our analysis of the cyclicality of small vs. large firms using the Cox model, we apply averaging techniques resembling the ones discussed for generalized linear models by, for instance, Wooldridge (2009, p. 582-83). The additive intensity regression model, which we also apply in the cyclicality analysis, was first used by Lando, Medhat, Nielsen, and Nielsen (2013) to study the time-variation of regression coefficients in default prediction for public US firms.

2 Data and methodology

Our raw data comprises 28,395 firms and 114,409 firm-year observations over the period 2003 to 2012, obtained from a large Danish financial institution. A firm is

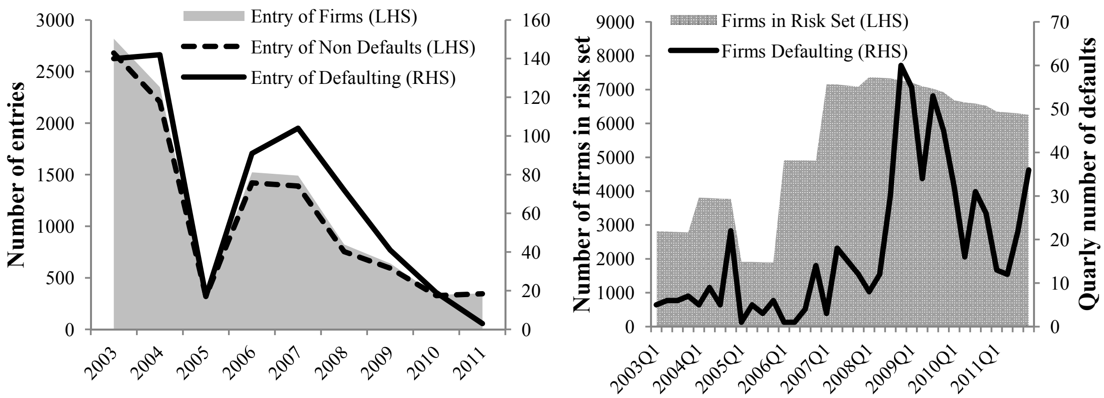

Figure 1. ***Entry and at-risk pattern in the sample.*** *The left panel shows the yearly number of firms entering the sample (grey mass) along with the yearly number of entries that do not default (black, solid line) or eventually do default (black, dashed line). The right panel shows the quarterly number of firms at risk of defaulting (i.e. in the "risk set"; grey mass) along with the actual number of defaulting firms in each quarter (black, solid line).*

included in this dataset if, in at least one of the years underlying the period of analysis, it has an engagement over DKK 2 million, which is the largest segmentation category used by the financial institution for its corporate clients. An engagement is defined in terms of loans or granted credit lines. After removing sole proprietorships, government institutions, holding companies without consolidated financial statements, firms that do not have Denmark as their residency, and firms that do not fulfill balance sheet checks, we are left with 10,671 firms and 48,703 firm-year observations. In the cleaned dataset, a total of 633 firms experienced a default event, defined by the Basel II Accord as more than 60 days delinquency. Moreover, 54 of the 633 defaulting firms experience a second default, in the sense that they became delinquent a second time during the sample period. Other default studies have treated a firm that re-emerges from default as a new firm, as merited by bankruptcy protection laws. However, due to the Basel II Accord's definition of a default event as a period of delinquency, we choose to disregard multiple default events, and our results should hence be interpreted as specific to a firm's initial default.

Figure 1 shows the patterns with which firms enter and potentially leave our final sample. The right panel shows the number of firms that enter the sample at each year along with an indication of the number of entries eventually corresponding to defaults and non-defaults. Despite discussions with the financial institution providing the data, the low number of firms entering the sample in 2005 remains a conundrum. It appears, however, that the firms that eventually default do not seem to differ systematically from the non-defaulting firms based on when they enter the sample. The right panel shows the number of firms at risk of defaulting, i.e. firms in the "risk set," at each quarter, along with the quarterly number of defaults. The risk set is seen to contain at least 2,000 firms at each quarter, and the 2008-09 financial crisis is readily visible from the sharp rise in the number of defaults.

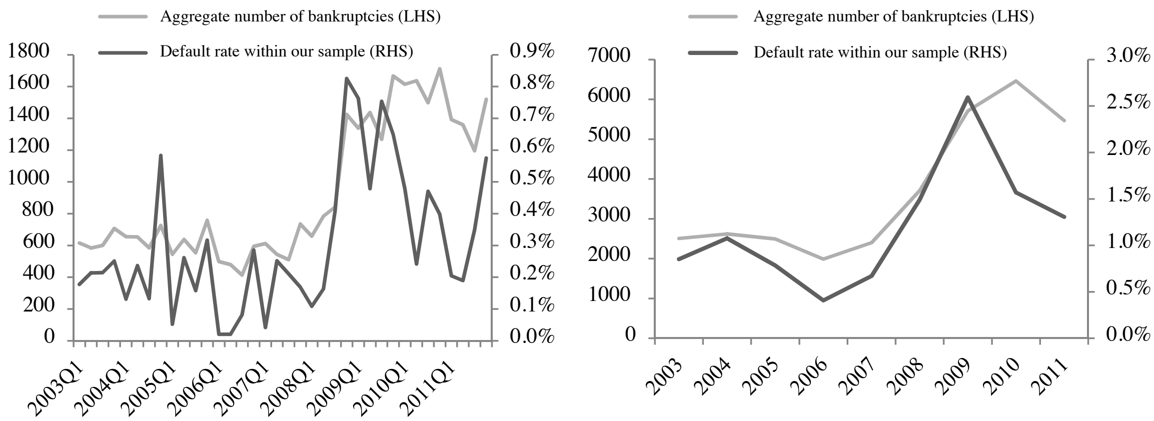

Figure 2. ***Default rates in the sample and the general Danish economy.*** *The left panel shows the quarterly default rate from the sample along with the corresponding aggregate number of quarterly bankruptcies in the general Danish economy. The right panel shows the yearly default rate from the sample along with the aggregate quarterly number of default in the general Danish economy.*

In order to incorporate quarterly macroeconomic factors, we re-code the financial numbers for each firm from annual to quarterly observations. This will naturally induce persistence in observations of accounting ratios from quarter to quarter, which we take into account by basing all inference on standard errors clustered at the firm-level. The resulting dataset is, as a result, enlarged to a total of 192,196 firm-quarter observations.

Figure 2 compares the observed default rate in our sample to the number of registered bankruptcies in Denmark. The comparison is feasible because the total number of firms at risk of default in Denmark is relatively stable over time. We see that, due to the relatively few incidences, the default rate in the sample fluctuates while still co-moving with the aggregate level in Denmark. This indicates that our results are not necessarily specific to the financial institution that provided us with data, but may be applied to Danish firms in general.

2.1 Internal covariates

Table 1 provides an overview of the internal (i.e. firm-specific) explanatory variables employed as covariates in our analysis. We use accounting ratios from each firm's balance sheet to measure firm size, age, leverage, profitability, asset liquidity, collateralization, and equity-value, and the table also gives the expected sign of each variable's effect on the probability of default. All our accounting ratios have been applied in previous default studies—see, for instance, Ohlsen (1980), Shumway (2001), Duffie et al. (2007), and Lando and Nielsen (2010). We also correct for industry effects as in Chava and Jarrow (2004). The main difference between our list of covariates and the covariates used in default studies of public firms is the lack of market-based measures like stock return and distance-to-default.

Table 1. ***Internal explanatory variables and corresponding accounting ratios.*** *The left column shows our list of internal explanatory variables, the center column shows the accounting ratios used to measure each internal explanatory variable, and the right column shows the expected effect of each accounting ratio on default probabilities. "Industry Effects" are included in the list for completeness, although this is only a control variable (see details in the text).*

Explanatory Variable	Investigated Measurements	Expected effect on probability of default
Size	Log of book value of assets	Negative
Age	Years active in the bank	Negative
Leverage	Short term debt to total assets	Positive
	Total debt to total assets	Positive
	Interest bearing debt to total assets	Positive
	Interest payments to total assets	Positive
Profitability	Net income to total assets	Negative
	EBIT to total assets	Negative
	EBITDA to total assets	Negative
Liquidity	Current ratio	Negative
	Quick ratio	Negative
Collateralization	Fixed assets to total assets	Negative
	PPE to total assets	Negative
Negative equity	Dummy for negative equity	Positive
Industry Effects	DB07 Sector affiliation	Control variable

In order to avoid discriminating against smaller companies that report financial statements in less detail, we use high-level, aggregated data to construct the accounting ratios. We control for industry effects since certain industry characteristics may prescribe a certain leverage structure, particularly linked to the volatility of cash flows. We use the sector affiliation by Statistics Denmark to identify a firm's primary industry as either "Construction," "Manufacturing," or "Wholesale and Retail," as these have above average default rates, but are at the same time coarse enough to ensure a sufficient number of firms in each sector.

