Risk allocation: The Double Face of Financial Derivatives

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Abstract

For the past two decades, derivatives provided the core financial innovation for risk-management and risk-sharing activities. However, in the aftermath of the 2007-2008 crisis, derivatives have started received, partly for good reason, an increasingly bad press. The main purpose of this paper is to lay the foundations for a theoretical framework in which systemic risk is centrally involved in the assessment of derivative usage. We begin by introducing a definition of systemic risk based on a Mixed Binomial model for the number of defaults. Then, we define an allocation to be efficient if it maximizes the Aggregate Sharpe ratio of the economy, i.e. if it allows to finance the maximum amount of productive investments while minimizing the overall systemic risk of the economy. We then say that a derivative is *socially efficient* or *cooperative* if it leads to an allocation having higher Aggregate Sharpe ratio. We illustrate the applicability of our model by means of a qualitative analysis of three types of derivatives, namely Plain vanilla, Asset backed securities and Credit default-swaps.

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1 Introduction and Motivation

An efficient allocation of the unavoidable risks generated by the business activity of individual agents is vital for the economic system. Insurance contracting and financial markets attain to this crucial goal. Motivated by this objective, which often takes the form of an attempt to achieve the simultaneous reduction of risk for all participants, insurance companies, as well as other financial risk-bearing entities, may enter into various kinds of formal risk-sharing agreements. Indeed, the idea that enhanced risk allocation should result in real economic benefits for society as a whole is deeply entrenched in political economic thought. To make just one notable example, Irving Fisher remarked, back in 1906, that

> [R]isk is one of the direst economic evils, and all of the devices which aid in overcoming it – whether increased guarantees, safeguards, foresight, insurance or legitimate speculation – represent a great boon to humanity. [24]

The question of determining optimal rules for sharing risks and constructing reinsurance has been long investigated in the theoretical actuarial literature. Around the 1930's, Medolaghi [29], de Finetti [19] and Ottaviani [30] developed the first linear reciprocal reinsurance models based on the minimization of individual and aggregate variance of risk. Some thirty years later, Borch [11, 12] took a major step forward by introducing a utility function into actuarial theory.[1]

Driven essentially by sheer increase in financial innovation, the practice of risk allocation performed by the financial system has rapidly evolved over the past few decades. Very substantial efforts have been devoted to the design of institutions and products enabling efficient risk sharing among individuals[2]. It is important to stress that innovation in the field

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[1] In the framework of a neoclassical perfect market, Borch sets up a n-person cooperative game and derives the condition for a Pareto-optimal risk exchanges in a reinsurance market. By means of the claim distribution for the risks insured during an insurance year, the best reinsurances, i.e. the one reaching Pareto-optimality, can be obtained.

[2]Representative examples can be found in [5], which also discusses a basic theory of risk sharing in an economy with incomplete markets, and in [22].

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of derivative products has significantly increased both the efficiency and the complexity of the allocation of risk. Today, market-based allocation of risk is primarily accomplished through the use of derivative products and *it is not an exaggeration to state that a considerable portion of financial innovation over the last 30 years has come from the emergence of derivative markets* (see [1]).

This owes directly to the fact that, for at least a couple of decades, the benefits of using derivatives, as well as of the associated market-based risk sharing model, appeared to be many-sorted and unquestionable. First of all, derivatives have been widely believed to cater for efficient risk management. Secondly, financial derivatives have been acknowledged an important informational role as they could be used as price discovery tools (see, e.g. [20]). Finally, derivatives appeared to provide more liquidity to the markets and also to the general economy.

For all those reasons, the trend in risk allocation has been marked by the securitization and distribution of risk. As this process started to unfold, various types of risks were originated, securitised, rated and allocated by different actors in the financial market. As a fundamental consequence of all this, risks were transferred from the regulated sector to unregulated one.

Institutional supervisors welcomed this market-based risk sharing model which, among other things, appeared to distribute risks away from a small number of large and systematically important actors, to a large number of smaller size investors. Similarly, it is now common for insurance firms to couple the standard approaches for managing underwriting risks, namely reinsurance, coinsurance, and geographic/product diversification, with the use of derivatives, just like other financial and non-financial firms.

