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Structural changes in cross-border liabilities: A multidimensional approach

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HIGHLIGHTS

- Time series of interbank liabilities by country are used to develop geometrical spaces.
- The systematic information of the interbank market is shown to populate a 3D ellipsoid.
- The shape of the ellipsoid is shrunk and distorted in periods of financial turbulence.
- Multivariate skewness and kurtosis quantify the shape deviations from normality.
- These coefficients and a measure of the space volume characterize systemic risk.

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ABSTRACT

We study the international interbank market through a geometric analysis of empirical data. The geometric analysis of the time series of cross-country liabilities shows that the systematic information of the interbank international market is contained in a space of small dimension. Geometric spaces of financial relations across countries are developed, for which the space volume, multivariate skewness and multivariate kurtosis are computed. The behavior of these coefficients reveals an important modification acting in the financial linkages since 1997 and allows us to relate the shape of the geometric space that emerges in recent years to the globally turbulent period that has characterized financial systems since the late 1990s. Here we show that, besides a persistent decrease in the volume of the geometric space since 1997, the observation of a generalized increase in the values of the multivariate skewness and kurtosis sheds some light on the behavior of cross-border interdependencies during periods of financial crises. This was found to occur in such a systematic fashion, that these coefficients may be used as a proxy for systemic risk.

1. Introduction

Globalization of economies leads to an ever-increasing interdependence of countries. The late 2000s financial crisis – considered by many economists to be the worst financial crisis since the Great Depression – resulted in the collapse of large financial institutions, the bailout of banks by national governments and downturns in stock markets around the world.

The recent turmoil in the international banking system has stressed the need for understanding financial systems as sets of countries where cross-border financial relations play the fundamental role. While some authors have investigated the role globalization plays in shaping the spread of financial crisis [1,2], studies on the consequences of financial crises to the international banking system are less prominent. As we recently argued [3], the adoption of a topological approach is

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recommended not only because of the proper emphasis on the financial interdependencies but also due to the possibility of describing how the structure of these interdependencies evolves in time. In so doing, we are able to address the role that the existing structure plays in the spread of shocks and conversely, the effectiveness of stress events and their impact on the structure of the cross-border interdependencies.

The high level of interdependency that characterizes financial institutions makes banking and financial crises different from other economic crises. It is mostly due to the associated fear of spreading or contagion, which is technically called systemic risk. Systemic risk refers to the risk (in probabilistic terms) of breakdowns affecting the system as a whole, in contrast to the idea of partial or isolated breakdowns, which are restricted to individual parts or to specific components.

Researchers have recently applied network tools to model systemic risk in financial systems [4–7]. The authors in Ref. [4] found that hedge funds and insurance companies have become highly interrelated over the past decade, likely increasing the level of systemic risk through a complex and time-varying network of relationships. Battiston and co-authors [5] introduced a novel measure of systemic impact (DebtRank) which was inspired by feedback-centrality. They analyzed a new and unique dataset on the USD 1.2 trillion FED emergency loans program to global financial institutions during 2008–2010. They found evidence that systemic default could have been triggered even by small dispersed shocks, suggesting that the debate on *too-big-to-fail* institutions should include the *too-central-to-fail* epithet [5].

Whenever risk has to be measured, the resource to the concept of standard deviation is the most common practice in the financial community. However, the fact that financial returns do not follow normal distributions raises many criticisms against, for instance, portfolio strategies based on the use of variance as a proxy for risk.

A step forward in the statistical setting in understanding how data differ from normality leads to the computation of higher moments of the distributions throughout the application of measures of skewness and kurtosis. The calculation of univariate measures in order to evaluate deviations of statistical indexes from normality is a common procedure. A certainly less common procedure is the evaluation of deviation from normality from a multidimensional perspective.

Here we address the impact of the recent financial crises in the international banking system from a geometric point of view. In so doing, we are able to evaluate systemic risk by a technique similar to classical multidimensional scaling. The resulting multivariate geometric spaces provide the basis for the computation of multivariate skewness and kurtosis in different time periods. These coefficients, while indicating deviations from multinormality allow for the characterization of systemic risk.

Geometric analysis of time series of stock returns was performed in the past for the characterization of stock market crises. It revealed that most of the systematic information of financial markets was contained in a space of small dimension [8–10] where one finds noticeable differences between *business-as-usual* and critical periods. During market crisis there is a contraction of volume in the reduced geometric space, corresponding to a greater synchronization of the market fluctuations. In addition, whereas the geometric "market cloud" of points in *business-as-usual* periods looks like a smooth ellipsoid, during some crises it displays distortions, which may be detected by computing higher moments of the distribution. Here we apply the same geometric analysis to flows of financial capital between countries.

During the last years, several authors have approached financial systems through a topological approach. Some papers have favored the study of interdependencies between credit banks [11], or focused on the analysis of shocks storming the financial systems of several countries [12]. The topological properties of some national interbank markets have been studied by Soramaki and co-authors [13], who analyzed the topology of a network of commercial banks. Another example is the work of Fujiwara [14] exploring the credit relationships that exist between commercial banks and large companies in Japan.

