

15 Jan 2009

1.
 - a) $\dim M = 2, \dim N = 2, \dim M \cap N = 1$
 - b) $p = (x, y, z), T_p(M \cap N) = \text{span}\{(x/y(y+v), v, 1)\}, v = -(2x^2 - 1)y/(2x^2 - 2y^2), T_p(M \cap N)^\perp = \text{span}\{(-1/x, 1/y, 1), (2x, -2y, -1)\}$
2.
 - a) $(0, 1/2, 0)$
 - b) $\pi^{1/2}$
 - c) mais perto: $(2 - 2a, 3 - 3a, 4, 4a)$, mais longe: $(2 + 2a, 3 + 3a, 4 + 4a)$;
 $a = 29^{-1/2}$
3.
 - a) 78π
 - b) 0
4. não é aditiva
5.
 - a) 2

30 Jan 2009

- 1.
- b) $\phi(\varphi) = (\cos \varphi, \cos \varphi, \sqrt{2} \sin \varphi), \varphi \in]-\pi/2, \pi/2[$
2.
 - a) $\sqrt{3}(e^{2\pi} - 1)$
 - b) $e^{2\pi-1}$
3. $x = y = z = 1$
4.
 - a) 0
 - b) 0
5. $\{\emptyset, \Omega, A_0, A_1, A_0^c, A_1^c, A_0 \cup A_1, (A_0 \cup A_1)^c\}, \mu(B) = \#B/\#\Omega$
6.
 - a) 0
 - b) $2 + f(0)$

6 Jan 2010

1.
 - a) $\dim M = 3$
 - b) $T_p M = \{(x, y, z, w) : x + w = 0\}, T_p M^\perp = \{(x, 0, 0, w) : x - w = 0\}$
2.
 - a) mais perto $(2/3, 0, 2/3)$, mais longe $(2, 0, -2)$
 - b) $(0, 1/2, 0)$
 - c) $1/2$
 - d) $(5 - \cos 2)/8$
3.
 - a) $\{\emptyset, \Omega, A_0, A_1, A_0 \cup A_1, A_0^c, A_1^c, (A_0 \cup A_1)^c\}$

27 Jan 2010

1.
 - a) $\theta \neq 0, \dim M_\theta = 2$
 - b) $T_p M = \text{span}\{(1, 0, 0, -1), (0, \theta, 1, 0)\}, T_p M^\perp = \text{span}\{(0, -1, \theta, 0), (1, 0, 0, 1)\}$
2.
 - b) $1/(e^\pi - 1) + \pi^3/24 - 1$
- 3.

a) $]0, 1[\times] - 1, 0[\times] 0, 2[$

c) $\pi/2$

4.

a) $0, m(E)$, decrescente

b) sse $m(f^{-1}(a)) = 0$

5 Jan 2011

2.

a) $2\pi(1 - 1/e)$

b) $45/56$

c) 2π

3. $(1 - e^{-1})/16$

4. $\pi\alpha^2/6$

5.

b) $1/2$

26 Jan 2011

1.

a) $(0, 3/8, 0)$

b) $\sqrt{\pi}$

c) $(-1, 0, 0)$

2. $\sqrt[4]{2}$

3. 0

4.

b) $e^{-\lambda}(1 + 2\lambda + 3\lambda^2/2)$

c) $(1 - e^{-\lambda})/\lambda$

6. integrável

9 Jan 2012

2.

a) $T_p M = \text{span}\{(1, 1, \sqrt{2}), (1, -1, 0)\}$, $T_p M^\perp = \text{span}\{(1, 1, -\sqrt{2})\}$

b) $\sqrt{2}/3$

3.

a) $2R/3$

b) $x^2 + y^2 = 1/2, z = 1/2$

c) $\frac{3}{8} \frac{R^2 - r^2}{R^{3/2} - r^{3/2}}$

4.

a) 3

b) $1 + 1/4 + 1/9$

25 Jan 2012

2.

a) $3R/4$

b) $(0, 3/4, 0)$

c) $\sqrt{\pi}$

3.

a) $(2x, 2y, -1)/\sqrt{4z + 1}$

b) 0

4.

a) 55

b) 10

11 Jan 2013

- 1.
- b) $2\pi^2$
- 2.
- b) $\frac{\pi}{2} \frac{e-1}{e^2}$
- 3.
- a) $3R/4$
- b) $(1, 1, 1, 0)$
- 4.
- a) $\dim=2$
- b) $(0, 0, \pm 1)$ em $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$ e $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 1\}$, $\pm(x, y, 0)$ em S , $(0, \pm 1, 0)$ em $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$
- c) $\pi/2$

31 Jan 2013

- 1.
- a) $\gamma(t) = (\sin(t), \sin(2t), 0)$, $t \in [0, 2\pi]$, $\Gamma = \gamma([0, 2\pi])$ não é uma variedade
- b) $p = \gamma(\pi/2) = (1, 0, 0)$, $T_p\Gamma = \text{span}\{(0, 1, 0)\}$, $T_p\Gamma^\perp = \text{span}\{(1, 0, 0), (0, 0, 1)\}$
- 2.
- b) $(\pi/2)^3$
- 3.
- a) $(3/2, 3/2)$
- b) $2\sqrt{R}/3$
- 4.
- a) $\dim=2$
- b) $(0, 0, \pm 1)$ em $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$ e $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : z = 1/2\}$, $(0, \pm 1, 0)$ em $\bar{D} \cap \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$, $\pm(x, y, 1-z)/[\sqrt{2}(1-z)]$ em S
- c) $\pi/4$