Things changed dramatically with the outbreak of the 2007-2008 global finiancial crisis when a number of serious issues started to be raised with respect to the use of derivative products. Paradigmatic cases have certainly been the instability of markets in the aftermath Lehman Brothers's failure and, to an even larger extent, the need of a bailout of AIG – the largest insurance company in the United States at the time [32].

This much certainly provided enough evidence to the effect that markets can, and sometimes do, fail to deliver the expected risk management performances and the associated desirable social outcomes which largely motivated the enthusiastic endorsement of financial derivatives[3]. Building on this line of thought, a number of authoritative commentators [17, 31, 34] now agree that the global economic and financial crisis was a direct consequence of specific failures of the market-based risk sharing process. It clearly emerges from those analyses that the unmanageable systemic risk produced by increasingly sophisticated financial derivatives played a fundamental role in the build-up of the crisis. To worsen the outlook, recent empirical studies established either negative or no relation between derivative usage and firm value.[4] It is therefore hardly surprising that financial derivatives have been put under the spotlight as the main culprits of the financial crisis inducing some theoreticians, practitioners, commentators all the way down to the general public, to invoke draconian measures for their regulation.

Whilst this sort of reaction is fully understandable from a behavioural point of view, it might not be pointing in the obviously right direction. The leading intuition articulated across this paper is that a proper evaluation of financial derivatives must take into account their impact on systemic risk. Despite the current ubiquity of the notion,[5] there is as yet no widespread consensus on a specific formalization of the concept of systemic risk. Needless

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[3]For instance, given their capital base and risk management policies, individuals choose their desired level of market risk. However, in setting their level of market risk it is doubtful that individuals internalize the systemic implications of the failure of their firms or the public cost associated with any implicit government safety net.

[4]For instance, [23] finds that in firms with greater agency and monitoring problems, derivative usage has a significantly negative impact on firm value measured by their Tobin's Q.

[5]The importance of the role of the systemic risk in financial crisis, is gaining increasingly recognition. Several definitions and several measures of systemic risk in the finance and insurance sectors have been proposed (see, e.g., [4, 9, 8, 26]), many of them based on the statistical properties of market data (stock returns, the prices of various kinds of derivatives, etc). These have the advantage that they should incorporate the most current information and expectations, since market returns reflect information and update more rapidly than accounting variables. On the other hand, accounting measures provide unique insights (see, e.g., [27]) and are less subject to behavioural biases.

to say, this complicates the task of devising efficient models of risk allocation.

The main contribution of this paper is the attempt to overcome those difficulties by providing a simple, yet in our opinion highly flexible, model. The theoretical framework proposed here differs substantially from the optimal risk sharing one to be found in the actuarial literature, for our main concern is the characterization of what we refer to as *socially efficient* or *cooperative* derivatives. The bottom line of our approach is, quite simply, that socially efficient derivatives provide the benefits which motivated the initial enthusiasm on derivatives without exposing the economy in which they are used to potentially fatal systemic failures.

The building block of our model is the definition of *systemic risk*, which we provide in terms of a Mixed Binomial model for the number of defaults. The key desideratum for the definition consists in the ability of the market to fund productive but risky projects without increasing the systemic risk of the whole financial system. This is justified by the obvious observation that capital is a limited resource and so is the capital available to cover for risky investments. Capital must therefore be allocated efficiently toward the highest risk-adjusted productive investments. While doing this, it should be borne in mind that not all risks are created equal, a fact that should be taken into account when risks are diversified and spread across the economy. In particular it is quite natural that those risks which relate to highly productive investments should to be preferred to those bringing lower or zero contribution to the gross national product.