Empirical studies have also been carried out on some European national interbank markets [15,16] throughout the analysis of the topological properties of the networks of Italian and Austrian banks.

Using the Bank for International Settlements (BIS) dataset, some papers have addressed the evolution of networks of bank transfers [16,17,14,11,18]. It is evident from the work of McGuire [18] that the international banking system has become an important conduit for the transfer of capital across countries. The work of Minoiu and Reyes [19] also explored the proprieties of the global banking networks; for several networked systems, they found evidence of important structural changes following the occurrence of stress events.

In this paper, from series of interbank liabilities over different time periods we have developed geometric spaces where each country is uniquely identified by a set of coordinates. For this purpose, a geometric approach is used, with a metric constructed from time series of amounts of liabilities by country. By a technique similar to classical multidimensional scaling, each country is mapped onto a point in Euclidean space and then the shape of the resulting "cloud of points" is analyzed. It turns out that, up to very small deviations, the market of cross-border liabilities is mostly concentrated in a low dimensional subspace. Furthermore, in this reduced subspace, the countries populate an elongated ellipsoid with faster decreasing axis.

Empirical results allow us to relate the shape of the geometric spaces to the characteristics of some relevant periods of financial turbulence, during which the shape of the ellipsoid is distorted, acquiring prominences in some particular directions. Measures of multivariate skewness and kurtosis are applied in order to quantify how far that shape deviates from multinormality. These statistical coefficients together with a measure of the space volume allow for the characterization of systemic risk.

The paper is organized as follows: Section 2 briefly presents the set of empirical data and the methodology. Section 3 presents the first results obtained from the application of a stochastic geometry technique. The main contributions of the paper are presented in Section 4, where measures of multivariate skewness and kurtosis are applied in order to test deviations from multinormality. The paper ends with appropriate conclusions.

Table 1
Reporting countries with available data from 1983 to 2012.

AT: Austria	IT: Italy
BS: Bahamas	JP: Japan
BH: Bahrain	LU: Luxemburg
BE: Belgium	NL: Netherlands
CA: Canada	AN: Netherlands Antilles
KY: Cayman Islands	NO: Norway
DK: Denmark	SG: Singapore
FI: Finland	ES: Spain
FR: France	SE: Sweden
DE: Germany	CH: Switzerland
HK: Hong Kong	GB: United Kingdom
IE: Ireland	US: United States

2. Data and methodology

The Bank for International Settlements (BIS) locational banking statistics (IBLR) – including international claims and liabilities of reporting banks by country of residence – provide a plentiful dataset of aggregate cross-border exposures for a set of reporting and non-reporting countries all over the world. These statistics were originally introduced in 1964 to monitor the development of euro-currency markets, starting to be available in 1977. The locational reporting system collects quarterly data on the gross international financial claims and liabilities of banks resident in a given country. The main purpose of the statistics is to provide information on the role of banks and financial centers in the intermediation of international capital flows.

Since the BIS locational banking statistics capture the net flows of financial capital between any two countries channeled through the banking system [18] this dataset is an appropriate source for the empirical study of temporal patterns arising from financial linkages across countries.

The locational statistics are intended to complement monetary and credit aggregates, being consistent with both the national balances of payments and the systems of national accounts. For a set of 57 reporting countries and aggregate zones, we consider a subset of 24 countries (see Table 1), each of them represented by the amounts of its liabilities *vis-a-vis* the other reporting countries, measured on a quarterly basis, from the last quarter of 1983 (Q4-1983) to the last quarter of 2012 (Q4-2012).[1]

Cross-border financial flows provide a global quantification of the size of transactions in financial assets, and thus, they are an aggregate measure of the intensity of financial linkages between different economies [20].

Empirical data provide the primary basis for the calculation of the distance matrix containing the distances between each pair of countries. The distance between each pair of countries is a function of the correlation coefficient computed over a given time interval. It so happens that from the matrix of distances and through the application of a stochastic geometry technique we are able to reconstruct the geometrical spaces where the given set of countries is supposed to exist.

It is accomplished by assuming that each country has coordinates in some finite dimensional space and that the coordinates represent quantitative measures of a set of characteristics which uniquely identify each country. However, initially, both the coordinates and the dimension of the space are unknown. The sole input from where to start is the distance matrix, where the distance is defined from the amounts of liabilities of banks resident in each reporting country *vis-a-vis* the other reporting countries. It is accomplished throughout the application of the stochastic geometry technique described in the following.

2.1. The method

Cross-correlation based distances, as applied in Ref. [21] to the study of stock market structures have been used in the analysis and reconstruction of geometric spaces in different fields [8–10]. The quantity

$$d_{ij} = \sqrt{2\left(1 - C_{ij}\right)} \tag{1}$$

where C_{ij} is the correlation coefficient of two time series $\vec{s}\,(i)$ and $\vec{s}\,(j)$ computed along a given time window

$$C_{ij} = \frac{\left\langle \vec{s}\,(i)\vec{s}\,(j)\right\rangle - \left\langle \vec{s}\,(i)\right\rangle\left\langle \vec{s}\,(j)\right\rangle}{\sqrt{\left(\left\langle \vec{s}^{\,2}(i)\right\rangle - \left\langle \vec{s}\,(i)\right\rangle^2\right)\left(\left\langle \vec{s}^{\,2}(j)\right\rangle - \left\langle \vec{s}\,(j)\right\rangle^2\right)}} \tag{2}$$

[1] The first data collection date back to 1977 but a significant amount of statistics can be obtained only from the eighties. The statistics prior to 1983, includes just 15 countries. The number of countries participating in the locational banking statistics increased from 15 in the mid-1970s to 44 in 2012. Here we consider 24 countries, for which the data are complete and reliable from 1983 to nowadays.

has been shown [22] to satisfy all the metric axioms. Therefore, it may be used to develop a geometrical analysis of the interbank market structure. To this end, we applied the following technique.