An analysis of the internal covariates revealed a few miscodings and extreme values. Due to the anonymized nature of the data, we were not able to check the validity of these data points manually, and we therefore choose to winsorize all the internal covariates at the 1st and 99th percentile—a practice also used by Chava and Jarrow (2004), Shumway (2001), and Bonfim (2009), among others. The winsorized summary statistics are presented in Table 2. The average firm has DKK 275 million assets, a ratio of 68% between total debt to total assets, and interest payments corresponding to 3% of total assets. Further, the average firm had a relationship with the bank for 23 years and remains in the sample for 7 out of the 9 years.

Due to Danish reporting standards, firms below a certain size may refrain from reporting revenue and employee count, and hence these variables are zero for a large proportion of firms in the sample. We therefore choose not to use these two

Table 2. ***Descriptive statistics for the internal covariates.*** *The table shows descriptive statistics for the firm-specific variables of the cleaned sample, winsorized at the* 1*st and 99th percentile. The total number of observations is 192,196 firm-quaters. Age is time since the bank recorded the first interaction with the client. Entry is the year where the firm entered the sample. Duration is the number of years the firm remains in the sample. All other variable have standard interpretations.*

Variable	Mean	Std	1%	5%	25%	50%	75%	95%	99%
Total assets (tDKK)	275.074	1.060.373	811	2.784	9.553	28.222	100.723	1.025.875	8.656.000
Revenue (tDKK)	242.031	885.431	0	0	0	0	70.925	1.097.486	6.733.409
Employees	113	348	0	0	1	18	64	484	2.666
Age (years)	23	20	1	3	9	18	30	71	97,25
Log(total assets) (tDKK)	10,46	1,81	6,70	7,93	9,16	10,25	11,52	13,84	15,97
Short term debt to total assets	0,51	0,28	0,01	0,09	0,30	0,49	0,69	0,96	1,58
Total debt to total assets	0,68	0,28	0,02	0,18	0,53	0,70	0,84	1,04	1,80
Interestbearing debt to total assets	0,39	0,28	0,00	0,00	0,17	0,37	0,56	0,87	1,38
Interest payments to total assets	0,03	0,03	0,00	0,00	0,01	0,02	0,03	0,07	0,17
Current ratio	1,66	2,42	0,03	0,28	0,88	1,17	1,59	3,80	20,21
Quick ratio	1,28	2,37	0,02	0,14	0,49	0,81	1,19	3,15	19,67
Fixed assets to total assets	0,40	0,29	0,00	0,01	0,14	0,36	0,63	0,93	0,99
Tangible Assets to total assets	0,30	0,28	0,00	0,00	0,06	0,23	0,49	0,87	0,97
Net Income to total assets	0,03	0,14	-0,68	-0,18	0,00	0,03	0,09	0,24	0,43
EBIT to total assets	0,06	0,15	-0,59	-0,16	0,00	0,05	0,12	0,29	0,51
EBITDA to total assets	0,10	0,15	-0,51	-0,12	0,02	0,09	0,17	0,34	0,54
Entry Year	2005	1,94	2003	2003	2003	2004	2006	2009	2010
Duration (Years)	7	2	1	2	5	7	9	9	9

variables in our further analysis in order not to discriminate against smaller firms. In the table, firm age is taken to be time since the bank recorded the first interaction with the client; entry year specifies the year at which the firm enters the sample; while duration is the number of years a firm is observed in the sample since its entry year.

2.2 External covariates

Our macroeconomic time-series were primarily obtained from Ecowin, with additional data from Statistics Denmark, OECD, as well as Stoxx. An overview of the macroeconomic variables employed in our analysis is presented in Table 3, along with the expected sign of influence on the probability of default. The macroeconomic covariates cover the stock market, interest rates, GDP, credit supply, inflation, industrial production, as well as demand of consumer goods.

The inclusion of lagged macroeconomic variables allows entering these variables as growth rates, differences, or levels. We select the appropriate form by 1) computing the correlation between each form of the macroeconomic variable and the observed default rate, and 2) visually inspecting the relationship of each form with the observed default rate. Note, however, that the macroeconomic variables

Table 3. ***External explanatory variables and corresponding macroeconomic variables.*** *The left column shows our list of external explanatory variables, the center column shows the macroeconomic variables used to measure each external explanatory variable, and the right column shows the expected effect of each macroeconomic variable on default probabilities.*

Explanatory Variable	Investigated Measurements	Expected effect on probability of default
Stock return	Return of OMX index	Negative
Stock volatility	Volatility of OMX index	Unknown
Interest rates	Slope of yield curve	Negative
GDP	Real growth in Danish GDP	Negative
Loan growth	Loan growth to non-financial firms	Positive
Credit availability	Funding costs	Positive
Aggregate defaults	Danish bankruptcies	Positive
Inflation	Headline CPI	Unknown
Demand side effects	Consumer confidence	Negative
	House prices	Negative
Supply side effects	Business indicator, manufacturing	Negative
	Capacity utilization	Negative
International exposure	Exports to Danish GDP	Unknown
	Return of Stoxx50 index	Negative
	EU 27 GDP growth	Negative

exhibit collinearity – for example, the Danish GDP growth and the European GDP growth rate, as well as the return on the OMX index and the Stoxx index have pairwise correlations of 0,92 and 0,77 respectively. The high degree of collinearity should be kept in mind when interpreting the results in the following section.

2.3 Estimation of default intensities

Suppose we have a sample of n levered firms observed over a time-horizon $[0, T]$, where firm i may default at a stochastic time τ_i. At each time t, the firm's financial state is determined by a vector $\mathbf{x}_{it}$ of internal covariates, with values specific to the firm, as well as a vector $\mathbf{z}_t$ of external, macroeconomic covariates, with values common to all firms in the sample. Default at time t occurs with intensity $\lambda_{it} = \lambda(\mathbf{x}_{it}, \mathbf{z}_t)$, meaning that λ_{it} is the conditional mean arrival rate of default for firm i, measured in events per time unit. Intuitively, this means that, given survival and the observed covariate histories up to time t, firm i defaults in the short time-interval $[t, t + dt)$ with probability $\lambda_{it}\, dt$.[1] We assume τ_i is doubly-stochastic driven by the combined history of the internal and external covariates (see for instance Duffie et al., 2007).

[1] Precisely, a martingale is defined by $1_{(\tau_i \leq t)} - \int_0^t 1_{(\tau_i > s)} \lambda_{is}\, ds$ with respect to the filtration generated by the event $(\tau_i > t)$ and the combined history of the internal and external covariates up to time t.

In our analysis of which accounting ratios and macro variables that significantly predict defaults below, we specify the firm-specific default intensities using the "proportional hazards" regression model of Cox (1972). The intensity of firm i at time t is thus modeled as

$$\lambda(\mathbf{x}_{it}, \mathbf{z}_t) = Y_{it} \exp\left(\boldsymbol{\beta}^\top \mathbf{x}_{it} + \boldsymbol{\gamma}^\top \mathbf{z}_t\right),$$

where Y_{it} is an at-risk-indicator for i, taking the value 1 if firm i has not defaulted "just before" time t and 0 otherwise, while $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of regression coefficients. The effect of a one-unit increase in the jth internal covariate at time t is to multiply the intensity by the "relative risk" e^{β_j}. The same interpretation applies to the external covariates. We let the first component of the vector $\mathbf{z}_t$ be a constant 1, so that the first component of the vector $\boldsymbol{\gamma}$ is a baseline intensity, corresponding to the (artificial) default intensity of firm i when all observable covariates are identically equal to zero.[2]

Following, for instance, Andersen et. al (1993), and under the standard assumptions that late-entry, temporal withdrawal, right-censoring, and covariate distributions are uninformative on regression coefficients, the (partial) log-likehood for estimation of the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ based on a sample of n firms becomes

$$l(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \sum_{i=1}^{n} \int_0^T \left(\boldsymbol{\beta}^\top \mathbf{x}_{it} + \boldsymbol{\gamma}^\top \mathbf{z}_t\right) dN_{it} - \int_0^T \sum_{i=1}^{n} Y_{it} \exp\left(\boldsymbol{\beta}^\top \mathbf{x}_{it} + \boldsymbol{\gamma}^\top \mathbf{z}_t\right) dt,$$

where $N_{it} = 1_{(\tau_i \leq t)}$ is the the one-jump default counting process for firm i. We investigate the assumption of independent censoring and entry-pattern in Section 5, and find that our parameter estimates are robust to the exclusion of firm-years that could potentially induce bias.