The next step consists in defining *efficient* an allocation which allows us to finance the maximum amount of productive investments while minimizing systemic risk. This simple approach gives us an opportunity to take into account a number of important factors in the evaluation of the efficiency of a risk allocation, namely (i) the quality of the diversification achieved by the individuals, (ii) the amount of funding of productive investments and (iii) the degree of leverage. Notice that within this framework we can explicitly model the systemic effects induced by the imprecise estimation of risk made by the individuals. Indeed, financial innovations can significantly alter the precision of the estimation of risk, a fact which

has clearly important implications for risk allocation. This adjustment of the subjective evaluation of risk is crucial if one aims, as we do, at modelling individual decisions under uncertainty as opposed to risk – the situation in which true and maximally precise evaluations of risks are given[6].

The final steps in the construction of our model consist in defining an *Aggregate Sharpe ratio* and identifying those channels through which a derivative may enhance or reduce the efficiency of the risk allocation in the economy. Having accomplished this we are in a position to say that *a derivative is socially enhancing or cooperative if it leads to a more efficient allocation of risk characterized by a higher Aggregate Sharpe ratio.*

The remainder of the paper is organized as follows. Our model is presented in Section 2. We illustrate in Section 3 its applicability by means of a qualitative analysis of three kinds of derivatives, namely Plain Vanilla, Asset Backed Securities and Credit Default Swaps. Section 4 concludes and points to future research directions.

2   Our Model

We begin by considering an economy characterized by a collection of risky investment projects and a group of individuals $i = 1, ..., N$ endowed with a given amount of initial (*equity*) capital C_i. Each individual constructs her own portfolio of risky assets and decides how much to invest on it. Let r_i be the gross return per unit of capital invested in the portfolio chosen by agent i (ROI) and let I_i be the amount agent i chooses to invest in the portfolio of risky assets. We assume that an individual i defaults when the loss on her risky asset position exceeds her initial capital C_i, which can then be interpreted as the *risk capacity* of agent i. Thus, the probability of default of agent i is given by

$$p_i = P(I_i r_i < -C_i).$$

A convenient measure that emerged as the industry standard for estimating and evaluat-

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[6]The reader who is unfamiliar with the topic may wish to consult [37] and the excellent selection of references therein contained.

ing the default risk in credit risk modelling (Moody's KMV model) is the so called Distance to Default (DD) that in our setting[7] becomes

$$DD_i = \frac{C_i}{I_i \sigma_i} = (\lambda_i \cdot \sigma_i)^{-1} \tag{1}$$

where $\lambda_i = I_i/C_i$ is the leverage of the position of agent i and σ_i is a measure of dispersion of the r_i distribution. Although DD can be defined for a broad set of probability distributions, in the following, to simplify computations and interpretations, we will consider the one-period return r_i to be normally distributed, i.e. $r_i \sim N(\mu_i, \sigma_i^2)$. It is, in fact, customary among practitioners to interpret DD as the "number of standard deviations a company is away from its default threshold". Obviously, the larger is DD the smaller is the probability of default. The DD measure does not include the contribution of the mean of r_i being typically negligible on the short-medium horizon and very difficult to estimate. More importantly, it does not consider the effects of the higher order moments of the distribution of r_i.

It is clear that the larger is the leverage λ_i the smaller will be DD and hence greater the risk of default. On the other hand, the expected Return On Equity (ROE) $I_i\mu_i/C_i = \lambda_i\mu_i$ is an increasing function of the leverage λ_i. As in standard mean-variance theory, each individual, after having selected the desired portfolio, decides how much to leverage his positions on the basis of his own risk preferences. Hence, each individual chooses the optimal leverage $\lambda^* = I_i^*/C_i$ (C_i is considered to be given) which maximizes her utility function. We assume that each agent has a Constant Absolute Risk Aversion (CARA) utility function[8] with different absolute risk aversion parameters α_i. Assuming for simplicity a zero risk free rate, each individual i then maximizes,

$$\max_{\lambda_i} \quad \lambda_i\mu_i - \frac{\alpha_i}{2}\lambda_i^2\sigma_i^2. \tag{2}$$