13 Jan 2014

- 1.
- a) $\dim=1$
- b) $T_{(1,1)}M^\perp = \text{span}\{(1, 0)\}$, $T_{(1,1)}M = \text{span}\{(0, 1)\}$
- 2.
- a) $(1 - \sqrt{30}/30)(1, 2, 3, 4)$
- b) $\sqrt{\pi}$
- 3.
- a) A componente com $x > 0$
- b) 1
- c) sim
- d) $3/16$

27 Jan 2014

- 1.
- a) $\dim=2$
- b) $T_{(1,1,1)}H^\perp = \text{span}\{(1, 1, -1)\}$, $T_{(1,1,1)}H = \text{span}\{(1, 0, 1), (0, 1, 1)\}$
- c) $(\sqrt{10}/5, 2\sqrt{10}/5, 1)$
- 2.
- a) $1/2$
- b) $(0, 0)$
- 3.

a) Em $\{(x_1, x_2, x_3) \in \mathbb{R}^3: x_1 = \pm 1, \max_{i=2,3} |x_i| \leq 1\}$, $\nu(x) = (\pm 1, 0, 0)$;
 em $\{(x_1, x_2, x_3) \in \mathbb{R}^3: x_2 = \pm 1, \max_{i=1,3} |x_i| \leq 1\}$, $\nu(x) = (0, \pm 1, 0)$; em
 $\{(x_1, x_2, x_3) \in \mathbb{R}^3: x_3 = \pm 1, \max_{i=1,2} |x_i| \leq 1\}$, $\nu(x) = (0, 0, \pm 1)$

b) 0

4.

b) 0

5. $2^{\#\Omega}$

12 Jan 2015

1.

a) $\dim=2$, $T_{(x,y,z)}M^\perp = \text{span}\{(x, y, z)\}$, $T_{(x,y,z)}M = \text{span}\{(-y, x, 0), (-z, 0, x)\}$

b) $\max_M f = \frac{1}{4}$, $\min_M f = \frac{1}{12}$

c) $3\pi/5$

2.

a) $\phi^{-1}(u, v) = (\sqrt{uv}, \log \sqrt{\frac{u}{v}})$

c) $\pi/2$

3.

a) $\{\emptyset, \mathbb{R}, \{\alpha\}, \{\alpha\}^c\}$

b) $\{\emptyset, \mathbb{R}\}$

4.

a) $\mu(D)$

26 Jan 2015

1.

a) $\text{div} \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$, $\nu(x, y, z) = (x, y, z)$

b) 2π

c) $(1, 0)$ min local, $(-1, 0)$ min global, $(1/2, \pm\sqrt{3}/2)$ max global

2.

a) $\phi^{-1}(u, v) = (\sqrt{uv}, \log \sqrt{\frac{u}{v}})$

c) $1/\pi$

3.

b) 0

4. $\mu = \sum_{n=1}^{10} \delta_{1/n}$

15 Jan 2016

1.

a) $\dim M = 2$, $T_{(x,y,z)}M = \text{span}\{(z, 0, x), (0, z, y)\}$, $T_{(x,y,z)}M^\perp = \text{span}\{(x, y, -z)\}$

b) Não existem

c) 0

2.

a) $(5 - \cos 2)/8$

b) $\sqrt{\pi}$

3.

a) $\sqrt{2}\pi e^{-\pi^2/4}$

b) 0

4.

b) 0

5. 2

2 Fev 2016

1.

a) $S: (x, y, 0)$; $V_1: (0, 0, 1)$; $V_2: (0, 0, -1)$

- b) 0
 c) S : não existem; C : $(\sqrt{2}/2, \sqrt{2}/2, 0)$ max, $(-\sqrt{2}/2, -\sqrt{2}/2, 0)$ min
 d) $1/2$
 e) S : $1/2$; C : 0

2.
 a) $\sqrt{2}\pi$
 b) $e - 1/e$
 3. 0
 4. constante
 5. 0

9 Jan 2017

1.
 a) $\dim M_2 = 1$, $T_{(x,y)}M_2 = \text{span}\{(-4y, x)\}$
 b) Para $(0, 1)$

$$\phi(t) = \begin{cases} (2 \cos(t/2 + \pi/4), \sin(t/2 + \pi/4)), & 0 < t \leq \pi/2 \\ (\cos t, \sin t), & \pi/2 < t < 3\pi/4 \end{cases}$$

Mesma ideia para $(0, -1)$

- c) $\dim M = 1$,

$$T_{(x,y)}M = \begin{cases} T_{(x,y)}M_2, & (x, y) \in M_2 \\ \text{span}\{(-y, x)\}, & (x, y) \in M_1 \setminus \{(0, 1), (0, -1)\} \\ \text{span}\{(1, 0)\}, & (x, y) \in \{(0, 1), (0, -1)\} \end{cases}$$

- d) $(4\sqrt{5}/5, \sqrt{5}/5)$
 e) $3\pi/2$
 2.
 a) $1/2 + \sin^2 1$
 b) $\sqrt{\pi}$
 3.
 a) $3/2$
 b) $\log 2$
 c) 1