Estimation, inference, and model selection for the Cox model may then be based on maximum likelihood techniques. Given maximum likelihood estimators (MLEs) of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, we can judge the influence of covariates on default intensities by judging the significance of the corresponding regression coefficients, and we can predict firm-specific and aggregate default intensities by plugging the MLEs back into intensity specification of the Cox model. Model check may be based on the so-called "martingale residual processes,"

$$N_{it} - \int_0^t Y_{is} \exp\left(\widehat{\boldsymbol{\beta}}^\top \mathbf{x}_{is} + \widehat{\boldsymbol{\gamma}}^\top \mathbf{z}_s\right) ds, \quad i = 1, \ldots, n, \quad t \in [0, T], \tag{1}$$

[2]Note that while the usual Cox model includes an (unspecified) time-varying baseline-intensity, thereby making it a semi-parametric survival regression model, we cannot simultaneously identify the vector $\boldsymbol{\gamma}$ of macroeconomic regression coefficients as well as a time-varying baseline-intensity – we therefore restrict to a fully parametric model with a constant baseline intensity.

which, when the model fit is adequate and in large samples, are asymptotic mean-zero martingales. Hence, when aggregated over covariate-quantiles or sectors, the grouped residuals processes should not exhibit any systematic trends when plotted as functions of time.

In addition to the Cox model, we will in our analysis of the macro-sensitivity of small and large firms in Section 4 also employ the additive regression model of Aalen (1980, 1989). This specifies the default intensity of firm i as a linear function of the covariates, allowing an easy comparison of regression coefficients across firm size-subsamples.

3 Default prediction

In this section, we provide and discuss the results of our empirical analysis of which accounting ratios and macrovariable that significantly predict defaults in our sample. First, we show the result from a model using only firm-specific variables. We will see that this model cannot adequately predict the cyclical variation in the aggregate default rate. We then add macroeconomic variables to the model and show that this addition allows the model to much more accurately predict the aggregate default rate over time. However, when judging the different models' ability to correctly rank firms with respect to default likelihood, we will see that macro variables only marginally improve the ranking based on accounting ratios alone. In summary, to capture cyclicality of default rates, it is sufficient to focus on macro variables—however, the accounting variables are necessary controls for variations in firm-specific default risk not related to size.

3.1 Using accounting ratios alone

Initially, we fit a model of firm-by-firm default intensities using only firm-specific variables. We will use this model to examine to what extent macroeconomic variables add additional explanatory power to default prediction of non-listed firms.

Estimation results for the intensity models using only firm-specific variables are provided in Table 4. Due to the high degree of correlation among the measurements within the same categories, we perform a stepwise elimination of variables in a given category, removing the least significant variables in each step. The outcome is that interest bearing debt to total assets, net income to total assets, quick ratio, and tangible assets (PPE) to total assets remain in the model, along with age of banking relationship, log of total assets, and a negative equity dummy.

Interpreting the preferred model (Model 5 in Table 4) the effect of age is negative, implying that the longer a firm has had a relationship with the bank, the less

Table 4. ***Estimation results for Cox models including only accounting ratios.*** *The table shows parameter estimates, standard errors, and model summary statistics for Cox models of the quarterly default intensity of firms in the sample. All variables are lagged one year to allow for one-year prediction. The full list of internal variables are included in model (1). Models (2) through (4) show the stepwise elimination, keeping only the most significant measure within the groups of (1) leverage, (2) profitability, (3) liquidity, and (4) collateralization. Model (5) (shaded grey) is the preferred specification when only firm-specific variables are used as covariates. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm-level.*

Dependent variable: Default (0/1)		(1)		(2)		(3)		(4)		(5)	
Variables (all lagged 1 year)		Coef		Coef		Coef		Coef		Coef	
	Intercept	-6,788	***	-6,494	***	-6,668	***	-6,684	***	-6,729	***
	Years active in the bank	-0,011	***	-0,011	***	-0,011	***	-0,011	***	-0,011	***
	Log of total assets	0,043	*	0,014		0,020		0,019		0,016	
(1)	Short term debt to total assets	-0,572	**								
	Total debt to total assets	0,644	**								
	Interest bearing debt to total assets	0,661	***	1,226	***	1,262	***	1,261	***	1,255	***
	Interest payments to total assets	7,843	***								
(2)	Net income to total assets	-0,961	**	-1,790	***	-2,012	***	-2,025	***	-2,015	***
	EBIT to total assets	1,403	*	1,963	**						
	EBITDA to total assets	-2,615	***	-2,514	***						
(3)	Quick Ratio	-0,078		-0,045		-0,057		-0,212	**	-0,202	**
	Current ratio	-0,192		-0,179		-0,161					
(4)	Fixed assets to total assets	-0,455		-0,288		-0,308		-0,280			
	PPE to total assets	0,529	**	0,571	**	0,473	**	0,466	**	0,255	
	Negative equity, dummy	0,467	**	0,547	***	0,555	***	0,563	***	0,570	***
	Construction, dummy	0,926	***	0,885	***	0,928	***	0,921	***	0,951	***
	Wholesale and retail trade, dummy	0,214		0,216	*	0,267	**	0,250	*	0,275	**
	Manufacturing, dummy	0,404	***	0,399	***	0,420	***	0,406	***	0,417	***
	Number of observations	192.196		192.196		192.196		192.196		192.196	
	Number of firms	10.671		10.671		10.671		10.671		10.671	
	Number of events	633		633		633		633		633	
	Sector effects	YES		YES		YES		YES		YES	
	QIC	7.677,2		7.716,4		7.724,8		7.722,8		7.721,9	
	QICu	7.669,2		7.710,5		7.719,2		7.717,8		7.717,5	

likely it is that the firm will default. The effect of size, as measured by book assets, appears insignificant in the specification. This might potentially be explained by the sample pertaining to only the largest corporate clients, where size is less relevant as an explanation of default. The leverage ratio of interest bearing debt to total assets is, as expected, positively related to default probability. Likewise, past profitability is negatively related to default probability. The quick ratio enters with a significant negative sign confirming the hypothesis that the more liquidity a firm has, the higher its ability to service unexpected cash shortfalls which would otherwise have resulted in a default. Tangible assets, or Plant Property and Equipments

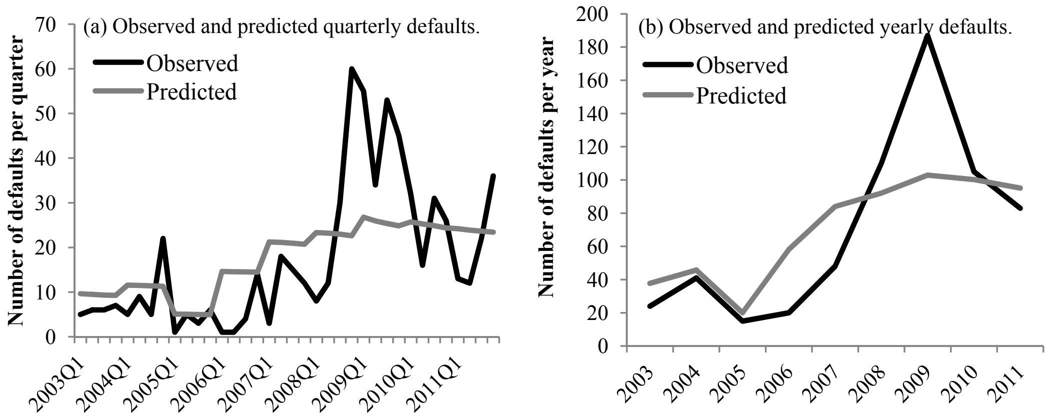

Figure 3. ***Default prediction based on the preferred Cox model only including accounting ratios.*** *Panel (a) shows the observed number of quarterly defaults in the sample along with the predicted number of defaults based on the preferred Cox model only including accounting ratios (Model (5) in Table 4). Panel (b) is similar, except that the aggregation is done on a yearly basis.*

(PPE), to total assets does not appear to have a significant effect, confirming the findings of Bonfim (2009) that tangible assets to total assets remain insignificant in explaining corporate defaults. The negative equity dummy enters with a positive sign in all specifications, confirming that a firm with negative equity is in fact a sign of a firm in trouble and at increased risk of default. The sign of the sectoral dummies are all positive and significant, confirming that these sectors have above average default rates.

Using the results of Table 4, we calculate a predicted quarterly default intensity for each firm in the sample, and then aggregate these to get a predicted aggregate intensity for each quarter. Figure 3 shows the observed number of quarterly defaults in the sample along with the predicted number of defaults based on the preferred Cox model only including accounting ratios. As evident in panel (a), the model based on accounting ratios alone is unable to explain the cyclical nature of the observed defaults. However, acknowledging that the firm-specific data can only change yearly through annual financial statements, it may be more appropriate to aggregate the predicted and observed number of defaults on a yearly basis. This is shown in panel (b), and the conclusion is the same: The model based solely on firm-specific variables is not capable of capturing the cyclical variation in defaults.

3.2 Including macroeconomic variables

Given that firm-specific variables are unable to explain the cyclical nature of defaults in our sample, this section attempts to incorporate macroeconomic effects. In order to assess if macroeconomic variables add explanatory power beyond what is implied by the firm-specific variables, the preferred model of the firm-specific variables is used as the basis of the covariate specification.