The first-order-condition for this utility maximization is

$$\mu_i - \alpha_i\lambda_i\sigma_i^2 = 0 \tag{3}$$

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[7] Notice that in a Gaussian world with $r_i \sim N(\mu_i, \sigma_i^2)$ the return obtained by investing I_i in such a portfolio is distributed as $I_i r_i \sim N(I_i\mu_i, I_i^2\sigma_i^2)$, and therefore the true probability of default is $P(I_i r_i < -C_i) = 1 - \Phi\left(\frac{C + I_i\mu_i}{I_i\sigma_i}\right) \simeq 1 - \Phi\left(\frac{C}{I_i\sigma_i}\right)$, i.e. a decreasing function of $\frac{C}{I_i\sigma_i}$.

[8] Hence a negative exponential utility function $U(X) = a - be^{-\alpha X}$.

so that the optimal leverage for individual i, λ_i^* becomes

$$\lambda_i^* = \frac{\mu_i}{\alpha_i \sigma_i^2} \tag{4}$$

In practice, however, the true value of the portfolio expected return μ_i and risk σ_i are not known. Let's call $\hat{\mu}$ the estimates made by agent i of the unknown expected return μ and $\hat{\sigma}_i$ the estimates of the unknown risk σ_i. Hence, the desired leverage can be written as

$$\widehat{\lambda_i^*} = \frac{\hat{\mu}_i}{\alpha_i \hat{\sigma}_i^2} = \lambda_i^* \frac{\sigma_i^2}{\hat{\sigma}_i^2} \frac{\hat{\mu}_i}{\mu_i} = \lambda_i^* e_i$$

where $e_i = \frac{\sigma_i^2}{\hat{\sigma}_i^2} \frac{\hat{\mu}_i}{\mu_i}$ is the estimation error of the agent i in evaluating the expected mean and risk of his position.

Then taking into account estimation errors, the "actual" or "realized" DD of individual i becomes:

$$DD_i = (\lambda_i^* \cdot \sigma_i \cdot e_i)^{-1} \tag{5}$$

Hence, the individual DD_i's are made of three components: the degree of leverage λ_i, the true riskiness of the portfolio σ_i, and the risk estimation error e_i. Being a decreasing function of DD_i, the probability of default p_i then increases with the degree of leverage λ_i, the true riskiness of the portfolio σ_i, and the underestimation of risk ($\hat{\sigma}_i < \sigma_i \Rightarrow e_i > 1$) or overestimation of the expected return ($\mu_i < \hat{\mu}_i \Rightarrow e_i > 1$).

Given the bijective relation between DD_i and p_i, the distribution of DD_i will determine that of p_i ($f(p)$ say) and, in particular, it will determine the mean $\bar{p}$ and variance of the p_i distribution $Var(p)$. Notice that if we define the default indicator X_i (which takes value 1 if firm i defaults and 0 otherwise) and consider even the case in which, conditional on the value of p_i, the random variables $X_1, ..., X_N$ are independent,[9] we have

$$\rho(X_i, X_j) = \frac{E(p_i^2) - \bar{p}^2}{\bar{p}(1 - \bar{p})} = \frac{Var(p)}{\bar{p}(1 - \bar{p})}.$$

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[9]Empirically, however, a direct contagion in which the actual default event causes a direct default of other firms or a deterioration of their positions is typically observed. This direct contagion could be easily incorporated in our framework by, for instance, having the individual DD_i being also a decreasing function of the total number (or value) of defaults of all the other firms in the system.

This shows that the default correlation is always non-negative in this setting and it increases with the variance of p_i and, hence, with that of DD_i. For future references, let us denote the average default-event correlation among all the individual pairs as $\bar{\rho}$.

