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A measure of multivariate kurtosis for the identification of the dynamics of a N-dimensional market

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HIGHLIGHTS

- We apply a stochastic geometry technique to reconstruct geometrical spaces from empirical data (stock returns).
- The shape of the geometrical spaces change with the occurrence of financial crises.
- We use extensions of the concept of kurtosis to measure distortions in the shape of the geometrical spaces.
- Modifications in the value of multivariate kurtosis correspond to changes in the shape of the market spaces.
- In relevant periods, the markets differentiate themselves from a random walk type of behavior.

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ABSTRACT

This paper investigates the common intuition suggesting that during crises the shape of the financial market clearly differentiates from that of random walk processes. In this sense, it challenges the traditional analysis of the nature of financial markets implicit in the most popular models. For this, a geometric approach is proposed in order to define the patterns of change of the market and a measure of multivariate kurtosis is used in order to test deviations from multinormality. The statistical difficulties of this approach are discussed and a new solution is proposed to the consideration of a large space of variables in an accurate measurement of the dynamics of the market. The emergence of crises can be measured in this framework, using all the available information about the returns of the stocks under consideration and not only a single index representing the market.

1. Introduction

The Samuelson theory of asset prices [1] was the basis of the Efficient Market Hypothesis [2–4], which assumes that the stock returns are approximated by a random walk and their changes are unpredictable, since prices are always supposed to fully reflect all the available information. This information is interpreted as such that the marginal benefit of adding new information is not larger than the marginal cost of obtaining it. Therefore, an hypothesis on the risk of the expected return of the stocks is required, given the condition that no investor can manage permanent excess returns and that their opportunities are unpredictable. As one anonymous referee mentioned, this means that a test of the EMH indeed means a joint-hypotheses test, that of an implicit model of returns, namely of adjustments of prices for risk, and the EMH. These joint-hypotheses are falsified by the periods of turbulence and distortion of the shape of the market, as opposed to periods of business-as-usual in which the appearance of randomness dominates, although no definitive statement is allowed as to each of the hypothesis in particular.

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From another point of view, many scholars criticized the theoretical assumptions of the EMH, namely its informational requisites (e.g. Ref. [5]), detected the existence of time dependencies violating the conditions of the theory (e.g. Ref. [6]), and identified the existence of asymmetry and fat tails in the distribution of returns (e.g. Ref. [7]). Consequently, due to these and other discrepancies, the EMH "is respected but not worshiped" in financial theory and statistics [8].

Tools for analysis of deviations from normality have existed for a long time, e.g. the concept of Kurtosis was introduced by Karl Pearson [9] in order to measure the size of the tails of a distribution as compared with those of the normal. It was assumed, by Pearson and then by "Student" (1927), who suggested a curious mnemonic based on the shape of animals in order to describe non-normal distributions, that kurtosis, as well as skewness, are common features of nature.

Fat tails are interpreted as the result of a larger part of the variance being provoked by rare extreme events, as compared to the normal distribution. In this sense, Pesaran discusses the effect of the recent punctuation of crashes – such as the dot-com crash of 2000 and the general financial crash after the subprime crisis of 2007 – and argues that periods of bubbles and crashes deviate from market efficiency [8]. This is precisely the intuition we pursue in this paper, proposing a new approach to the measurement of the dynamics of changes of the distributions representing the stock returns. In the following, we describe the emergence of crises amidst long periods of normal trading and, as we are interested in major changes that occur at the fat tails of the distribution, we use extensions of the concept of kurtosis to the realm of a n-dimensional object.

For this, we proceed as follows: first, we briefly define the concepts used to describe the dynamics of the market; the following section presents the geometry applied to the measurement of the distortion in certain periods; finally the next sections describe the results of the statistics applied to relevant periods, when the market differentiates itself from a random walk type of behavior.