2.2. The geometry of cross-border liabilities

Using the BIS quarterly based time series of liabilities of each country ($\overrightarrow{l}(i)$) *vis-a-vis* the other reporting countries, we define a normalized vector

$$\overrightarrow{\rho}(i) = \frac{\overrightarrow{l}(i) - \langle \overrightarrow{l}(i) \rangle}{\sqrt{n\left(\langle \overrightarrow{l}^2(i) \rangle - \langle \overrightarrow{l}(i) \rangle^2\right)}} \tag{3}$$

n being the number of components (number of time labels) in the vectors $\overrightarrow{l}(i)$. With this vector one defines the *distance* between the countries i and j by the Euclidean distance of the normalized vectors,

$$d_{ij} = \sqrt{2\left(1 - C_{ij}\right)} = \left\| \overrightarrow{\rho}(i) - \overrightarrow{\rho}(j) \right\| \tag{4}$$

where C_{ij} is the correlation coefficient of the liabilities $\overrightarrow{l}(i)$ and $\overrightarrow{l}(j)$, respectively of countries i and j computed along a time window of n (quarterly) observations.

Having computed the matrix of distances for the set of N reporting countries, one obtains coordinates in R^{N-1} compatible with these distances ($d_{i,j}$) through the following algorithm:

$$\begin{aligned} \overrightarrow{x_1} &= \{0\} \\ \overrightarrow{x_2} &= \{d_{1,2}, 0\} \end{aligned} \tag{5}$$

then, when $i > 2$ and $j < i - 1$, the coordinate $x_i(j)$ is given by

$$x_i(j) = \frac{d_{i,j}^2 + x_{j+1,j}^2 - d_{i,j+1}^2 + \sum_{z=1}^{j-1}(x_i(z) - x_{j+1}^2(z)) - \sum_{z=1}^{j-1}(x_i(z) - x_j^2(z))}{2x_{j+1,j}} \tag{6}$$

and when $j = i - 1$

$$x_i(j) = x_i(i-1) = \sqrt{d_{i,1}^2 - \sum_{1}^{i-2}(x_{i,j}^2)}. \tag{7}$$

If $x_i(i-1)$, so obtained, is smaller than a small quantity ε no new dimension is added and the country is projected on the already obtained subspace. The main difference between this embedding algorithm and classical multidimensional scaling (MDS) relies on the allowance for the ε-fuzziness in the definition of the hyperplanes. In so doing, the embedding dimension is, in general, smaller than the one obtained from classical MDS.

For the chosen time window the liabilities of the countries are now represented by a set $\{x_i\}$ of points in Euclidean space. Then, the center of mass $\overrightarrow{R}$ and the center of mass coordinates $\overrightarrow{y}(k) = \overrightarrow{x}(k) - \overrightarrow{R}$ are calculated.

The covariance distance matrix

$$T_{ij} = \sum_k \overrightarrow{y_i}(k)\overrightarrow{y_j}(k) \tag{8}$$

is diagonalized to obtain the set of eigenvalues and normalized eigenvectors $\{\lambda_i, \overrightarrow{e}_i\}$. The eigenvectors $\overrightarrow{e}_i$ define the characteristic directions of this geometric space. Their coordinates $z_i(k)$ along these directions are obtained by projection

$$z_i(k) = \overrightarrow{y}(k) \bullet \overrightarrow{e}_i. \tag{9}$$

The same analysis is performed for random and time permuted data and the relative behavior of the eigenvalues is compared.

The characteristic directions correspond to the eigenvalues λ_i that are clearly different from those obtained from surrogate data. They define a subspace S_f of dimension f which contains the systemic information related the interbank market structure.

This corresponds to the identification of empirically constructed variables that drive the cross-border market and, in this framework, the number of surviving eigenvalues (f) is the effective characteristic dimension of this economic space.

Having found the number of characteristic dimensions (f) we are able to denote by $\overrightarrow{z}(k)^{(f)}$ the restriction of the k-country to the subspace S_f and by $d_{ij}^{(f)}$ the distances between countries i and j restricted to this low-dimensional space.

This procedure is the key for the following method, since it allows for the consideration of populations of 24 countries, given that only a very small number of coordinates describing their distances is used in the computation of our measures of the multivariate space.