Following [25] we define systemic risk as the probability of failure of a sufficiently large fraction of the total population of financial institutions. Unlike other definitions proposed in the literature (such as the Systemic Expected Shortfall of [2] or the CoVaR of [3]) this definition quantifies the systemic risk of the financial sector as a whole and it is consistent with the characterization of systemic risk given by the U.S. Commodity Futures Trading Commission, the Bank for International Settlements, the International Monetary Fund, the Financial Stability Board, and other major institutions (see [25]). Specifically, Systemic Risk (SR) is the probability of the fraction of the number of Defaults $(D_N = \sum_{i=1}^{N} X_i)$ over the whole population (N) exceeding a certain threshold (ϑ)

$$SR = P\left(\frac{D_N}{N} > \vartheta\right).$$

Hence, ceteris paribus SR is an increasing function of the individual values σ_i and λ_i (because by reducing DD_i they increase the probability of default p_i) and of the average default-event correlation $\bar{\rho}$.

Concerning the estimation error components e_i it seems, in general, reasonable to assume that individual errors are uncorrelated and hence their estimation errors tend to average out, i.e. $E[e] = 1$. However, at the aggregate level, the variance of the estimation error $Var[e]$ starts to play an important role since the higher the variance of e, the higher will be the probability of having more than ϑ defaultsand hence the greater the SR of the economy.

In fact, in our definition, SR is based on the distribution of the fraction of defaults which can be interpreted as a Mixed Binomial model for the fraction of defaults, where the mixing distribution p (which randomizes the default probability of the binomial process) is driven by the distribution of the individual distance to default DD_i. It can then be shown that in a Mixed Binomial model with $N \to \infty$ the distributions of the fraction of defaults becomes

$$P\left(\frac{D_N}{N} > \vartheta\right) \xrightarrow[N \to \infty]{} \int_{\vartheta}^{1} f(p)dp \equiv 1 - F(\vartheta).$$

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Hence, for a sufficiently large population N, the default distribution is determined by the distribution of p; i.e. when considering the default fraction for N large, the only remaining variability is that of the distribution of p. In fact, intuitively, when N is large, the realized frequency of default is close to the realized value of p so the distribution of the fraction of default becomes that of p. Then, the more the variability in the distribution of DD and hence in that of p, the more the weight in the tails of the default distribution and the more the correlation of default events. Since our systemic risk definition is a quantile of the default distribution, more weight in the tails caused a larger systemic risk. Therefore, the larger is the variance of the estimation error $Var[e]$, the larger is the variance of the mixing distribution, and hence the higher will be the SR of the economy.

Let us now denote

$$Y = \sum_{i=1}^{N} I_i r_i$$

to be the total gross return of the whole economy. Notice that, since derivative contracts are zero-sum transactions[10], the portfolio returns coming from derivatives positions exactly offset each other at aggregate level, so that they bring no contribution to the total return of the economy Y. Therefore, contributions to Y only come from the investments in the risky projects of the "real sector".

We are now in the position to present our definition of a socially efficient allocation. We say that an allocation is *socially efficient* if, for any given level of $E[Y]$, it minimizes SR,

$$\min SR \quad \text{subject to} \quad E[Y] = E[Y_0],$$

Note that this is equivalent to requiring that for any given level of SR, it maximizes the expected value of $E[Y]$, i.e.

$$\max E[Y] \quad \text{subject to} \quad SR = SR_0,$$

Hence, the socially efficient frontier will be a curve in the plane SR–$E[Y]$ which maximizes $E[Y]$ for any level of SR or, alternatively, which minimizes SR for any level of $E[Y]$.

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[10]The net monetary gain earned by one counterparty of the derivatives contract is exactly compensated by the net monetary loss paid by the other counterparty.
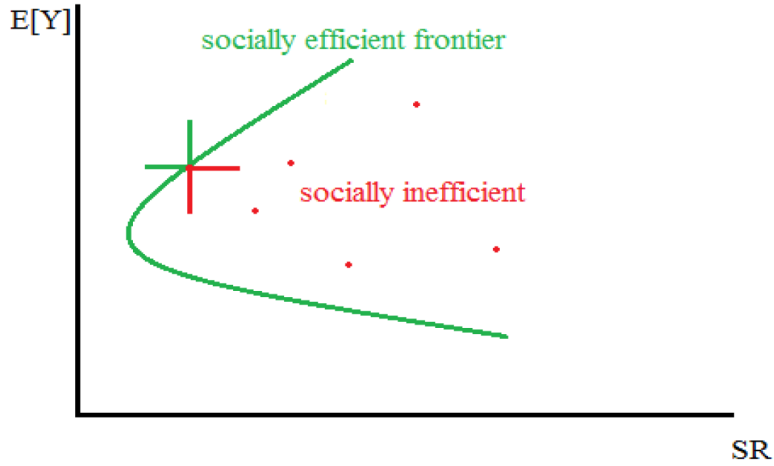
Figure 1: A stylized representation of the efficient frontier in the aggregate SR–$E[Y]$ plane.