2. The identification of the dynamics of change of the market

In the province of statistical research on financial data, the evidence of fat tails is expressed in typical leptokurtic distributions. Recently, Pesaran [8] proposed an empirical verification of four indexes (S&P, FTSE 100, DAX, NIKKEI 225), for 2000–2009, and found evidence of kurtosis, rejecting the null hypothesis of a normal distribution using a Jarque–Bera test [10]. In the same paper, originated as a contribution to a seminar in honor of Fama, the author mentions the historical data of the monthly returns measured by the S&P, as compiled by Shiller [11]: from 1871 to 2009 there is an impressive evidence of kurtosis, and even when shorter periods are chosen, deviations from normality are typical.

As a consequence of this common empirical evidence of non-normality and asymmetric and heavy tailed distributions of financial data, anticipated by no less than Fama himself (Fama, [2,12]), several adjusted models for skewness and kurtosis have been proposed in financial statistics, namely in the asset pricing applications (e.g., Refs. [13–17]). In this sense, for instance Dufour, Beaulieu and Khalaf proposed the incorporation of asymmetry of the return distribution on asset evaluation [12,18,19].

The problem we address in this paper follows these lines of research but, instead of describing the market with the recourse to a single measurement of an index, we propose to capture the whole available empirical information on the dynamics of a population of stocks through time, considering as a consequence the multivariate process. There are sound reasons for this option since, in general, the statistical experiments and empirical approaches "are multivariate by nature" ([20]: 783).

By using a stochastic geometry technique, we found that the dynamics of the S&P500 set of stocks defines market spaces as low-dimensional entities and that this low-dimensionality is caused by the small proportion of systematic information present in correlations among stocks in normal periods of trade. However, this situation changes dramatically in periods of crashes or crises.

This is verified with extensive data representing different sets of the daily returns, namely that of the 236 S&P500 stocks that remained in the market from 1973 to 2009, and then verified by that of the 471 S&P500 stocks surviving from 2005 to 2009 (see Fig. 1). These dates are chosen to maximize the number of firms included in our population, since for larger periods the numbers will be reduced by bankruptcies and fusions.

These populations are used to study the market dynamics: in both cases, in the subperiods of *business-as-usual*, the geometric object defined by the dynamics of the market approaches the spherical configuration, typical of a Gaussian distribution; conversely, when a subperiod includes relevant crashes, the shape of that geometric object is distorted, acquiring prominences in some particular directions. Moreover, we found that, during crashes, market spaces contract along their effective dimensions. In this, we also follow a definition by R.A. Fisher [21], establishing that, measured in an Euclidean space, the multivariate normal errors are described by the surface of a sphere, and suggesting that, whenever large errors occur, the topological deviations have to be considered.

In order to capture the contracting and distortion effects in the market shape, we measure multivariate kurtosis (b_2, p) as presented in the next section.

3. The measure of the distortion of the space of the market

The strategy of measurement of the space of the financial market follows the strategy suggested by Mantegna [22,23] and developed by some of the authors of this paper, and can be simply stated in the following terms. From the set of returns

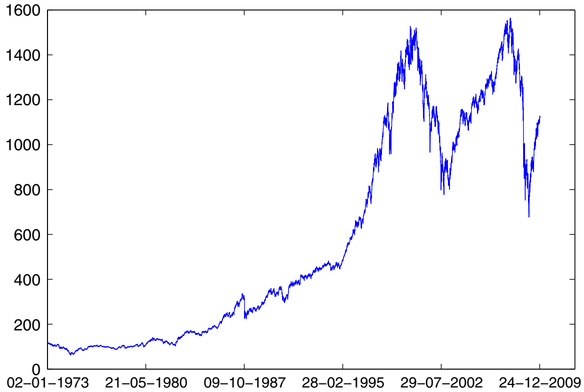

Fig. 1. The evolution of the S&P500 for 1973–2009.