Table 5 presents estimation results for the models incorporating macroeco-

Table 5. ***Estimation results for Cox models with both accounting- and macroeconomic variables.*** *The table shows parameter estimates, standard errors, and model summary statistics for Cox models of the quarterly default intensity of firms in the sample. All variables are lagged one year to allow for one-year prediction. The full list of internal and external variables are included in model (6), and model (7) is the preferred specification after stepwise elimination of variables. Model (8) is the preferred specification in Model (7) excluding the firm-specific variables. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm-level.*

Dependent variable: Default (0/1)	**(6)**		**(7)**		**(8)**	
Variables (all lagged 1 year)	**Coef**		**Coef**		**Coef**	
Intercept	-13,341	***	-8,051	***	-7,206	***
Years active in the bank	-0,011	***	-0,011	***		
Log of total assets	0,007		0,007			
Interest bearing debt to total assets	1,232	***	1,231	***		
Net income to total assets	-1,877	***	-1,877	***		
Quick Ratio	-0,200	**	-0,200	**		
PPE to total assets	0,282	*	0,281	*		
Dummy for negative equity	0,592	***	0,591	***		
Aggregate quarterly number of Danish bankruptcies	0,005					
Danish Real GDP growth	-0,027					
Export / GDP	9,633	*				
Inflation, pct point	-0,124					
OMX stock market return	-0,052	***	-0,045	***	-0,038	***
OMX stock market volatility	-0,119	***	-0,101	***	-0,099	***
Difference CIBOR - policy rate, pct. Point	1,837	***	2,006	***	2,117	***
Yield curve slope 10y - 3m, pct. Point	0,986	***	0,588	***	0,634	***
Growth in house prices	-0,114	***	-0,104	***	-0,115	***
Change in consumer confidence Indicator	-0,110	***	-0,099	***	-0,110	***
Change in cyclical indicator, construction	0,002					
Change in capacity utilization in the industrial sector	-0,338	***	-0,300	***	-0,292	***
Loan growth to non-financial institutions	0,023	**				
Stoxx50 stock market return	0,031	***	0,034	***	0,029	***
EU27 Real GDP growth	0,250	***	0,227	***	0,213	***
Number of observations	192.196		192.196		192.196	
Number of firms	10.671		10.671		10.671	
Number of events	633		633		633	
Sector effects	YES		YES		YES	
QIC	7.513		7.513,6		8.265,0	
QICu	7.518		7.508,6		8.264,7	

nomic variables. The selection procedure has been to perform a stepwise elimination of insignificant variables until only significant macroeconomic variables remain in the model. Model (7) is the preferred model including both firm-specific and microeconomic variables, while Model (8) is this preferred model excluding the firm-specific variables.

The effects of the firm-specific variables remain robust to the inclusion of the macroeconomic variables. In the preferred model (Model (7) of Table 5), the significant macroeconomic variables are as follows: The return of the OMX stock market index, the volatility of OMX index, the difference between CIBOR and the policy rate, slope of the yield curve, change in consumer confidence, change in the capacity utilization, the return of the Stoxx 50 index, and, finally, the European GDP growth rate. On the other hand, the aggregate number of defaults, the Danish real GDP growth, exports as a fraction of GDP, inflation, changes in the cyclical indicator for construction, as well as the loan growth to non-financials are all insignificant in predicting default events.

When interpreting the coefficients of the macroeconomic variables in multivariate intensity regression models, one should bear in mind that it would be unrealistic to obtain a complete *ceteris paribus* effect of one macroeconomic variable, as this variable cannot be viewed in isolation from other macroeconomic variables. While not done here, an appropriate interpretation would involve testing the model from the perspective of internally consistent scenarios of macroeconomic variables. For instance, a further analysis shows that the volatility of the stock market, the slope of the yield curve, the return of the Stoxx 50 index, and the European GDP growth rate appear with an opposite sign in the preferred model compared to a model where they enter separately.

Nonetheless, a positive return of the OMX stock market would, controlling for other macroeconomic effects, imply a lower number of default occurrences one year after. An increased spread between CIBOR and the policy rate would be associated with an increased number of default occurrences, thereby supporting the notion that the higher funding costs of the banks would generally be passed through to clients. Both growth in house prices and changes in the consumer confidence index tend to be negatively linked to defaults, illustrating the importance of demand side effects. Capacity utilization is also negatively associated with default occurrences, meriting the interpretation that higher level of idle capacity could result in price competition that would ultimately lead a number of firms to default.

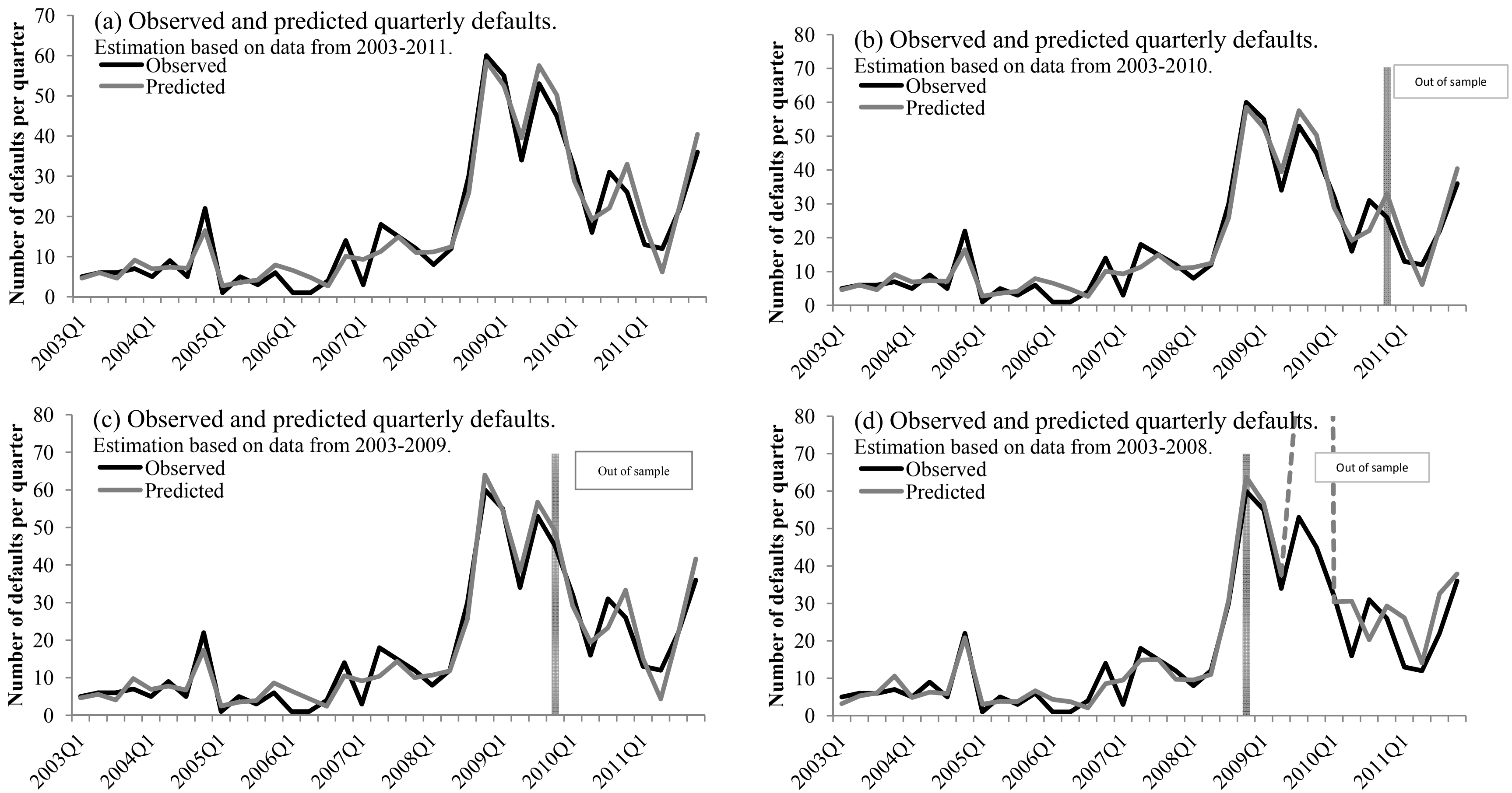

Figure 4. ***Default prediction based on the preferred Cox model with both accounting ratios and macroeconomic variables.*** *All panels show the observed quarterly number of defaults in the sample along with a predicted number of defaults based on the preferred Cox model with both internal and external covariates (Model (7) in Table 5). In Panel (a), the model is estimated using the full sample from 2003 to 2011. In Panels (b), (c), and (d), the model is estimated on shorter subsamples, allowing in each case out-of-sample prediction on the remaining part of the full sample.*

Figure 4 illustrates the relationship between the observed and predicted number of defaults taking into account both the firm-specific variables and the macroeconomic variables in Model (7) of Table 5. Adding macroeconomic variables as explanatory factors improves the model's ability to predict the cyclical variation in quarterly default occurrences. While there is a potential that the good fit of the preferred model may be a result of the numerical optimization techniques deployed for estimating the parameters, the out of sample prediction obtained from estimating the same model on only part of the data puts comfort in the chosen model. Panel (b) and (c) of the figure estimates the model on the sample excluding observations from 2010 and both 2010 and 2011 respectively. The obtained coefficients from the models estimated on the reduced samples are then used to estimate the aggregate intensities for all 36 quarters, thereby generating out of sample predictions. Hence, Panel (b) of the figure shows one and Panel (c) shows two years of out of sample prediction. The out of sample prediction based on the reduced sample estimation adequately captures both the level and cyclical variation in default rates.