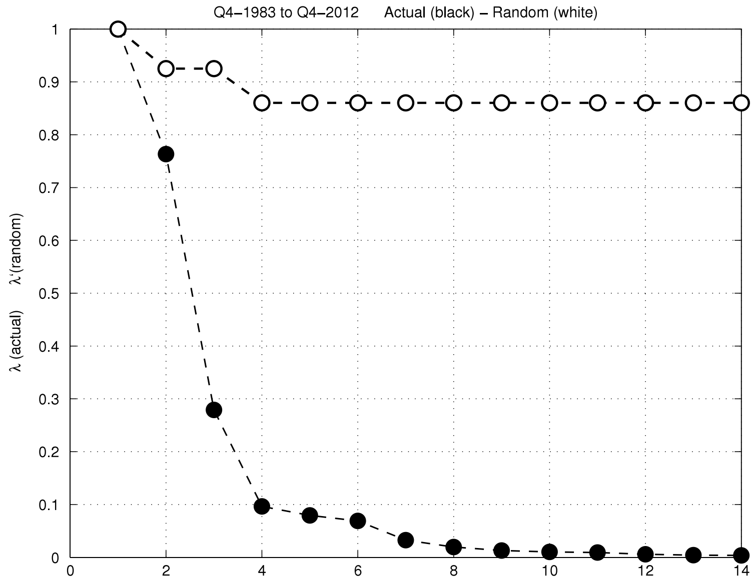

Fig. 1. The ordered eigenvalue distributions (actual/black and random/white) for the 24 countries.

3. The multidimensional spaces

Our study starts with the application of the stochastic geometry technique to the time series of interbank liabilities, each time series corresponding to a reporting country and providing quarterly observations over 29 years, from the last quarter of 1983 to the last quarter of 2012. We also consider the results calculated from surrogate data, namely from data obtained by independent time permutation for each country and to random data with the same mean and variance.

3.1. The dimensional reduction

Fig. 1 shows the ordered eigenvalue distributions of actual (λ_i black) and random (λ'_i white) data.

From the difference between the decay of actual and surrogate (time-permuted) eigenvalues we conclude that the first three dimensions capture the structure of the deterministic correlations and economic trends that are driving the cross-border financial system, whereas the remainder of the space may be considered as being generated by random fluctuations.

Eigenvalues from the fourth to the 23rd have a quite similar behavior in both actual and time-permuted cases, showing that for this cross-border system the three largest dimensions define our empirically constructed variables and that the dimensions of higher order essentially represent noise.

3.2. The three-dimensional spaces

The plots in Figs. 2, 4 and 5 show the three-dimensional geometric spaces populated by the 24 countries in different time periods while Fig. 3 displays the application of the stochastic geometry technique to surrogate data.

Fig. 2 shows the projections ($\vec{z}\,(k)^{(3)}$) of the 24 countries along the first three eigenvectors obtained from the entire 29-year actual data ($N = 24$ and $n = 116$ quarters), over the period 1983–2012. Different colors are used to identify European (blue) and non-European (red) member states.

It is evident from Fig. 2 that the most eccentric positions in the reduced subspace are occupied by countries outside the European Union (EU). Countries like Japan (JP), Hong Kong (HK) and the Netherlands Antilles (AN) are far away from all other countries, whereas a much greater centrality (and closeness) is characteristic of many EU countries. Notice that the closeness among them in the three-dimensional space comes from positive and strong correlations between country debts. Conversely, the countries that split apart from the others have debt positions that are negatively (and strongly) correlated to the debt positions of the other countries. In this example and when the whole 29 years are considered (Fig. 2), Finland (FI), Japan (JP), the Netherlands Antilles (AN) and Hong Kong (HK) are strongly and negatively correlated to the great majority of reporting countries.

Fig. 3 shows the geometric space obtained from the application of the stochastic geometry technique to surrogate data, namely from data obtained by independent time permutation for each country. In this plot we observe that the space generated from surrogate data is quite different from the cloud of points obtained from real data (Fig. 2), being less concentrated around the most exocentric positions and occupying a much greater volume in the three-dimensional space.

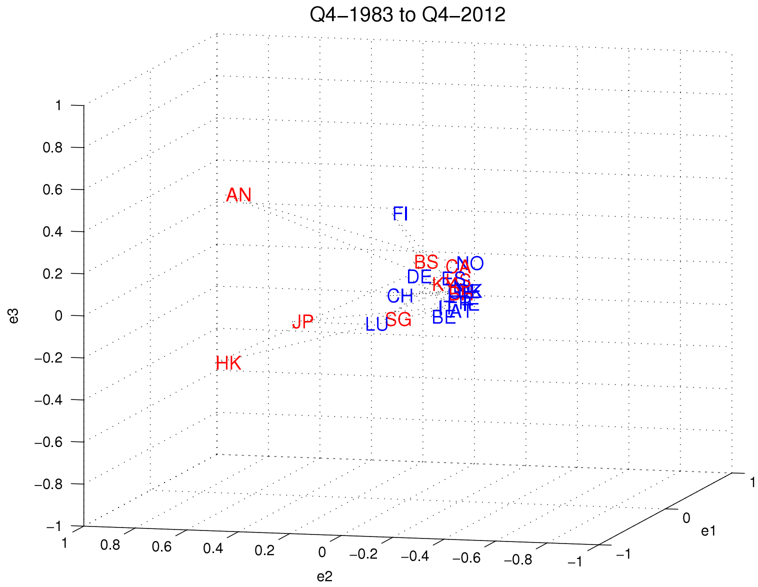

Fig. 2. The three-dimensional space in 1983–2012 for the 24 countries. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