of the stocks and their historical data of returns over the time interval, and using an appropriate metric, we compute the matrix of distances between the stocks. Considering the returns for each stock,

$$r(k) = \log(p_t(k)) - \log(p_{t-1}(k)) \tag{1}$$

a normalized vector

$$\vec{\rho}(k) = \frac{\vec{r}(k) - \langle \vec{r}(k) \rangle}{\sqrt{n\left(\langle r^2(k)\rangle - \langle r(k)\rangle^2\right)}} \tag{2}$$

is defined, where n is the number of components (number of time labels) in the vector $\vec{\rho}$. With this vector the distance between the stocks k and l is defined by the Euclidean distance of the normalized vectors.

$$d_{kl} = \sqrt{2\left(1 - C_{ij}\right)} = \|\vec{\rho}(k) - \vec{\rho}(l)\| \tag{3}$$

with C_{kl} being the correlation coefficient of the returns $r(k)$, $r(l)$.

$$C_{kl} = \frac{\langle \vec{r}(k)\vec{r}(l)\rangle - \langle \vec{r}(k)\rangle\langle \vec{r}(l)\rangle}{\sqrt{\left(\langle \vec{r}^{\,2}(k)\rangle - \langle \vec{r}(k)\rangle^2\right)\left(\langle \vec{r}^{\,2}(l)\rangle - \langle \vec{r}(l)\rangle^2\right)}}. \tag{4}$$

As the distance is properly defined according to the due metric axioms, it is possible to obtain, from the matrix of distances, the coordinates for the stocks in a Euclidean space of dimension smaller than N. The standard analysis of reduction of the coordinates is applied to the center of mass and the eigenvectors of the inertial tensor are then computed.

The same technique is also applied to surrogate (time-permuted and random) data, namely to data obtained by independent time permutation for each stock, and these eigenvalues are compared with those obtained from actual data n order to identify the characteristic directions for which the eigenvalues are significantly different. They define a reduced subspace of dimension f, which carries the systematic information related to the market correlation structure.

This corresponds to the identification of empirically constructed variables that drive the market and, in this framework, the number of surviving eigenvalues is the effective characteristic dimension of this economic space (f). This procedure is the key for the following method, since it allows for the consideration of populations of hundreds of stocks, given that only a very small number of coordinates describing their distances is used in the computation of our measures of the multivariate space.

This suggests the definition of a *systematic covariance*. For this, we denote by $\vec{z}(k)^{(f)}$ the restriction of the k-asset to the subspace V_f. and by $d_{kl}^{(f)}$ the distances restricted to this space. Then using Eqs. (3) and (4) we may define a notion of *systematic covariance* $\sigma_{kl}^{(f)}$

$$\sigma_{kl}^{(f)} = \mu_k\sqrt{\sigma_{kk} - \bar{r}_k^2}\,\mu_l\sqrt{\sigma_{ll} - \bar{r}_l^2}\left(1 - \frac{1}{2}\left(d_{kl}^{(f)}\right)^2\right) \tag{5}$$

where $\mu_k = |\vec{z}(k)^{(f)}|/|\vec{z}(k)|$, $\bar{r}_k = \langle \vec{r}(k)\rangle$ and $\sigma_{kk} = \langle \vec{r}(k)\vec{r}(k)\rangle$.

The plots of Fig. 2 show that while in the first days of January 2007 there is no relevant difference in relation to surrogate (time-permuted) data, new shapes emerge in March 2007 as well as in November 2008, two periods of strong turbulence in the financial markets, indicated by a distortion in the shape of the sphere.