Panel (d) of Figure 4 shows prediction based on excluding the years 2009, 2010 and 2011 from the estimation. For the out of sample prediction in Panel (d), large deviations occur in 2009 (which pertains to 2008 covariates observations because of the one year lag). However, it should be emphasized that the latter model has been fitted to a period of economic expansion, and therefore it is of little surprise that the model cannot be used to predict future defaults in a period of economic contraction. This finding also highlights the importance of estimating default predicting occurrences on a full business cycle.

3.3 Ranking firms with respect to default likelihood

The out of sample estimation results presented in Figure 4 showed that the preferred model including both firm-specific and macroeconomic variables adequately captures the level and cyclicality of defaults. However, given the stochastic nature of a default event, it will never be possible to completely predict which firms will default in a given quarter. Nonetheless, by specifying a particular cut-off point for the intensities, the model's ability to correctly discriminate between defaults and non-defaults can be evaluated in terms of how many outcomes that are correctly predicted and how many outcomes that are incorrectly predicted. This, however, necessitates arbitrarily specifying the cut-off point.

A more general approach is to plot the Receiver Operating Curve (ROC), as shown in Figure 5. The curve illustrates the percentage of defaults that are correctly classified as defaults on the vertical axis against the percentage of non-defaults that are mistakenly classified as defaults on the horizontal axis for all possible cutoff points. The area under the curve (AUC) is then used as measure of the model

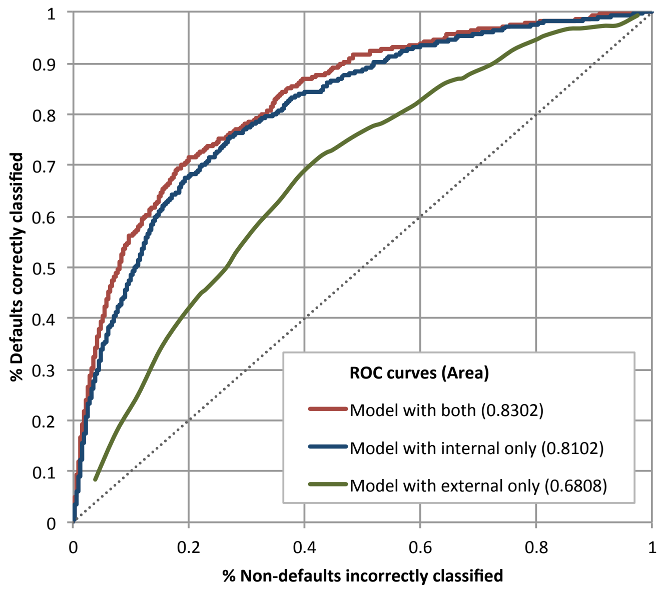

Figure 5. ***Comparison of firm-ranking accuracy for different covariate-specifications.*** *The figure shows receiver operating characteristic (ROC) curves for Cox models with different covariate-specifications fitted to the sample. Each curve illustrates the model's ability to correctly discriminate between defaults and non-defaults, and plots the percentage correctly classified defaults (true positives) against the percentage incorrectly classified non-defaults (false positives) at all possible cut-off points of default intensity. The area under each curve serves as a goodness-of-fit measure, where a value of 1 means a model with perfect discriminatory ability, while a value of 0.5 means a model that discriminates based on a random guess.*

goodness of fit where a value of 1.0 implies a model with perfect discriminatory ability and a value of 0.5 is a completely random model.

In terms of discriminatory power, the addition of macroeconomic variables does not improve the model's ability to effectively determine which firms eventually default beyond what is implied by the accounting ratios. From the ROC curves, it can be seen that the model with both external and internal covariates is only marginally better in correctly determining defaults compared to the model with just internal covariates. This is consistent with the notion that it is the firm-specific characteristics that provide the ordinal ranking of firms, and therefore also ultimately determine *which* firms that actually default. Including the macroeconomic factors only improves the model's ability to capture the cyclicality in the aggregate default rate, which is related to *when* defaults occurs.

4 The macroeconomy's impact on small and large firms

The results of the previous section show that our final model specification including both firm-specific and macroeconomic variables is able to both accurately rank firms and predict the aggregate default rate over time. We now use the model to investigate whether there is empirical support in our data for the provisions in the Basel II Accord that implicitly assume that smaller firms are less impacted by the macroeconomy compared to larger firms (The Basel Committee on Banking Supervision, 2006, Articles 273-74). The same provisions are carried forward and extended in the Basel III Accord and the recently adopted CRD VI (The European Parliament and the Council of the European Union, 2013, Articles 153.4 and 501.1). Specifically, Basel II and III allow banks to estimate capital requirements for small and medium-sized corporations (SMEs) using a risk weight formula that includes a lower asset correlation with macroeconomic risk-drivers compared to larger corporations. This allows banks to have effectively smaller capital reserve for this particular segment than would have been the case if they were treated as standard large corporations.

Precisely, the Basel II and III Accords specify the correlation between obligor i's assets and a common (macroeconomic) risk-driver as

$$\rho_i = 0.12 \times \frac{1 - e^{-50 \times PD_i}}{1 - e^{-50}} + 0.24 \times \left(1 - \frac{1 - e^{-50 \times PD_i}}{1 - e^{-50}}\right) - 0.04 \times \left(1 - \frac{S_i - 5}{45}\right),$$

where PD_i is the one-year probability of default, $S_i = \min\{50, \max\{5, S_i^*\}\}$, and S_i^* is total revenue in millions of Euros. The last term is the deduction in asset correlation specific to SMEs, and equals zero for firms with annual revenue above 50 million euros. To gauge the economic significance of the reduction, note that with an annual default probability of 1%, an SME with an annual revenue of 25 million Euro achieves a deduction in asset correlation of around 11.5% relative to an equally risky non-SME.

Before moving forward, let us underline that the specific asset correlation formula from the Basel II and III Accords is *not* in itself the focus of our analysis, as the capital reduction for SMEs can be achieved in other ways—for instance through the use of capital multipliers, as in CRD IV.[3] Instead, we focus on testing the implicit assumption that smaller firms are less cyclical compared to larger firms—that is, whether the the capital reduction for SMEs *can* be merited by the fact that they are less cyclical than larger firms, and therefore should have lower correlation with the common risk-driver compared to larger firms.

Our tests examine the extent to which default intensities for small and large firms are impacted differently by macro variables. First, we split the sample into two subsamples corresponding to "small" and "large" firms. Based on the year a

[3]Comment (44) on p. 6 of The European Parliament and the Council of the European Union's (2013) report states the following: *"Small and medium sized enterprises (SMEs) are one of the pillars of the European economy given their fundamental role in creating economic growth and providing employment. [...] The limited amount of alternative sources of funding has made EU SMEs even more sensitive to the impact of the banking crisis. It is therefore important to fill the existing funding gap for SMEs and ensure an appropriate flow of bank credit to SMEs in the current context. Capital charges for exposures to SMEs should be reduced through the application of a supporting factor equal to 0.7619 to allow credit institutions increase lending to SMEs."*

firm enters the sample, it is classified as “large” if its first-year asset level is above the median asset level that year. Similarly, a firm is classified as “small” if its asset value at the time of entry is below the median asset level that year. This particular division is done so as to ensure approximately equal sample sizes with a sufficient amount of default events in each category and to allow for the classification of firms in a predicable manner. In untabulated tests we, however, find that our results are robust to a division into four subsamples based on the quartiles of the asset value at the year of entry.

4.1 Macro-sensitivity analysis based on the Cox model

Table 6 shows estimation results for the final model specification including both firm-specific and macroeconomic covariates fitted to each of the two subsamples. With the exception of OMX stock market volatility, the magnitude of all macroeconomic effects are larger for large firms compared to small firms. The difference between the coefficients of macroeconomic factors for the two subsamples is significant for the slope of the yield curve, growth in house prices, and European GDP growth, and all are larger in magnitude in the subsample of large firms.[4] Hence, if we suppose there exists a large and a small firm whose only difference is their size (which is reasonable since all accounting ratios in our models are relative to total assets), the apparent interpretation of these results might be that the small firm’s default intensity is less exposed to macroeconomic fluctuations.

