Análise Matemática III

LISTA 2

(1) Esboce detalhadamente os seguintes conjuntos:
 (a) $S = \{(x, y) \in \mathbb{R}^2 \colon 0 < x < 1,\ x^2 < y < 2x^2\}$.
 (b) $S = \{(x, y) \in \mathbb{R}^2 \colon -1 < y < 1,\ -\sqrt{1 - y^2} < x < 2y^2 - 1\}$.
 (c) $S = \{(x, y, z) \in \mathbb{R}^3 \colon 0 < x < 1,\ x^2/2 < y < x^2,\ 0 < z < x^2\}$.
 (d) $S = \{(x, y, z) \in \mathbb{R}^3 \colon 0 < x < 1,\ 0 < y < 1 - x,\ 0 < z < x + y\}$.
 (e) $S = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2/4 < 1\}$.
 (f) $S = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 < z^2 + 1\}$.

(2) Considere as variedades seguintes e determine as suas dimensões e espaços tangente e normal no ponto p:
 (a) $M = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 = 1,\ z = x^2 - y^2\}$, $p = (1, 0, 1)$
 (b) $M = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 = z^2 + 1,\ 0 < z < 2\}$, $p = (0, \sqrt{2}, 1)$

(3) Determine os extremos de f em M:
 (a) $f(x, y) = x$, $M = \{(x, y) \in \mathbb{R}^2 \colon x^2 + 2y^2 = 3\}$.
 (b) $f(x, y) = x^2 + y^2$, $M = \{(x, 2) \in \mathbb{R}^2 \colon x \in \mathbb{R}\}$.
 (c) $f(x, y) = x^2 - y^2$, $M = \{(x, \cos x) \in \mathbb{R}^2 \colon x \in \mathbb{R}\}$.
 (d) $f(x, y, z) = x$, $M = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 = 2,\ x + z = 1\}$.

(4) Determine e classifique os extremos de φ em M:
 (a) $\varphi(x, y) = x^2 - y^2$, $M = \{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 = 1\}$
 (b) $\varphi(x, y) = xy$, $M = \{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 = 2a^2\}$
 (c) $\varphi(x, y) = x^{-1} + y^{-1}$, $M = \{(x, y) \in \mathbb{R}^2 \colon x^{-2} + y^{-2} = a^{-2}\}$
 (d) $\varphi(x, y, z) = x + y + z$, $M = \{(x, y, z) \in \mathbb{R}^3 \colon (1/x) + (1/y) + (1/z) = 1\}$
 (e) $\varphi(x, y, z) = x^2 + 2y - z^2$, $M = \{(x, y, z) \in \mathbb{R}^3 \colon 2x - y = 0, x + z = 6\}$
 (f) $\varphi(x, y, z) = x + y$, $M = \{(x, y, z) \in \mathbb{R}^3 \colon xy = 16\}$
 (g) $\varphi(x, y, z) = xyz$, $M = \{(x, y, z) \in \mathbb{R}^3 \colon x + y + z = 5, xy + xz + yz = 8\}$

(5) *Calcule o espaço tangente e o espaço normal num ponto do gráfico de uma função $f \colon W \to \mathbb{R}^{n-m}$, C^1, com $W \subset \mathbb{R}^m$ aberto.