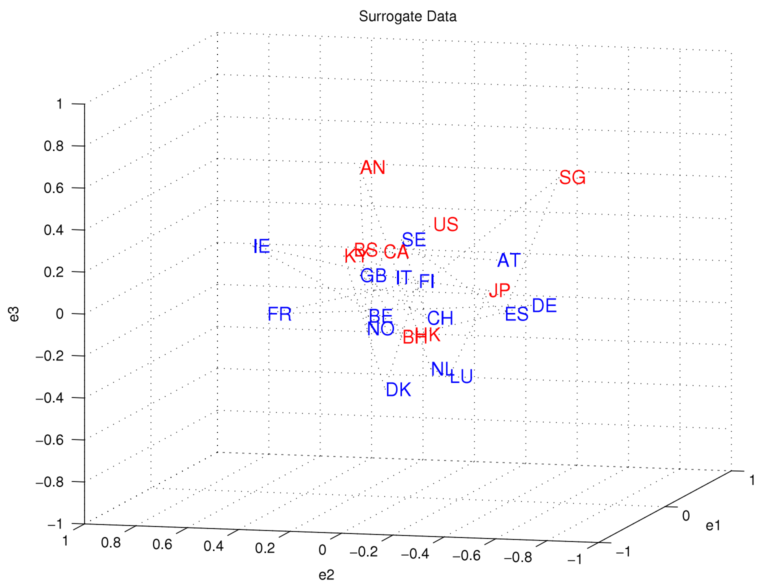

Fig. 3. The three-dimensional space for surrogate data.

To have a qualitative idea concerning the structure of the characteristic dimensions of the spaces obtained from actual data, we have divided the 29 years' data into two batches and performed the same operations. The plots in Figs. 4 and 5 show the reduced subspaces associated with the three largest eigenvalues obtained over the 12-year periods 1987–1999 and 1997–2009, respectively.

The plot in Fig. 4 shows that years going from 1987 to 1999 can be defined as a period of *business as usual*, with the geometric object close to a spherical configuration, typical of a Gaussian distribution, where the market space is similar to that of a random universe, as in Fig. 3.

Conversely, a new shape emerges in the period 1997–2009 as reported in the plot presented in Fig. 5. In this period, the shape of the geometric space is distorted, acquiring prominences in some particular directions.

The geometrical spaces in Figs. 4 and 5 show that there are some important differences in the interbank market along different time periods. The main difference seems to rely on the *clustering trend* that characterizes the 1997–2009 geometric space, where the distances between countries seem to be shortened; except for the few examples of the Netherlands Antilles (AN), Japan (JP) and Hong Kong (HK) that, like in the plot presented in Fig. 2, remain far from almost all other countries.

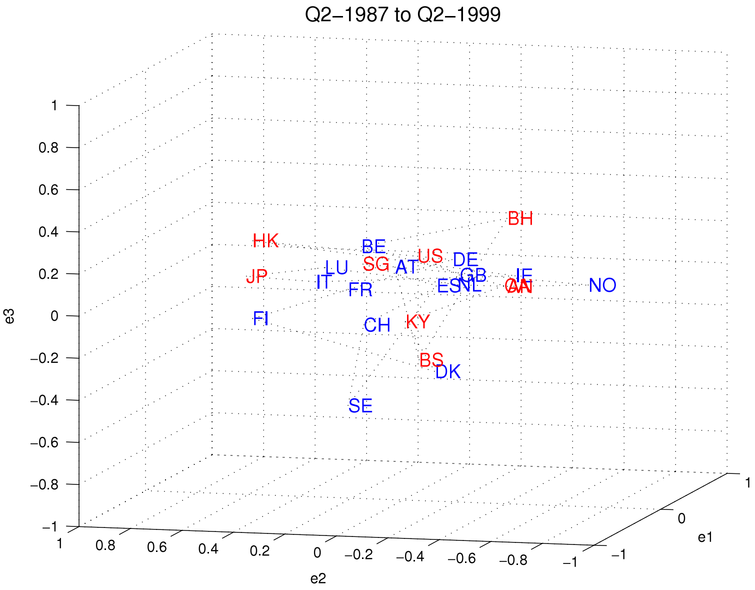

Fig. 4. The three-dimensional space in 1987–1999 for 24 countries.