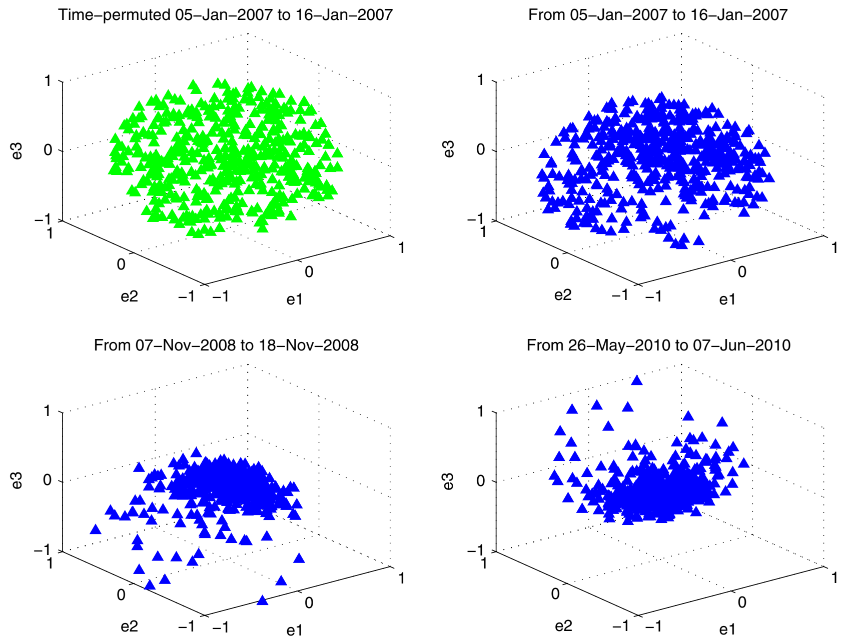

Fig. 2. As Fig. 2 clearly indicates, in periods of normal trade the spherical configuration is maintained, similar to that of surrogate data, whereas in periods of turbulence new shapes emerge.

4. A test based on multivariate kurtosis

Given this evidence of contraction and distortion of the market space in different periods, we proceed to test deviation from normality. The computation of univariate measures and tests of deviations of general indexes from normality is a common procedure (e.g., Ref. [8]). Instead, we propose to consider the available information on the richest detail of a large population of stocks. For this, we recur to heuristic concepts and a test of multivariate kurtosis (b_2, p), such as proposed by Mardia [24]. In this case, multivariate kurtosis is defined as

$$b_2, p(t) = \frac{1}{N} \Sigma_i [(z_i(t) - \bar{z})(\sigma^{(f)})^{-1}(z_i(t) - \bar{z})]^2 \tag{6}$$

where $\sigma^{(f)}$ is the systematic covariance, p is the number of variables and N the number of observations. In the calculation of $\sigma^{(f)}$ $f = 6$ since six was found [25,26] to be the number of effective dimensions of the S&P500 market space.

Although the statistical properties of tests of multivariate normality are not as established as those applied to univariate normality [27,28], a large body of literature was built in the last decades on the topic. Mardia [24,29,30], using Arnold's results [31] proposed the affine invariant measures as previously discriminated, established their asymptotic distributions and formulated tests for the null of multivariate normality. Different scholars discussed the limit distributions of Mardia's tests [32–35] and investigated their consistency [36,37]. Others, such as Koziol [38–41], Srivastava [42] and Henze [43], proposed alternative approaches to test multivariate normality.

Several authors surveyed the available procedures and tests of multivariate normality and identified more than fifty alternatives, although only some qualified as generally accepted methods [44–49]. Through the comparison of the power of the different tests, using extensive Monte Carlo simulations, some of these authors argued that the Mardia tests have low power [49] in particular against the BHEP test proposed by Henze and his collaborators [46,47], whereas others obtained an opposite conclusion, favoring the Mardia test [12]. Bai and Ng, considering the fact that the sampling distributions of these coefficients is not well known for serially correlated data, proposed a strategy of generalization of the Jarque–Bera test in order to account for these problems [50,51].

Considering the stationarity of our series of returns, by construction, we are nevertheless confronted with evidence of serial and cross correlation. It is well known that daily returns tend to be negatively serially correlated, that their statistical significance tends to be greater in periods of unrest and that their cross correlation increases with volatility [8]. A novel approach to deal with this problem, considering serial correlation in the residuals of overlapping observations, was proposed recently by Pesaran and his colleagues, using a new version of a seemingly unrelated regression equations based estimation [52]. Furthermore, there is evidence of cross correlation, which has been rarely discussed in the framework of multivariate analysis, with some exceptions (e.g., Ref. [53]).