On the other hand, the estimation results for the two subsamples also show substantial differences with regards to the coefficients of the firm-specific variables: Firm size appears with a significant positive coefficient for small firms, but an insignificant (yet negative) coefficient for large firms; neither quick ratio nor the ratio of tangible assets to total assets have significant effects for small firms, whereas they have significant effects for large firms; and, finally, negative equity has a significant effect for small firms, but not for large.

While the results for the macroeconomic variables may corroborate the lower asset correlation adopted in Basel II for SMEs, the direct comparison of coefficients in the two subsamples is arguably naïve, because it ignores that a covariate’s marginal effect in a non-linear model, like Cox regression, actually depends on the values of all the other covariates. This implies that comparing the coefficients for the macroeconomic variables in the two subsamples is potentially problematic, because such a comparison fails to take into account that the firm-specific characteristic for the small and large firms are generally different and have different effects on default intensity.

[4] In an unreported analysis, we find that the importance of the European GDP growth for the large firms is merited by the tendency of large firms in the sample to engage more actively in exports.

Table 6. ***Estimation results for Cox models fitted to the subsamples of small and large firms.*** *The table shows parameter estimates, standard errors, model summary statistics, and comparison criteria for Cox models of the quarterly default intensity for small and large firms in the sample. The covariate list corresponds to the preferred model including both firm-specific and macroeconomic covariates (Model (7) in Table 5). All variables are lagged one year to allow for one-year prediction. In each year, an entering firm is deemed "small" if its book value of assets is under the median assets level for that particular year—otherwise, it is deemed "large." Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on standard errors clustered at the firm level. "Same sign" indicates whether or not the estimated coefficients are of the same sign for small and large firms; "Magnitude" indicates which type of firm has the largest estimated coefficient; and "Sig. Diff." indicates whether or not the coefficients for small and large firms are significantly different at the 5% level using a Welch t-test. Finally, "PEA" indicates whether the partial effect at the average is largest for the large firms or the small firms, while "APE" indicates whether the average partial effect is largest for the large firms or the small firms.*

	Small firms			Large firms			Comparison				
Variables (all lagged 1 year)	Coef		Se	Coef		Se	Same sign	Magnitude	Sig. diff.	PEA	APE
Intercept	−8.290	***	0.676	−9.057	***	0.765	Yes	Large	No	Small	Small
Years active in the bank	−0.008	**	0.004	−0.012	***	0.004	Yes	Large	No	Small	Small
Log of total assets	0.122	**	0.056	−0.006		0.043	No	NA	No	NA	NA
Interest bearing debt to total assets	1.048	***	0.224	1.628	***	0.351	Yes	Large	No	Small	Small
Net income to total assets	−1.788	***	0.302	−2.432	***	0.422	Yes	Large	No	Small	Small
Quick ratio	−0.112		0.074	−0.462	***	0.137	Yes	Large	Yes	Large	Large
PPE to total assets	0.040		0.197	0.508	**	0.255	Yes	Large	No	Large	Large
Dummy for negative equity	0.782	***	0.199	0.269		0.291	Yes	Small	No	Small	Small
OMX stock market return	−0.045	***	0.015	−0.045	**	0.018	Yes	Large	No	Small	Small
OMX stock market volatility	−0.102	***	0.038	−0.100	**	0.049	Yes	Small	No	Small	Small
Difference CIBOR - policy rate, pct. point	1.655	***	0.447	2.463	***	0.553	Yes	Large	No	Small	Small
Yield curve slope 10y - 3m, pct. point	0.372	***	0.130	0.869	***	0.154	Yes	Large	Yes	Small	Large
Growth in house prices	−0.040		0.032	−0.200	***	0.047	Yes	Large	Yes	Large	Large
Change in consumer confidence indicator	−0.083	***	0.026	−0.122	***	0.036	Yes	Large	No	Small	Small
Change in capacity utilization	−0.194	**	0.086	−0.420	***	0.106	Yes	Large	No	Small	Large
Stoxx50 stock market return	0.027	***	0.009	0.043	***	0.013	Yes	Large	No	Small	Small
EU27 Real GDP growth	0.101	*	0.055	0.410	***	0.070	Yes	Large	Yes	Large	Large
PEA scaling factor	0.0023			0.0009							
APE scaling factor	0.0041			0.0025							
Number of observations	91,182			101,014							
Number of firms	5,333			5,338							
Number of defaults	376			257							
Sector effects	Yes			Yes							

To elaborate, note the marginal effect in a Cox regression of a change in the jth macroeconomic variable on the default intensity of firm i is given by

$$\frac{\partial \lambda(\mathbf{x}_{it}, \mathbf{z}_t)}{\partial z_{jt}} = \gamma_j \, Y_{it} \exp\left(\boldsymbol{\beta}^\top \mathbf{x}_{it} + \boldsymbol{\gamma}^\top \mathbf{z}_t\right) = \gamma_j \, \lambda(\mathbf{x}_{it}, \mathbf{z}_t),$$

which depends on all the characteristics of firm i through $\mathbf{x}_{it}$, as well as all other macroeconomic variables through the dependence on $\mathbf{z}_t$.

A somewhat crude way to facilitate comparison between subsamples in non-linear models, like Cox regression, is to compute the marginal effect of a covariate at "average levels" in each of the subsamples. This gives rise to the *partial effect at the average* (PEA) and the *average partial effect* (APE)—see, for instance, Wooldridge (2009, p. 582-83). In the setting of an intensity model, the PEA plugs a subsample's average covariate values into the subsample's estimated intensity, while the APE takes the average across the estimated intensity values for each subsample. Due to the non-linearity of the intensity, Jensen's inequality implies that the two ways of averaging will generally produce different results.

In our analysis, the PEA is a measure of a covariate's marginal effect for the "average firm" and at "average macroeconomic levels" in each of the two subsamples of small and large firms. We thus compute the PEA for the jth macroeconomic variable in subsample k as

$$\mathrm{PEA}_{kj} = \gamma_{kj} \underbrace{\exp\left(\widehat{\boldsymbol{\beta}}_k \bar{\mathbf{x}}_k + \widehat{\boldsymbol{\gamma}}_k^\top \bar{\mathbf{z}}_k\right)}_{= s_k^{\mathrm{PEA}}},$$

where $k \in \{\text{small}, \text{large}\}$, $\bar{\mathbf{x}}_k$ is the average internal covariate vector for firms in subsample k, $\bar{\mathbf{z}}_k$ is the average macroeconomic covariate vector in subsample k, $\widehat{\boldsymbol{\beta}}_k$ and $\widehat{\boldsymbol{\gamma}}_k$ are the estimated regression coefficients in subsample k, while s_k^{PEA} is a subsample-specific scaling factor for each PEA. On the other hand, the APE is a measure of a covariate's marginal effect at the "average intensity level" across firms and time in each of the two subsamples. The APE for the jth macroeconomic variable in subsample k is thus computed as

$$\mathrm{APE}_{kj} = \gamma_{kj} \underbrace{\frac{1}{T} \int_0^T \frac{1}{|k(t)|} \sum_{i \in k(t)} \exp\left(\widehat{\boldsymbol{\beta}}_k \mathbf{x}_{it} + \widehat{\boldsymbol{\gamma}}_k^\top \mathbf{z}_t\right) dt}_{= s_k^{\mathrm{APE}}},$$

where $k(t)$ denotes the firms belonging to subsample $k \in \{\text{small}, \text{large}\}$ at time t, and s_k^{APE} is again a subsample-specific scaling factor for each APE.

The two right-most columns of Table 6 show the PEAs and APEs for each covariate in each of the two subsamples. Focusing on the effects of the macro variables, the PEA seems to suggest that most macro effects are, on average, strongest in the sample of small firms. On the other hand, the APEs for the macro variables suggest that it is entirely dependent on the macro variable at hand whether its average effect is strongest for the smaller or the larger firms.

In sum, while the naïve comparison of regression coefficients seems to indicate that there is merit to the assumption that smaller firms are less cyclical than larger firms, the more refined analysis based on the PEA and APE, which takes the non-linearity of the Cox model into account, indicates that smaller firms may "on average" be as cyclical, or perhaps even more cyclical, than larger firms. We are thus reluctant to draw any conclusions from a comparison of the subsamples based on the Cox regression model. We, therefore, in the following section, perform an additional macro-sensitivity analysis based on a model that allows for an easier comparison of coefficients across subsamples.

4.2 Macro-sensitivity analysis based on the additive Aalen model

A direct comparison of coefficients in subsamples is possible using the additive survival regression model. This model was first proposed by Aalen (1980, 1989) and has recently been applied in a default study of public US corporations by Lando et al. (2013). The Aalen model specifies the default intensity for firm i as

$$\lambda(\mathbf{x}_{it}, \mathbf{z}_t) = \boldsymbol{\beta}(t)^\top \mathbf{x}_{it} + \boldsymbol{\gamma}^\top \mathbf{z}_t,$$

where $\boldsymbol{\beta}(t)$ is a vector of unspecified regression *functions*, giving the linear effects of the firm-specific covariates at time t, while $\boldsymbol{\gamma}$ is a (constant) vector of regression coefficients for the macroeconomic covariates. In contrast to the multiplicative effects in the Cox model, covariate effects in the additive model are easy to interpret and compare across subsamples. We will therefore use the additive model to check the assumption that smaller firms are less sensitive to the macroeconomy compared to larger firms. Note, however, that the Cox model still has the advantage that it automatically produces nonnegative intensities and its constant regression coefficients allow out of sample prediction.