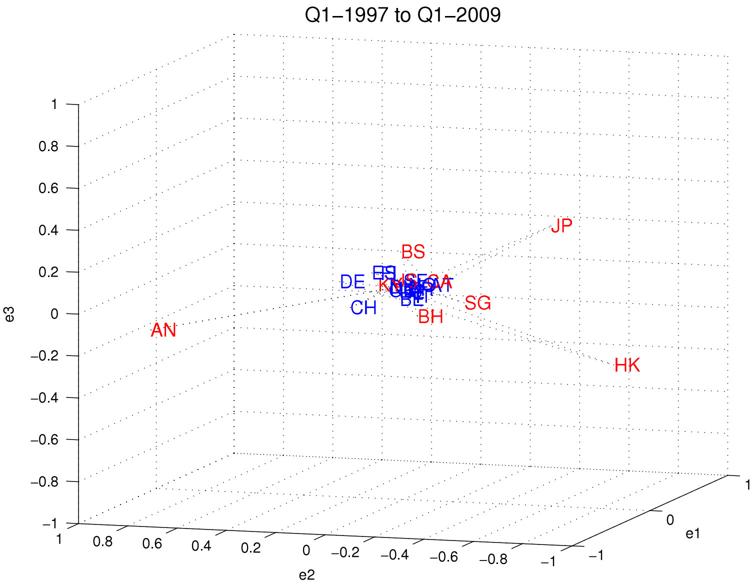

Fig. 5. The three-dimensional space in 1997–2009 for 24 countries.

These ordering changes have an economic meaning, being related to the relative amounts of financial flows of specific groups of countries in different periods. We envision that the *clustering trend* and the distorted shape are due to the shocks that stormed the financial systems of several countries in that period. The 1997–2009 time window besides including the second *Black Monday*, also includes the recent turmoil in the international banking system of the late 2000s.

3.3. The space volume

The observation of the plot presented in Fig. 5 shows that a generalized shortening of most of the distances leads to a contraction effect in the volume of the three-dimensional space. To capture the extent of a contraction (or expansion) effect, we take the largest three eigenvalues that define the effective dimension of the cross-border market space and compute its volume (V) as

$$V_t = \sqrt[3]{\prod_{i=1}^{3} \lambda_t(i)}. \tag{10}$$

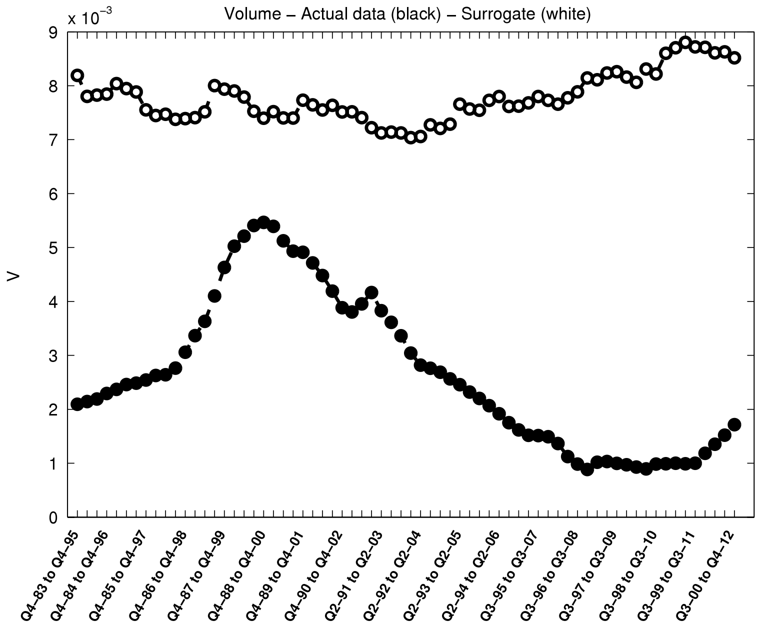

Fig. 6. The evolution of the volume of the reduced three-dimensional spaces over the last 29 years for both actual data (black) and surrogate data (white).

This measure V is used as a reference for the identification of the abnormal periods. The volume expands whenever the cloud of points represents a situation of *business as usual* and the market space is similar to that of a random universe. However, in critical periods, the volume of the geometric object severely contracts, leading to the emergence of distorted shapes [9,10].

As the plot in Fig. 5 clearly indicates, clustering emerges together with a contraction effect in the shape of the reduced space. Such a contraction effect is registered as a lower value of V.

Fig. 6 shows the behavior of V over the 29 years for the actual data and for time permuted observations. The results obtained from actual data are represented with black circles while those obtained from surrogate data are colored white. At each time interval, V is computed from the distances measured over a moving window of length 48 (12 years). The choice of such a wide window ensures statistical robustness and allows for capturing the main differences in the behavior of V when the late 1990s start to be taken into account.

The lowest values in the evolution of V for the period under consideration correspond to the 12-year slot Q3-1996 to Q3-2008. Fig. 6 also shows that the highest values of V correspond to the time slot Q4-1988 to Q4-2000. After that and up to 2010, one observes a persistent decreasing trend in the values of V and a contraction effect in the corresponding three-dimensional space.

The evolution of V confirms our previous results, identifying the major changes in the 29-year period and highlighting the impact of the recent financial crises to the structure of the cross-border interdependencies.

It can be noticed that after 2011, the value of V starts to increase, meaning that a small recovery from the recent crises – translated into a return to the situation of *business as usual* – is taking place.