Considering these suggestions, we checked our data applying the Mardia measure to the stocks in our population, for each period, in order to diagnose deviations from normality and to describe the dynamics of the market in different periods.

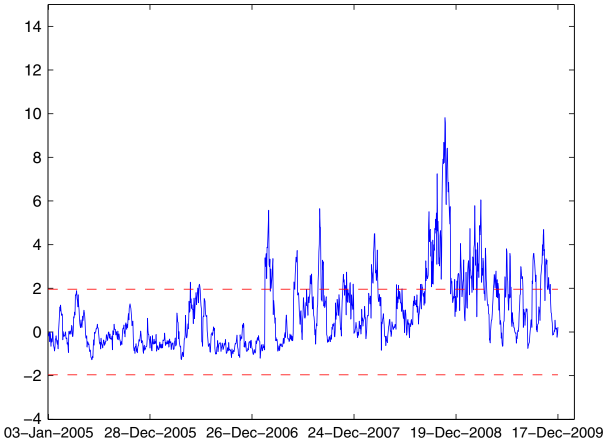

Fig. 3. The evolution of $g(t)$ from 2005 to 2009.

In each case we proceeded to systematic comparisons with the measures of series of random data obtained from a Gaussian distribution with the same average and variance as in our population.

Mardia's test of multivariate normality is performed in order to determine if the null hypothesis of multivariate normality is a reasonable assumption regarding the population distribution of a random sample.

Mardia proved that, under the null hypothesis, the statistics

$$t2 = \frac{b_2, p(t) - (p^2 + 2p)}{\sqrt{\frac{8p^2+16p}{N}}} \tag{7}$$

is asymptotically distributed as a standard normal. Since our aim is not only to test the data but essentially to distinguish the periods of *business-as-usual* from the periods of crises and, in that sense, to test the distortions in the dynamics of the markets, we replace the expected value and standard deviation used in Mardia's standardization with the empiric counterparts as obtained from the observed values of the statistics in a *business-as-usual* period. In this sense, we consider *business-as-usual* periods 1973–1995 for the longer series and January 2005–June 2007 for the shorter series. The modified statistics is then

$$g(t) = \frac{b_2, p(t) - \overline{b_2, p(t)}}{\widehat{\sigma}(b_2, p(t))} \tag{8}$$

where $\overline{b_2, p(t)}$ and $\widehat{\sigma}(b_2, p(t))$ are the estimated values of, respectively, the mean and the standard deviation. In this case, our variables are the six coordinates identifying the relevant dimensions, which represent the relevant information about the market. This is a robust result, unaltered even when the next dimensions are considered, confirming our previous result indicating that such dimensions essentially represent noise. Even if the Mardia test – or other tests on multivariate normality – cannot be applied to a very large number of variables, since the asymptotic properties are not known for those cases, this strategy allows both for considering a large population (236 and 471 firms), represented by the coordinates of the space they define, and to test the described dynamics of the market, considering all the available information and not just a single index averaging through the market.

5. Results and discussion

In previous empirical work, we found a robust result: markets of different sizes, ranging from 70 to 424 stocks across different time windows (from one year to 35 years), and using different market indexes for different markets, may be described by six effective dimensions [25,26]. A striking characteristic of the data is also that, as expected, whenever the market suffers a crash, there is a contracting effect provoking a clear distortion in the dominant directions of the market space. If the volume expands whenever the cloud of points represents a situation of *business-as-usual* and the market space is similar to that of a random universe, whenever a crisis occurs the volume of the geometric object severely contracts, leading to the emergence of characteristic and distorted shapes.