The linearity of the additive model allows for estimation of both time-varying and constant parameters using ordinary least squares-methods. For the time-varying coefficients, the focus is on the *cumulative regression coefficients*, $B_j(t) = \int_0^t \beta_0(s)\, ds$, which are easy to estimate non-parametrically. Further, formal tests of the significance and time-variation of regression functions is possible through resampling schemes. We refer to Aalen, Borgan, and Gjessing (2008), Martinussen and Scheike (2006), and Lando et al. (2013) for a detailed presentation of estimation and inference procedures for the additive model.

Initially, we fit an additive model for our entire sample of firms, including the same covariate specification as our final Cox model (Model (7) of Table 5), and

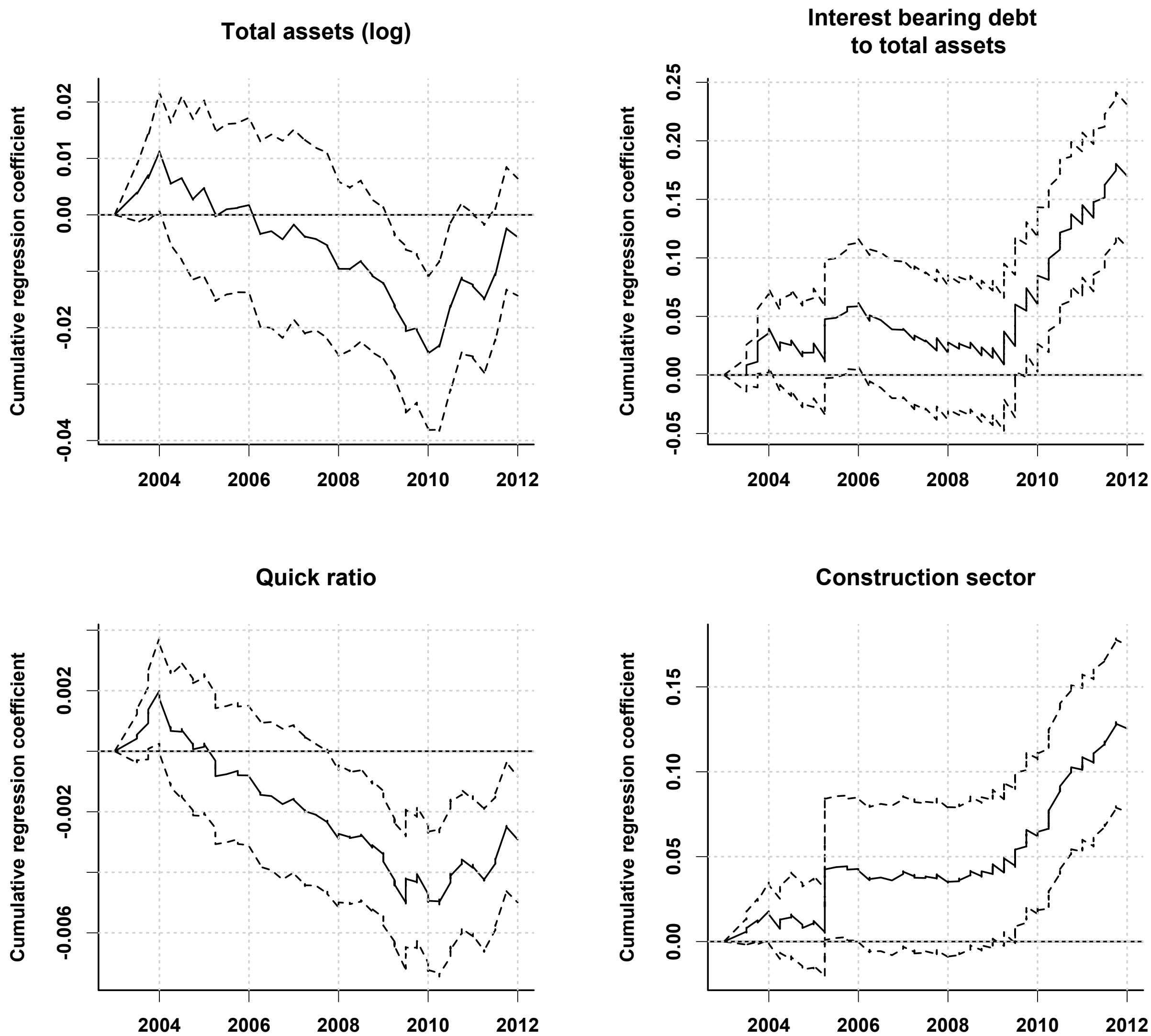

Figure 6. ***Cumulative regression coefficients from Aalen analysis of full sample.*** *The panels show (cumulative) estimation results for the significantly time-varying firm-specific covariates from an analysis based on the additive Aalen model including the same covariate specification as our final Cox model (Model (7) of Table 5). All variables are lagged one year to allow for one-year prediction. The dotted lines are asymptotic 95% pointwise confidence bands.*

with time-varying coefficients for the firm-specific covariate. The hypothesis of a time-constant effect could not be rejected for all firm-specific covariates but (log of) total assets, interest bearing debt to total assets, quick ratio, and the construction sector indicator. The time-varying effects of these four firm-specific covariates are shown as cumulative regression coefficients with 95% pointwise confidence bands in Figure 6. When interpreting these effects, one should focus on the *slope* of the cumulative coefficients, which estimates the regression coefficients themselves. We

Table 7. ***Constant regression coefficients from Aalen analysis of full sample.*** *The table shows the estimation results for the time-constant regression coefficients for the firm-specific and macroeconomic covariates from an analysis based on the additive Aalen model including the same covariate specification as our final Cox model (Model (7) of Table 5). All variables are lagged one year to allow for one-year prediction. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on robust standard errors.*

Variables (all lagged 1 year)	Coef		Se
Intercept	0.0025		0.0407
Years active in the bank	-2.05×10^{-5}	***	5.34×10^{-6}
Net income to total assets	−0.0140	***	0.0017
PPE to total assets	−0.0014	**	0.0006
Dummy for negative equity	0.0078	***	0.0012
Wholesale and retail trade, dummy	0.0005		0.0004
Manufacturing, dummy	0.0012	**	0.0004
OMX stock market return	−0.0036	**	0.0016
OMX stock market volatility	−0.0064		0.0080
Difference CIBOR - policy rate, pct. point	0.0059		0.0510
Yield curve slope 10y - 3m, pct. point	0.0110		0.0111
Growth in house prices	−0.0006		0.0024
Change in consumer confidence indicator	−0.0105	**	0.0047
Change in capacity utilization	−0.0077		0.0085
Stoxx50 stock market return	0.0042	***	0.0012
EU27 Real GDP growth	0.0050		0.0077

see, for instance, that interest bearing debt to total assets has a negligible effect up to around the year 2009, before the effect becomes quite strong and associated with higher default intensity for the rest of the sample period. All four covariate effects have the expected signs in periods where they have a non-negligible influence on the default intensity.

The estimation results for the time-constant regression coefficients from the additive model fitted to the entire sample are given in Table 7. We see that all coefficients corresponding to firm-specific variables have the same sign as in our final Cox model (Model (7) of Table 5) and roughly the same significance level. The macroeconomic covariates, however, appear to have lost much of their importance compared to the analysis based on the Cox models. In the additive setting, only OMX stock market return, change in consumer confidence indicator, and Stoxx50 stock market return have significant effects. The latter is of the reversed sign compared to intuition, but is nonetheless consistent with results for public-firms found by Duffie et al. (2007); Duffie, Eckner, Horel, and Saita (2009), Lando and Nielsen (2010), and Figlewski et al. (2012), amongst others.[5] The general lack of importance of macroeconomic effects is a consequence of the significantly time-varying effects for four of the firm-specific variables: Allowing firm-specific effects to be time-varying implies that the effects will to some extent vary with the macroecon-

[5]Recent work by Giesecke, Lando, and Medhat (2013) shows that univariately significant but multivariately insignificant or even reversed effects may be observed for macroeconomic covariates if these have indirect effects mediated through other covariates included in the models. This is in particular the case for stock market returns.