4. The multivariate coefficients

Having applied the stochastic geometry technique to the set of 24 reporting countries, we concluded that the systematic information related to the interbank market structure is contained in a reduced subspace of just three dimensions, meaning that the number of characteristic dimensions of the geometric space built from this data source is three ($f = 3$). Hence, we denote by $\overrightarrow{z}(k)^{(f)}$ the restriction of the k-asset to the subspace S_f and by $d_{ij}^{(f)}$ the distances restricted to this space. Then using Eqs. (3) and (4) we apply the notion of *systematic covariance* $\sigma_{ij}^{(f)}$ as in Ref. [8]:

$$\sigma_{ij}^{(f)} = \mu_i \sqrt{\sigma_{ii} - \bar{l}_i^2}\, \mu_j \sqrt{\sigma_{jj} - \bar{l}_j^2} \left(1 - \frac{1}{2}\left(d_{ij}^{(f)}\right)^2\right) \tag{11}$$

where $\mu_i = |\overrightarrow{z_i}(k)^{(f)}| / |\overrightarrow{z_i}(k)|$, $\bar{l}_i = \left\langle \overrightarrow{l}(i) \right\rangle$ and $\sigma_{ii} = \left\langle \overrightarrow{l}(i)\, \overrightarrow{l}(i) \right\rangle$.

Given the evidence of contraction and distortion of the geometric spaces in different periods, besides the calculation of the space volume, we proceed to test the deviation from multinormality occurring in the shape of these spaces.

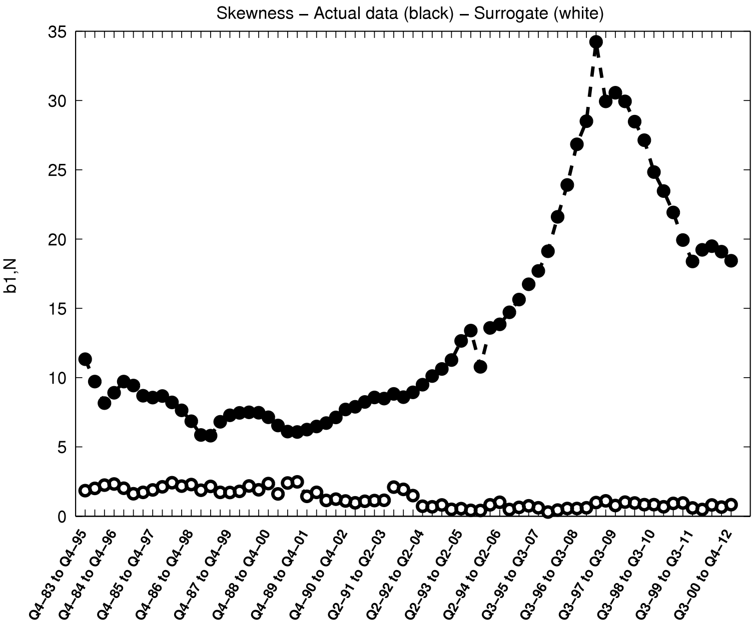

Fig. 7. The evolution of multivariate skewness of the distribution of the coordinates of the countries in the three-dimensional spaces over the last 29 years for both actual data (black) and surrogate data (white).

4.1. Multivariate skewness and kurtosis

The statistical properties that characterize multivariate normality are not as investigated as those applied to univariate normality [23,24]. Even so, a large body of literature was built in the last decades on the topic of multivariate normality [25,26].

Here we propose to consider the available information on the 24 reporting countries in order to apply the heuristic concepts of multivariate skewness (b_1, N) and multivariate kurtosis (b_2, N), such as proposed by Mardia [25]. In this case, they are defined as:

$$b_1, N(k) = \frac{1}{n^2} \sum_i \sum_j [(z_i(k) - \bar{z})(\sigma^{(f)})^{-1}(z_j(k) - \bar{z})]^3 \tag{12}$$

$$b_2, N(k) = \frac{1}{n} \sum_i [(z_i(k) - \bar{z})(\sigma^{(f)})^{-1}(z_i(k) - \bar{z})]^2 \tag{13}$$

where $\sigma^{(f)}$ is the systematic covariance (Eq. (11)), N is the number of variables and n the number of observations. When $\sigma^{(f)}$ is computed, $f = 3$ since three was found to be the number of effective dimensions of the interbank market space.[2]

In the present case, the variables are the three coordinates identifying the effective dimensions of each county, since these coordinates are enough to represent the relevant information about the cross-border market.

In so doing, we are allowing both for considering the whole set of countries, represented by the coordinates of the space they define, and for testing the described dynamics of the interbank market while considering all the available information of the entire interbank market. This is a well established result, remaining the same even when the dimensions of order higher than the third are considered, confirming our previous findings that indicate that the dimensions of higher order essentially represent noise [34]. In each case we performed systematic comparisons with the measures of series of random data obtained from a Gaussian distribution with the same mean and variance of real data.

Plots in Figs. 7 and 8 present the evolution of multivariate skewness $b_1, N(k)$ and multivariate kurtosis $b_2, N(k)$ calculated over a moving window of 48 quarters. Both coefficients were calculated from actual data (black) and from surrogate data (white).

They display an increasing trend with a steeper slope starting in the time slot Q2-1993 to Q2-2003 and reaching the highest value in 2008. After that, the values of both coefficients decrease, indicating that the situation is moving back to the one that characterizes the less turbulent period of the earlier nineties. The relative increase in kurtosis (b_2, N) reaches less than 20% along the first 20 years and after a stationary period that ends in 2005, the kurtosis shows a fast increase of

[2] We have previously considered the work of different authors on the limit distributions of Mardia's tests [27–32]. A detailed discussion on alternative approaches to test multivariate normality can be found in Ref. [33].