Fig. 3 describes the evolution of $g(t)$ for 2005–2009 and Fig. 4 the same statistic for the larger period of 1973–2009, and the typical limits of the interval corresponding to a level of significance of 5% are indicated. The hypothesis of normality is clearly rejected in the periods of turbulence, as expected.

In Fig. 5 the results of the test for random data with the same standard deviation and expected value as the true data are presented, highlighting by comparison the presence of structure in the market described by our data.

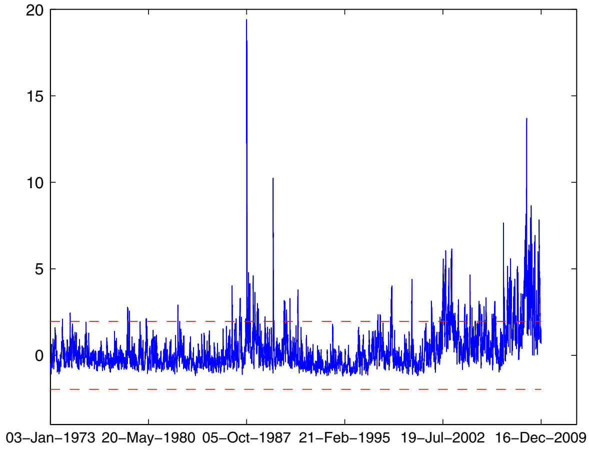

Fig. 4. The evolution of $g(t)$ from 1973 to 2009.

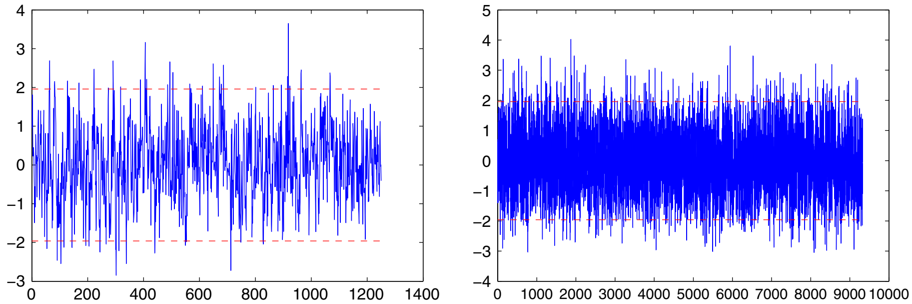

Fig. 5. The evolution of $g(t)$ corresponding to random data with the same mean and standard deviation as in Fig. 3 (l.h.s.) and Fig. 4 (r.h.s.)

6. Conclusion

The multivariate analysis has rarely been applied to stock market series. In this paper, we propose a novel approach to that application, which permits the consideration of the evolution of a large population of stocks for different periods and describes the deformations in the geometry of the market, while discussing some of the technical difficulties involved in this enterprise. This is possible since the evolution of the distances among all the firms is encapsulated by a small manifold of coordinates in the market space. Although this presents some statistical difficulties we are addressing in the current research, this avenue favors a new approach to the multidimensional objects constructed by the dynamics of complex markets and interactions among many firms and decisions.

As it was previously found, during periods of financial turbulence the shape of the market changes dramatically, whereas in periods of normal business it resembles the spherical form typical of a random distribution. This result is strongly confirmed by our data. A moment, kurtosis, is used in order to measure the distortions of the distribution, and the results highlight how the distances among stocks contract in periods of unrest. Given this, the hypothesis of well behaved random distribution of the stock returns can be challenged and should be challenged. It does not correspond to the state of nature during a crisis or an episode of turbulence in the financial markets we investigated.

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References

[1] P. Samuelson, Proof that properly anticipated prices fluctuate randomly, Industrial Management Review 6 (1965) 41–49.
[2] E.F. Fama, The behaviour of stock prices, Journal of Business 60 (1965) 401–424.
[3] E.F. Fama, Efficient capital markets: a review of theory and empirical work, Journal of Finance 25 (1970) 383–420.
[4] E.F. Fama, Efficient capital markets: II, Journal of Finance 46 (1991) 1575–1617.
[5] S.J. Grossman, J. Stiglitz, On the impossibility of informationally efficient markets, American Economic Review 70 (1980) 393–408.