Table 8. ***Constant regression coefficients from Aalen analysis of small and large firms.*** *The table shows the estimation results for the time-constant regression coefficients for the firm-specific and macroeconomic covariates from an analysis based on the additive Aalen model for the two subsamples of small and large firms. All variables are lagged one year to allow for one-year prediction. Significance of parameters is indicated at the 10% (*), 5% (**), and 1% (***) levels. Parameter significance is based on robust standard errors.*

	Small firms			Large firms		
Variables (all lagged 1 year)	Coef		Se	Coef		Se
Intercept	0.0597		0.1883	0.0600		0.1918
Years active in the bank	-1.83×10^{-5}	*	1.14×10^{-5}	-1.70×10^{-5}		1.18×10^{-5}
Net income to total assets	−0.0160	***	0.0025	−0.0168	***	0.0025
PPE to total assets	−0.0021	**	0.0010	−0.0021	**	0.0010
Dummy for negative equity	0.0082	***	0.0016	0.0085	***	0.0016
Wholesale and retail trade, dummy	0.0003		0.0006	0.0003		0.0006
Manufacturing, dummy	0.0011		0.0007	0.0011		0.0007
OMX stock market return	-0.0084		0.0066	−0.0135	*	0.0069
OMX stock market volatility	0.0192		0.0324	0.0089		0.0317
Difference CIBOR - policy rate, pct. point	−0.3409		0.2317	−0.3048		0.2416
Yield curve slope 10y - 3m, pct. point	0.0003		0.0524	0.0134		0.0560
Growth in house prices	0.0136		0.0105	0.0098		0.0115
Change in consumer confidence indicator	−0.0324	*	0.0191	−0.0283	*	0.0196
Change in capacity utilization	−0.0625		0.0402	−0.0713	*	0.0404
Stoxx50 stock market return	0.0111	**	0.0050	0.0146	**	0.0054
EU27 Real GDP growth	0.0049		0.0351	0.0131		0.0370

omy, and this reduces the added explanatory power of macroeconomic variables.

We now fit the additive model including both the preferred firm-specific and macroeconomic covariates to each of the two subsamples containing small and large firms. Even though the macroeconomic variables did not have much explanatory power in the additive model fitted to the whole sample, an additive model, due to its linearity in the regression coefficients, still allows us to directly compare effects for macroeconomic variables in each subsample. One could perhaps imagine that some macroeconomic covariates had significant additive effects in the subsample of large firms, but not in the subsample of small firms – this would be evidence that macro-dependence differs for small and large firms, as is the working assumption in the Basel II and III Accords.

Estimating the additive model on each of the two subsamples did not change the conclusions regarding which firm-specific covariates have significant time-varying effects compared to the full sample. The estimation results for the time-constant regression coefficients in each of the two subsamples is shown in Table 8. We see no difference in sign, and virtually no difference in significance or magnitude for small firms compared to large firms. Hence, from the analysis based on the additive model, we find no evidence that small firms are less sensitive to macroeconomic variables.

5 Robustness and model check

In this section, we perform robustness checks of some our results and examine the model fit of the final model including both internal and external covariates. First, the estimation period is altered, in order to examine to what extent the particular time periods chosen for analysis were driving the results. Second, the chosen lag length of the macroeconomic-variables is discussed. Lastly, the final model's ability to predict outcomes within different sectors and assets sizes of firms is investigated through grouped martingale residual processes.

5.1 Independent censoring and entry-pattern

Given that data is only available from 2003 onwards, the existing stock of firms entering the sample in 2003 may potentially be of better average quality than the firms entering at a later point in time. This bias would violate the assumption of independent censoring. To address this issue, estimation was done on a reduced sample that excludes firms entering the sample in 2003 (where a considerable part of these entries ties to the existing stock of the bank clients). The results (not presented here, but available upon request) are that all estimated coefficients remain significant and of the same sign as the final model (Model (7) in Table 5). To address the concern that the very low number of entries in 2005 might have an impact on the results, the final model specification was re-estimated using two samples: One that exclude entries from 2005, and another that excludes all entries up until 2006. The estimates from these model fits (not presented here, but available upon request) are still all significant and of the same sign as the model estimated on the full sample.

5.2 Lag length

We have throughout chosen to focus on a lag length of one year for the covariates employed in our intensity models. One may, however, believe that for macroeconomic variables, this is not the appropriate lag, as aggregate changes may take longer to impact firms, since these operate with a capital buffer that allow them to operate though an extended period of time before a default is observed. To address this concern, we estimated the preferred model with all macro variables lagged eight quarters instead of four. The results (not presented here, but available upon request) showed that the macroeconomic variables are generally less able to explain defaults when lagged eight quarters, as indicated by the loss in significance for most of the coefficients.

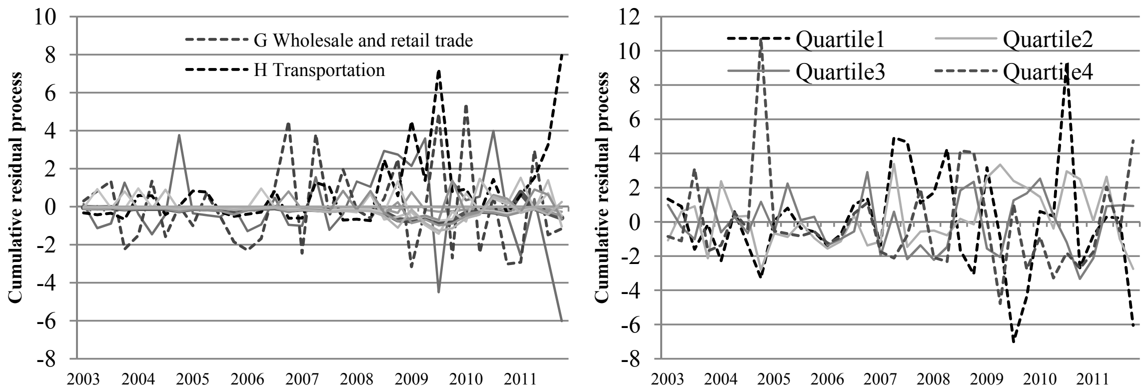

Figure 7. ***Model check based on grouped martingale residual processes.*** *The figure shows cumulative martingale residual processes for the preferred Cox model including both firm-specific and macroeconomic covariates (Model (7) in Table 5). In the left panel, grouping corresponds to each firm's sector, which is stationary across the sample period, while in the right panel, the grouping is time-varying and done According to asset quartiles.*

Still, this analysis does not consider the possibility that different macroeconomic variables are operating though different lag periods. Considering all possible combinations of lag periods would result in an extensive number of permutations of the model for us to check. Instead, we construct a correlation matrix for the observed default rate and each of the macroeconomic time series lagged from zero to eight quarters. It generally shows that, while the lag length of four quarters does not provide the highest correlation with the default rate for all macroeconomic variables, it appears that a unified lag period of four quarters is at least a very appropriate choice.

5.3 Grouped martingale residual processes

We check the fit of our final model using the martingales residual processes (1). Specifically, we consider to what extent the model is systematically over or under estimating the default frequency in different sectors and size-groups.

By definition, the martingale residual processes are the difference between the observed default frequency and the default frequency predicted by the model. Since a single firm can at most have one default event in our estimation setup, the firm-specific processes contain too little information. However, when grouped in sufficiently large clusters, the increments of the grouped processes should not be systematically positive or negative if the model fit is adequate. An increasing grouped residual process would imply that the model is under-predicting the number of defaults this particular group, whereas a decreasing grouped residual process would imply that the model is predicting too many defaults for this group.

The left panel of Figure 7 shows grouped residual processes by sector as a function of time. We see that the residual processes fluctuate around zero with both positive and negative increments for all sectors. This is support for the model performing equally well for all sectors. However, noting that the sectors “wholesale and retail trade” and “transportation” have the largest deviances, we re-estimated our final model excluding firms in these two sectors – the results (not presented here, but available upon request) do not change.

The right panel of the figure shows grouped residual processes by asset quartiles as a function of time. We again observe no truly systematic deviations. We note, however, that the largest quarterly deviances occur in the largest and smallest quartiles, further motivating the point that default prediction models should take firm size into account.

6 Concluding remarks

The Basel II and III Accords award preferential treatment to bank loans to SMEs on the basis that smaller firms as less sensitive to macroeconomic cyclicality compared to larger firms, effectively and significantly lowering capital charges for lending to the SME-segment. This paper investigates, in an intensity regression framework, whether there is empirical support for the hypothesis that smaller firms are less sensitive to macroeconomic cyclicality compared to larger firms.

Using a Cox regression setup, we find that solely discriminating with respect to firm size, and keeping all other firm characteristics equal, default intensities for smaller firms do in fact exhibit less sensitivity to macro variables. However, when we account for the Cox model’s non-linearity, and use averaging techniques adapted from other non-linear regression models, there seems to be no clear-cut conclusion from the analysis. We therefore also conduct the investigation using an additive specification that allows us to directly compare the coefficients of macro variables for small and large firms. In this setting, we find no evidence that there is different sensitivity to macro variables for smaller firms compared to larger firms.

Our tests control for variations in default risk not related to firm size by including accounting ratios in all our regression specifications. Furthermore, our regression results indicate that our method of focusing on the coefficients of the macro variables for small vs. large firms is adequate, as it is the macro variables that add the cyclicality component to our prediction models.

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