

Análise Matemática III

LISTA 9

(1) Indique quais as funções simples e calcule os seus integrais¹:

(a) $f = \chi_{[1,+\infty[} + \chi_{]-\infty,-1]}$

(b) $f = 2\chi_{[0,+\infty[} - 3\chi_{]1,+\infty[}$

(c) $f(x) = \begin{cases} x^{-1}, & x \in \mathbb{N} \\ 0, & \text{c.c.} \end{cases}$

(d) $f(x) = \begin{cases} x, & x^{-1} \in \mathbb{N} \\ 0, & \text{c.c.} \end{cases}$

(e) $f(x, y) = \begin{cases} 1, & 0 \leq x \leq 1/2, 0 \leq y \leq 1 \\ -3, & 1/2 < x < 1, 0 \leq y \leq 1/2 \\ 2, & 1/2 < x < 1, 1/2 < y < 1 \\ 0, & \text{c.c.} \end{cases}$

(f) $f = \sum_{k \in \mathbb{N}} (-1)^k k^{-1} \chi_{]0,1/k]}$

(g) $f(x) = [x] \chi_{[-100,100]}$

(h) $f(x, y) = ([x] + [y]) \chi_{[0,2] \times [0,2]}(x, y)$

(i) $f(x, y) = \left(\left[\frac{3}{1+x} \right] \chi_{]0,2[}(y) - \left[\frac{2}{1+y} \right] \chi_{]0,3[}(x) \right) \chi_{]0,+\infty[\times]0,+\infty[}(x, y)$

¹Note que $[x]$ é a parte inteira de x , i.e. $[x] = \sup\{k \in \mathbb{Z}: k \leq x\}$.