In this economy an efficient allocation is reached when both the idiosyncratic risks of the single projects are optimally diversified (reducing σ_i's) and the remaining non–diversifiable risk is optimally estimated (minimum $Var(e)$ and $E(e) = 1$) and optimally shared among the investors according to their risk capacities (minimization of the aggregate leverage of the economy). This definition of an efficient risk allocation makes also clear that only those risks which directly or indirectly contribute to elevate $E[Y]$ should be born by the society, the others, by only increasing SR but not $E[Y]$, lead to a less efficient allocation point in the SR–$E[Y]$ plane.

We can now state our definition of a socially efficient derivative: *a derivative is said to be socially efficient or cooperative if it permits to move from an interior and inefficient point to a superior aggregate risk allocation having a higher "Aggregate Sharp ratio" $E[Y]/SR$.*

3 Evaluation of the social efficiency of derivatives

The theoretical framework developed in Section 2 allows us to identify the various channels through which a given type of derivative enhances or reduces the efficiency of the risk allo-

cation according to the Aggregate Sharpe ratio of the economy. In this Section we present a qualitative analysis of some types of derivatives in order to show how our framework could be applied to evaluate different types of derivative contracts. A more detailed quantitative analysis of the various types of derivatives will be the topic for future, empirically oriented, work.

3.1   Case 1: Plain vanilla derivatives

The main characteristic of the most standard and simple derivative contracts (such as futures, plain vanilla options, or swaps) is the possibility to transfer specific types of risks. This possibility of relocating risks from one subject to another, who is more willing to bear it, is often considered sufficient to enhance risk allocation and, hence, to be socially beneficial. However, whether this turns out to be the case, depends on whether the final allocation entails a lower systemic risk compared to the initial one. The reduction of systemic risk, in turn, depends on whether risk is transferred to individuals with larger DD, i.e. with either higher risk capacity (lower leverage λ), better portfolio diversification (lower σ_i), or better information (smaller estimation error e). This is particularly unlikely to be the case when risk transfer is motivated by regulatory arbitrages or moral hazard behaviors.

Moreover, even in many plain vanilla derivatives, it is well possible that none of the counterparties is transferring a preexisting risk. In this case, since no risk is reallocated, there cannot be any social efficiency gain coming from a better risk allocation. However, new risks are created. In fact, although such contracts are zero-sum transactions, they are certainly not "zero-sum risk", in the sense that the risk generated by the two-side bets does not compensate. Hence, even though the risks produced by these speculative bets does not contribute to $E[Y]$, they have to either absorb capital diverted from other investments projects (which instead contribute to Y) or increase the position leverage of the agents and, therefore, the systemic risk of the economy. So, even simple and usually beneficial derivatives products, if misused, can considerably increase systemic risk and thus reduce the Aggregate Sharpe ratio of the economy.

3.2 Case 2: Asset backed securities

An Asset Backed Security (ABS) is a security whose cash flows are derived from, and colletaralized by, a specified pool of receivables or other financial assets. Those underlying assets are typically represented by illiquid and risky assets which would individually have a very low rating score. The securitization process precisely consists in pooling various types of contractual debt (mortgages, loans or credit card debt) and selling those debts to various investors. Pooling those assets together and slicing such a pool into different risk classes, or tranches, categorized into varying degrees of subordination, reduces the risk of the so called "senior" tranches, since these rely on the protection of the junior tranches which would be the first to suffer the losses in the pool. Through this complex mechanism of pooling and splitting into different risk classes, the senior tranches were able to achieve AAA-ratings. Prior to the 2007-2008 financial crisis, this extremely high rating induced many banks and other investors to hold large amount of AAA-tranches of a vast variety of ABS products. As it was apparent after the burst of the crisis, those risk evaluations of ABS were severely biased, since the products turned out to be much riskier than implied by their credit scores.

