LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics II

Undergraduate Degrees in Economics and Management
Second Exam, June 25, 2015

1. Show that $\lambda = 2$ is an eigenvalue of the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ and compute the corresponding eigenvectors.

2. Let Ω be the domain of $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = \left(\dfrac{\sqrt{1-x^2}}{\ln(x-y)}, \dfrac{y}{x^2+y^2} \right)$. Determine Ω analytically and sketch a graphical representation. Briefly justify that Ω is not open nor convex or compact.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$

(a) Compute $\dfrac{\partial f}{\partial x}(x,y)$, for all $(x,y) \in \mathbb{R}^2$.

(b) Show that $\dfrac{\partial f}{\partial x}$ is continuous at $(0,0)$.

(c) If $f(a,b) \neq 0$, the partial elasticity of f with respect to y is defined by $\mathrm{El}_y f(a,b) = \dfrac{b}{f(a,b)} \cdot \dfrac{\partial f}{\partial y}(a,b)$. Compute $\mathrm{El}_y f(1,1)$.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a C^1 function and consider $g(x,y) = f(x^2y^2, y \sin x)$. Compute $\nabla g(0,0)$.

5. Consider the function $f(x,y) = \dfrac{1}{2} + \dfrac{x^2}{2} + \dfrac{y^2}{2} - x^2 y$.

(a) Determine and classify all critical points of f.

(b) Justify that f attains a global maximum and minimum over $M = \{(x,y) : x^2 + 2y^2 \leq 4\}$ and determine them.

6. Compute $\displaystyle\iint_S (1 + x^2 y)\, dx\, dy$, where $S = \{(x,y) \in \mathbb{R}^2 : x \leq y \leq \sqrt{x}\}$.

7. Consider the differential equation $y''(x) + 2y'(x) - 3y(x) = 2x$.

(a) Solve the initial value problem $y(0) = -4/9$, $y'(0) = 1/3$.

(b) Does the differential equation admit any bounded solutions? Justify.

Point values: **1**. 2,0 **2**. 2,0 **3**. (a) 2,0 (b) 2,0 (c) 1,5 **4**. 1,5 **5**. (a) 2,0 (b) 2,0 **6**. 2,0
7. (a) 2,0 (b) 1,0