[6] A. Lo, A.C. MacKinley, Stock prices do not follow random walks: evidence from a simple specification test, Review of Financial Studies 1 (1988) 41–66.
[7] V. Plerou, P. Gopikrishnan, X. Gabaix, L.A. Amaral, H.E. Stanley, Price fluctuations, market activity and trading volume, Quantitative Finance 1 (2001) 262–269.
[8] M.H. Pesaran, Predictability of asset returns and the efficient market hypothesis, IZA Discussion Papers, Institute for the Study of Labor, 2010.
[9] K. Pearson, Das Fehlergesetz und Seine Verallgemeiner-ungen Durch Fechner und Pearson, a Rejoinder, Biometrika 4 (1905) 169–212.
[10] C.M. Jarque, A.K. Bera, A test for normality of observations and regression residuals, International Statistical Review 55 (1987) 163–172.
[11] R. Shiller, 2011. http://www.econ.yale.edu/~shiller/data.htm.
[12] J.-M. Dufour, L. Khalaf, M.-C. Beaulieu, Exact Skewness–Kurtosis tests for multivariate normality and goodness-of-fit in multivariate regressions with application to asset pricing models, Oxford Bulletin of Economics and Statistics 65 (2003) 891–906.
[13] R. Jarrow, A. Rudd, Approximate option valuation for arbitrary stochastic processes, Journal of Financial Economics 10 (1982) 347–369.
[14] C. Corrado, T. Tsu, Skewness and Kurtosis in S&P500 index returns implied by option prices, Journal of Financial Research 19 (1996) 175–192.
[15] C. Brown, D. Robinson, Skewness and Kurtosis implied by option prices: a correction, Journal of Financial Research 25 (2002) 279–282.
[16] S. Vahamaa, Skewness and Kurtosis adjusted Black–Scholes model: a note on hedging performance, Finance Letters 1 (2003) 6–12.
[17] J. Iqbal, R. Brooks, D. Galagedera, Multivariate tests of asset pricing: simulation evidence from an emerging market, Applied Financial Economics 20 (2010) 381–395.
[18] M.-C. Beaulieu, J.-M. Dufour, L. Khalaf, Exact multivariate tests of asset pricing models with stable asymmetric distributions, in: Numerical Methods in Finance, Kluwer, 2005.
[19] M.-C. Beaulieu, J.-M. Dufour, L. Khalaf, Finite sample multivariate tests of asset pricing models with coskewness, Computational Statistics and Data Analysis 53 (2009) 2008–2021.
[20] R. Liu, J. Parelius, K. Singh, Multivariate analysis by data depth: descriptive statistics, graphics and inference, Annals of Statistics 27 (1999) 783–840.
[21] R.A. Fisher, Dispersion on a sphere, Proceedings of the Royal Society 217 (1953) 295–305.
[22] R.N. Mantegna, Hierarchical structure in financial markets, The European Physics Journal B 11 (1999) 193–197.
[23] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, United Kingdom, 2000.
[24] K.V. Mardia, Measures of multivariate Skewness and Kurtosis with applications, Biometrika 57 (1970) 519–530.
[25] T. Araújo, F. Louçã, The geometry of crashes—a measure of the dynamics of stock market crises, Quantitative Finance 7 (2007) 63–74.
[26] T. Araújo, F. Louçã, The seismography of crashes in financial markets, Physics Letters A 372 (4) (2008) 429–434.
[27] K.V. Mardia, J. Kent, J. Bibby, Multivariate Analysis, Academic Press, New York, 1979.
[28] R. Gnanadesikan, Methods for Statistical Data Analysis of Multivariate Observations, Wiley, New York, 1997.
[29] R.V. Mardia, Applications of some measures of multivariate skewness and kurtosis in testing normality and robustness studies, Sankhyā B 36 (1974) 115–128.