Hence, the complex and opaque structure of the ABS and the related Collateralize Debt Obligations (CDOs) [13] not only increased uncertainty on the level of risk of the underlying assets, thus increasing the variance of the estimation error e, but also caused a systematical underestimation of such a risk so that a strong bias in e arose; this in turn led to an unwarranted rise in leverages which further increased individual risk. The separation between the subject originating the risk and those bearing it, also gave rise to large scale moral hazard problems, which greatly increased the size and misspricing of the generated risks. Moreover, being present in the portfolio of many institutions, ABSs and CDOs highly increased the correlation among the individual probability of default.[11]

Therefore, in the light of our model, this type of products exposed financial institutions to more systemic risk through:

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[11]See [7] for a very elementary, readable but efficient discussion of the increasing asset correlations and of the interconnectedness of markets (and the world economy).

(i) an increase in the variance of the estimation error $Var(e)$ (lack of transparency);

(ii) a positive bias in e, i.e. $E[e] > 1$ (misspricing and moral hazard);

(iii) an increase in individual leverages λ_i and

(iv) an increase in the correlation of the probability of default $\bar{\rho}$ which magnify the probability $P(D_N/N > \vartheta)$.

As a consequence, according to our definition, this type of derivatives were not (in that form) cooperative or socially efficient: even if $E[Y]$ might increase, SR is also increased, and the net change in the position of the economy in the $E[Y] - SR$ space is not superior to (i.e. does not dominate) the initial one.

3.3 Case 3: Credit Default Swaps

A Credit Default Swap (CDS) is a contract in which one party, the protection seller, sells protection to a second party, the protection buyer, against a credit event of a third party issuing a debt. In case of a default the protection buyer is compensated for the loss generated by the failure of the third party. The protection buyer pays, on a regular basis, a premium to the protection seller usually expressed in percentage points of the notional, the so called CDS spread. CDS are then simply a form of insurance against default. However, unlike insurance contracts, CDS do not require an exposure to the underlying credit risk. If the protection buyer does not hold the underlying security, CDS are said to be *naked*. Naked CDS can then be used to build speculative bets on the default of the third party: the naked protection buyer is betting on default while the other is betting against. The large amount of volume of the CDS market (in principle even larger than the total debt of the underlying entity) indicates that a substantial portion of contracts are naked [28, 38].

CDS have come to play an important role in conveying information about the market consensus on the creditworthiness of the underlying[12], especially in cases when the underlying

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[12] Empirical studies on how new information is incorporated in bond and CDS prices shows that information mainly flows from CDS to bond prices (see [10]).

debt market is not particularly liquid. This price discovery properties of the CDS, if viewed in the light of our model, translate in a reduction of the variance (and possibly bias) of the estimation errors e.

Non-naked CDS simply imply transferring credit risk from the protection buyer to the protection seller, leaving the total amount of credit risk in the economy unaltered. So the systemic risk implication of this relocation of risk depends on the relative risk capacity of the protection seller compared to the one of the protection buyer. If the protection seller has a greater risk capacity (in terms of larger capital, smaller leverage or better diversified portfolio which results in greater DD) than the protection buyer, systemic risk is reduced, otherwise it will be increased.

However, in the presence of naked CDS where no risk shifting is involved, the total amount of counterparty risk in the economy is increased by the two-sided bets on the default event. These newly created risk are unproductive and need to either absorb risk capacity or increases the systemic risk. Naked CDS can also encourage investors to divert their capital away from financing real investment leading in some cases to the selection of riskier ventures with lower expected returns [14].