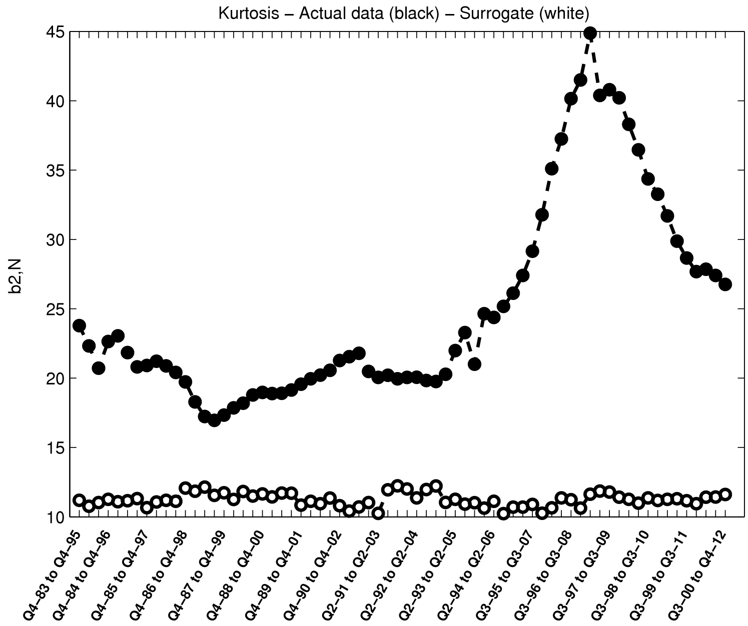

Fig. 8. The evolution of multivariate kurtosis of the distribution of the coordinates of the countries in the three-dimensional spaces over the last 29 years for both actual data (black) and surrogate data (white).

more than 50% in just two years. The increase in skewness (b_1, N) is also strong, with a rise of more than 40% in the last few years. After 2008, however, the two measures – and thus the distorted shapes of the geometric object – start to behave in the opposite way, helping to illustrate how the recent financial crisis affected the interbank market.

This was found to occur in such a systematic fashion that measuring those higher moments of the distributions of the 3*D* coordinates of the countries may be used as a proxy for systemic risk.[3]

5. Concluding remarks

In this paper we have investigated the modifications that may occur in the structure of the cross-border financial interdependencies as a consequence of financial crises. Using time series of interbank liabilities conducted through the international banking system, we have developed geometric spaces of debt positions between 24 countries. These structures were developed for (i) a 29-year period that goes from 1983 to 2012 and two 12-year periods (ii) from 1987 to 1999, and (iii) from 1997 to 2009.

The geometrical analysis of (i)–(iii) implied that most of the systematic information of the interbank international market was contained in a three-dimensional space, from which a multidimensional statistical characterization could be conveniently carried out.

From the geometrical perspective, the reduced subspaces of countries showed that the most eccentric positions are occupied by countries outside the European Union. Moreover, it also showed that comparing (ii) and (iii), the main difference relies on the *clustering trend* that characterizes the later period, where the distances between countries seem to be shortened, except for the few examples of the Netherlands Antilles, Japan and Hong Kong. Such a clustering effect is followed by a contraction of the volume (V) of the corresponding geometrical space. When V is computed over (i), its evolution confirms our previous results, identifying the major changes in the 29-year period and highlighting the relevance of the recent economic crises.

Empirical results allow us to relate the shape of the geometric spaces to the characteristics of some relevant periods of financial turbulence, during which the shape of the ellipsoid is distorted, acquiring prominences in some particular directions.

Our approach is complemented by a statistical analysis of the three-dimensional spaces. Measures of multivariate skewness and kurtosis are applied in order to quantify how far the shape of the geometrical spaces obtained in different time periods deviates from multinormality. These coefficients, together with a measure of the space volume allow for the characterization of systemic risk. It so happens that, in the cross-border market we are dealing with, our results show the existence of a relevant increase in the values of multivariate kurtosis and multivariate skewness of the distributions of

3 These results are in line with those obtained from the application of the same multivariate measures to the evolution of the stock returns [33]. It was shown that a relevant increase in the value of multivariate kurtosis of the stock return distributions is empirically related to periods of turbulence, displaying a completely different behavior in normal periods.

the coordinates of a set of countries once they are embedded in a low dimensional space. The increase in those values is empirically related to periods of financial turbulence.

Hence, the construction of the three-dimensional spaces is twofold: they do not only provide the basis for the characterization of country interdependencies based on the positions of each country in a well defined metric space, as they also allow for the application of statistical coefficients that evaluate deviations from multinormality using the distribution of the coordinates of the reporting countries in those metric spaces.

As the data and the method suggest, such a feature has been part of a mutation in the structure of the cross-border interdependencies since the late 1990s. Results highlight an important modification acting in the financial linkages across countries in the period 1997–2009, and situate the recent financial crises as a counterpart of a larger structural change going on since 1997.

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