

Análise Matemática III

LISTA 2

- (1) Esboce detalhadamente os seguintes conjuntos:
- (a) $S = \{(x, y) \in \mathbb{R}^2: 0 < x < 1, x^2 < y < 2x^2\}$.
 - (b) $S = \{(x, y) \in \mathbb{R}^2: -1 < y < 1, -\sqrt{1-y^2} < x < 2y^2 - 1\}$.
 - (c) $S = \{(x, y, z) \in \mathbb{R}^3: 0 < x < 1, x^2/2 < y < x^2, 0 < z < x^2\}$.
 - (d) $S = \{(x, y, z) \in \mathbb{R}^3: 0 < x < 1, 0 < y < 1 - x, 0 < z < x + y\}$.
 - (e) $S = \{(x, y, z) \in \mathbb{R}^3: x^2 + y^2 + z^2/4 < 1\}$.
 - (f) $S = \{(x, y, z) \in \mathbb{R}^3: x^2 + y^2 < z^2 + 1\}$.
- (2) Considere as variedades seguintes e determine as suas dimensões e espaços tangente e normal no ponto p :
- (a) $M = \{(x, y, z) \in \mathbb{R}^3: x^2 + y^2 = 1, z = x^2 - y^2\}, p = (1, 0, 1)$
 - (b) $M = \{(x, y, z) \in \mathbb{R}^3: x^2 + y^2 = z^2 + 1, 0 < z < 2\}, p = (0, \sqrt{2}, 1)$
- (3) Determine os extremos de f em M :
- (a) $f(x, y) = x, M = \{(x, y) \in \mathbb{R}^2: x^2 + 2y^2 = 3\}$.
 - (b) $f(x, y) = x^2 + y^2, M = \{(x, 2) \in \mathbb{R}^2: x \in \mathbb{R}\}$.
 - (c) $f(x, y) = x^2 - y^2, M = \{(x, \cos x) \in \mathbb{R}^2: x \in \mathbb{R}\}$.
 - (d) $f(x, y, z) = x, M = \{(x, y, z) \in \mathbb{R}^3: x^2 + y^2 = 2, x + z = 1\}$.
- (4) Determine e classifique os extremos de f em M :
- (a) $f(x, y) = xy, M = \{(x, y) \in \mathbb{R}^2: x^2 + y^2 = 2a^2\}$
 - (b) $f(x, y) = x^{-1} + y^{-1}, M = \{(x, y) \in \mathbb{R}^2: x^{-2} + y^{-2} = a^{-2}\}$
 - (c) $f(x, y, z) = x + y + z, M = \{(x, y, z) \in \mathbb{R}^3: (1/x) + (1/y) + (1/z) = 1\}$
 - (d) $f(x, y, z) = xyz, M = \{(x, y, z) \in \mathbb{R}^3: x + y + z = 5, xy + xz + yz = 8\}$
 - (e) $f(x, y, z) = x + y, M = \{(x, y, z) \in \mathbb{R}^3: xy = 16\}$
 - (f) $f(x, y, z) = x^2 + 2y - z^2, M = \{(x, y, z) \in \mathbb{R}^3: 2x - y = 0, x + z = 6\}$
- (5) *Calcule o espaço tangente e o espaço normal num ponto do gráfico de uma função $f: W \rightarrow \mathbb{R}^{n-m}, C^1$, com $W \subset \mathbb{R}^m$ aberto.