Moreover, being the market of CDS highly concentrated in few large protection sellers, the counterparty risk generated by the default of one of these dominant actors can generate default contagion and domino effects. In the presence of a large market of naked CDS, the risk is not only given by the loss generated by the default of the issuer of debt, but also by the counterparty risk of all the protection sellers who wrote CDS on that entity. In other words, if CDS protection sellers have insufficient capital to cover CDS losses, the default of the issuer of the debt also causes the default of protection sellers, hence widening the scope for contagion and systemic risk [15].

This risk was clearly illustrated by the AIG case during the 2008 crisis which exerted domino effects through enormous CDS contractual links. In fact, AIG's collapse was caused largely by its $526 billion portfolio of CDSs. Federal Reserve Chairman Ben Bernanke has characterized AIG operations in derivative markets as the behaviour of a "quasi-hedge fund"

that "made irresponsible bets and took huge losses"[13].

Therefore, the introduction of CDS have both positive and negative effects on systemic risk and $E[Y]$:

(i) they allow a better price discovery on the creditworthiness of the issuer so that, in the notation of our model, $Var(e)$ and hence SR is reduced;

(ii) if credit risk is allocated to more capitalized and diversified subjects, it could permit a better allocation of risk, i.e. either decreasing SR or increasing Y;

(iii) if, however, CDS transfer credit risk to more leveraged and systemically important institutions SR will increase;

(iv) in addition, CDS can magnify the underlying credit risk by compounding it with the counterparty risk of the protection sellers, thus increasing the total amount of risk in the system (without financing more productive projects) and the possibility of contagion ($\bar{\rho}$ which elevates $P(D_N/N > \vartheta)$).

Therefore, the social efficiency of CDS remains unclear, depending on which of the above effects eventually dominates. However, some of the negative effects which tend to increase SR could be mitigated by moving the trading of CDS from the OTC market to a centralized clearinghouse which would greatly reduce the counterparty risk of CDS.

4   Conclusions

Today, the first and most fundamental problem in risk allocation is to understand the complex effects induced by financial derivatives and how they could be so badly misused to ignite a global crisis. We argued that part of the answer lies in the relation among individual risk, systemic risk and the related notions of correctly assessing risk capacity and leverage.

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[13]David Stout and Brian Knowlton, Fed Chief Says Insurance Giant Acted Irresponsibly, N.Y. TIMES, Mar. 3, 2009, available at http://www.nytimes.com/2009/03/04/business/economy/04webecon.html

In this paper we put forward a new theoretical framework within which the social efficiency of the risk allocation determined by a given derivative product can be assessed. To this purpose, we introduced a convenient measure, the distance to default, for estimating and evaluating the default risk of a single agent and identify its three components, namely the degree of leverage, the riskiness of the individual portfolio and the estimation error in evaluating the expected return and risk of the portfolio. Then, we proposed a definition of systemic risks based on the distribution of the fraction of defaults modeled as a Mixed Binomial process with the mixing distribution driven by the distance to default of the individual agents. Next, we introduced an aggregate risk-return plane for the risk allocations and define an Aggregate Sharpe ratio for the whole economy. Finally, we proposed our criteria to evaluate the social efficiency of a derivative by looking at whether it increases or reduces the Aggregate Sharpe ratio of the system.

This approach, albeit simple, permits to take into account many important factors in the evaluation of the efficiency of a risk allocation: the quality of the diversification achieved by the individuals, the amount of funding of productive investments, the degree of leverage and, importantly, it brings to the foreground the systemic effects induced by the imprecise estimation of risk made by the individuals.

Applying this theoretical framework to some of the most controversial derivatives we can conclude that a generic derivative contract is socially cooperative if : (i) informational gains are provided by improving transparency and price discovery, (ii) preexisting risks are reallocated only to more capitalized, diversified or informed subjects, (iii) new risks are not created.

We believe that our framework may be employed by regulators and supervising authorities for policy evaluations as well as in the design of future financial reform. In order to fully explore such a possibility, however, we must bring to completion the empirically oriented work which is currently under way on this topic.

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