[30] K.V. Mardia, M. Kanazawa, The null distribution of multivariate Kurtosis, Communications in Statistical—Simulation and Computation 12 (1983) 569–576.
[31] H.J. Arnold, Permutation support for multivariate techniques, Biometrika 51 (1964) 65–70.
[32] S.J. Schwager, Multivariate Skewness and Kurtosis, in: Encyclopaedia of Statistical Sciences, Vol. 6, Wiley, New York, 1985.
[33] L. Baringhaus, N. Henze, Limit distributions for Mardia's measure of multivariate Skewness, Annals of Statistics 20 (1992) 1889–1902.
[34] T. Kariya, E. George, LBI tests for multivariate normality in curved families and Mardia's test, Sankhyā 57 (1995) 440–451.
[35] Y. Zhao, S. Konishi, Limit distributions of multivariate Kurtosis and moments under watson rotational symmetric distributions, Statistics and Probability Letters 32 (1997) 291–299.
[36] L. Baringhaus, N. Henze, A consistent test for multivariate normality, Metrika 35 (1988) 339–348.
[37] N. Henze, On Mardia's Kurtosis test for multivariate normality, Communications in Statistics—Theory and Methods 23 (1994) 1031–1045.
[38] J. Koziol, Exact slopes of certain multivariate tests of hypotheses, Annals of Statistics 6 (1978) 546–558.
[39] J. Koziol, A class of invariant procedures for assessing multivariate normality, Biometrika 69 (1982) 423–427.
[40] J. Koziol, On assessing multivariate normality, Journal of the Royal Statistical Society: Series B 45 (1983) 358–361.
[41] J. Koziol, Probability plots for assessing multivariate normality, Journal of the Royal Statistical Society, Series D 42 (1993) 161–173.
[42] M.S. Srivastava, A measure of Skewness and Kurtosis and a graphical method for assessing multivariate normality, Statistics and Probability Letters 2 (1984) 263–267.
[43] N. Henze, B. Zirkler, A class of invariant and consistent tests for multivariate normality, Communications in Statistics—Theory and Methods 19 (1990) 3595–3617.
[44] R. Horswell, S. Looney, A Comparison of tests for multivariate normality that are based on measures of multivariate Skewness and Kurtosis, Journal of Statistical Computation and Simulation 42 (1992) 21–38.
[45] S.W. Looney, How to use tests for univariate normality to assess multivariate normality, The American Statistician 49 (1995) 64–70.
[46] C. Mecklin, D. Mundfrom, An appraisal of bibliography of tests for multivariate normality, International Statistical Review 72 (2004) 123–138.
[47] C. Mecklin, D. Mundfrom, A Monte Carlo comparison of the type I and Type II error rates of tests of multivariate normality, Journal of Statistical Computation and Simulation 75 (2005) 93–107.
[48] G. Szekely, M. Rizzo, A new test for multivariate normality, Journal of Multivariate Analysis 93 (2005) 58–80.
[49] P. Farrell, M. Salibian-Barrera, K. Naczk, On tests for multivariate normality and associated simulation studies, Journal of Statistical Computation and Simulation 77 (2007) 1065–1080.
[50] J. Bai, S. Ng, Tests for Skewness, Kurtosis and normality for time series data, Journal of Business and Economic Statistics 23 (2005) 49–60.
[51] J. Doornik, H. Hansen, An omnibus test for univariate and multivariate normality, Oxford Bulletin of Economics and Statistics 70 (2008) 927–939.
[52] M.H. Pesaran, A. Pick, A. Timmermann, Variable selection, estimation and inference for multi-period forecasting problems, Journal of Econometrics 64 (2011) 173–187.
[53] M. Richardson, T. Smith, A test for multivariate normality in stock returns, Journal of Business 66 (1993) 295–321.